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An Example of Creation of Matter in a Gravitational Field

R. I. KHRAPKO

Moscow Aviation Institute

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An example is constructed for the creation of matter in the form of a compact body, and of the subsequent destruction of the matter in an external nonstationary gravitational field, within the framework of classical general relativity without using ideas of quantum theory.

PARTICLE production in a gravitational field, i.e., the transformation of the energy of a gravitational field into usual mass-energy described by an energy-momentum tensor, was discussed in several papers,[1-4] which made use of field quantization in a Riemannian space. On the other hand, the change of mass of an isolated body due to emission (reception) of gravitational waves, in the linear approximation and under the condition of Euclidean metric at infinity, has been computed, e.g., in [5,6] within the framework of classical general relativity theory.

It is interesting to demonstrate that this theory allows, without resort to ideas from quantum theory, for the possibility of creation of matter in a nonstationary gravitational field. In other words, it seems desirable to construct an example of a material system, for which the world tube starts on some space-like hypersurface and extends into the future. It is understood that from the first instant, i.e., starting from this hypersurface the system should be subject to (mechanical) stresses, necessary for absorbing energy from the gravitational field. Thus, the condition of energy dominance will be violated in some region, in agreement with Hawking's theorem, which asserts the impossibility of matter production if this condition is satisfied.[7] If at a later time the sign of these stresses is changed into the opposite one, the system begins to lose mass, and its world tube may end. The present paper is devoted to the construction of such a system.

Consider the "interior of the singular Schwarzschild ball," i.e., the region of space-time admitting a coordinate system with the metric

$$ds^2 = \frac{tdt^2}{1-t} - \frac{1-t}{t}dy^2 - t^2(d\theta^2 + \sin^2\theta d\varphi^2) \qquad (1)$$

with $0 < t < 1$, and inside it a region situated between the space-like hypersurfaces

$$t = 1 + \frac{t_1 - 1}{\cos\theta}, \quad t = 1 + \frac{t_2 - 1}{\cos\theta} \quad (t_2 > t_1), \qquad (2)$$

bounded in the angle coordinate $\theta < \theta_m$, and arbitrarily bounded in the y-coordinate (cf. Eqs. (4) and (6) in [8]). We fill this region with a substance having the energy-momentum tensor:[1)]

$$T^{ij} = \varepsilon u_0^i u_0^j + \sigma_2 u_2^i u_2^j + \sigma_3 u_3^i u_3^j. \qquad (3)$$

where ϵ, σ_2, and σ_3 are functions of t and θ, u_0^i is a timelike unit vector ($u_0^i u_0^j g_{ij} = 1$):

$$u_0^i = \{\sqrt{(1-t)/t}, 0, 0, 0\}, \qquad (4)$$

u_2^i and u_3^i are spacelike unit vectors ($u_2^i u_2^j g_{ij} = u_3^i u_3^j g_{ij} = -1$) tangent to hypersurfaces of the form $t = 1 + (t_0 - 1)/\cos\theta$:

$$u_2^i = \left\{\frac{(t-1)\sin\theta}{\sqrt{t(t-\sin^2\theta)}}, 0, \frac{\cos\theta}{\sqrt{t(t-\sin^2\theta)}}, 0\right\}, \qquad (5)$$

$$u_3^i = \{0, 0, 0, 1/t\sin\theta\}. \qquad (6)$$

We note that we should have $\cos^2\theta_m > 1 - t_1$, otherwise the hypersurface (2) will not be everywhere spacelike.

It is essential to assume that the mass density and the pressures σ_2 and σ_3 are small, so that one can neglect the distortions of the metric field (1) (in the same manner one always neglects the distortion of the usual Schwarzschild field in calculating the motion of dust or of planets in it).

Necessary and sufficient requirements on the tensor (3) are: the vanishing of the covariant derivative with respect to the metric (1)[2)]

[1)] i,j=0,1,2,3 and the coordinates are labeled as follows: $x^0=t$, $x^1=y$, $x^2=\theta$, $x^3=y$.

[2)] The Christoffel symbols for this metric are obtained from those for the Schwarzschild metric by means of the substitutions $t \to y$ and $r \to t$.

$$\nabla_i T^{ij} = 0 \tag{7}$$

and the boundary condition

$$T^{ij} f_j = 0, \tag{8}$$

where f_i is a covector corresponding to the boundary hypersurfaces. Of the four equations (7), two are identically satisfied owing to homogeneity in y and φ, and, after elimination of σ_3, the other two lead to the relation

$$\frac{\partial \varepsilon}{\partial t} + \frac{(4t-3)\varepsilon}{2t(t-1)} = \frac{3\sigma_2 \sin^2\theta}{2t(t-1)}, \tag{9}$$

which expresses σ_2 in terms of ϵ.

If one defines $\epsilon(t, \theta)$ in the form of the product $\epsilon = a(\theta) b(1 + (t-1)\cos\theta)$, where $a \geq 0$, $b \geq 0$ are smooth functions of the variables indicated, with $a(\theta_m) = 0$ and $b(t_1) = b(t_2) = 0$, then the conditions (8) are satisfied for the hypersurfaces (2) owing to the fact that u_2^i and u_3^i are tangent to them and $\epsilon = 0$ on these hypersurfaces, as well as for the hypersurface $\theta = \theta_m$, owing to the fact that $\epsilon(\theta_m) = \sigma_2(\theta_m) = 0$, and u_3^i is tangent to it. This proves the correctness of the construction.

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