Interaction between a Large-amplitude Direct Magnetosonic Wave of Frequency $\Omega < \sqrt{\omega_{He}\omega_{Hi}}$ and a Plasma

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The problem is discussed of the interaction between a plasma and a direct magnetosonic wave of sufficiently large amplitude with frequency $\omega_{Hi} \ll \Omega \ll \omega_{He}$ and $\Omega \sim \omega_{Hi}$. It is shows that such a wave can parametrically excite in the plasma short-wave potential oscillations with frequency $\omega \sim \Omega$; in this case the threshold buildup velocity u (u is the velocity of the electrons in the field of the magnetosonic wave) can be much smaller than the velocity of sound. Interaction of the short-wave potential oscillations with the plasma leads to efficient heating of the plasma as the result of resonance absorption of the waves by particles; the Q of such a system turns out to be very low and does not exceed about 10 because of the large damping of the potential waves.

1. INTRODUCTION

T the present time a large number of papers have been published on investigation of parametric excitation of various types of plasma oscillations by external high-frequency electromagnetic fields of rather large amplitude. Silin and co-workers[1, 2] in a series of papers have studied excitation of ion-acoustic and plasma oscillations by an external high-frequency electric field with frequency $\Omega \gtrsim \omega_{pe}$ (ω_{pe} is the plasma electronic frequency).

Zyunder and Gradov[3] have discussed the parametric buildup of ion-acoustic oscillations under the influence of an external electric field with frequency $\Omega \sim \omega_{He}$ ($\omega_{He} = eH_0/mc$ is the electron cyclotron frequency). In all of these studies the frequency of the external electric field was assumed to be rather high, namely: $\Omega \gtrsim \omega_{pe}, \omega_{He}$. On the other hand, interest has recently increased substantially in the problem of plasma heating by an external magnetosonic wave of large amplitude with frequency $\Omega \ll \omega_{He}$. Thus, Kovan and Spektor[4] have considered the question of interaction with a plasma of slow magnetosonic wave of large amplitude with frequency $\Omega \gg \omega_{Hi}$. Stepanov and co-workers[5-7] have studied both theoretically and experimentally the interaction with a plasma of a fast magnetosonic wave (waves of the atmospheric whistler type) of large amplitude with frequency $\omega_{Hi} \ll \Omega \ll \omega_{He}$. It turns out that for a sufficiently large amplitude (electron velocity in the electric field of the wave $u \gtrsim v_{Ti}$) such a wave can excite efficiently small-scale potential oscillations with frequency $\Omega \ll \omega \ll \omega_{He}$ which interact with the plasma, and thus give up its energy to heating of the plasma. Recently papers have appeared (see, for example, Vdovin et al.[8]) in which it is shown experimentally that the Q of a plasma interacting with a magnetosonic wave depends weakly on the amplitude of the wave and remains rather low even when the inequality $u \ll v_{Ti}$ is satisfied. It therefore seems interesting to study the possibility of parametric excitation of low-frequency oscillations in a plasma by external magnetosonic waves, since such instabilities, as a rule, have a rather low threshold.

In the present work we investigate the possibility of parametric excitation of short-wave potential waves in a plasma by an external magnetosonic wave of sufficiently large amplitude propagating transverse to the external magnetic field H_0 with a frequency $\Omega < \sqrt{\omega_{Hi}\omega_{He}}$. It is known[9] that in such a wave when the condition $\Omega > \omega_{Hi}$ is satisfied the electric field is directed mainly along the direction of propagation of the wave; the frequency itself is related to the magnitude of the wave vector by the following dispersion relation:

$$\Omega^2 = \omega_{Hi}\omega_{He}\frac{k^2}{k^2+\omega_{pe}^2/c^2}. \tag{1}$$

In derivation of Eq. (1) it was assumed that the wave is propagated strictly transverse to the external magnetic field H_0, i.e., $k_z \equiv 0$. In what follows we will assume that the magnitude of the wave vector k in the magnetosonic wave is much smaller than the magnitudes of the wave vectors of the excited oscillations.

It is not difficult to show that in this case in derivation of the dispersion laws for the excited oscillations we can assume that in the plasma there exists only the external electric field $E_x = E_0 \cos \Omega t$ (in what follows we will assume that the magnetosonic wave is propagated along the x axis) which does not depend on x. Below we will discuss two cases separately: $\Omega \gg \omega_{Hi}$ and $\Omega \sim \omega_{Hi}$, since both the decay threshold and the types of oscillations excited are substantially different for the cases $\Omega \gg \omega_{Hi}$ and $\Omega \sim \omega_{Hi}$.

1. INTERACTION WITH A PLASMA OF A MAGNETOSONIC WAVE WITH FREQUENCY $\omega_{Hi} \ll \Omega < \sqrt{\omega_{Hi}\omega_{He}}$

a. Derivation of the dispersion laws. In a plasma located in a magnetic field H_0 directed along the z axis, let a direct magnetosonic wave of sufficiently large amplitude be propagated with frequency

$$\omega_{Hi} \ll \Omega < \sqrt{\omega_{Hi}\omega_{He}}.$$

The plasma is assumed to be weakly inhomogeneous along the x axis.

In solution of this problem we used the collisionless kinetic equations for the particle distribution functions

$$\frac{\partial f_\alpha}{\partial t}+\mathbf{v}_\alpha\frac{\partial f_\alpha}{\partial \mathbf{r}}+\frac{e_\alpha}{m_\alpha}\left(\mathbf{E}+\frac{1}{c}[\mathbf{v}_\alpha\mathbf{H}]\right)\frac{\partial f_\alpha}{\partial \mathbf{v}_\alpha}=0 \qquad (2)^*$$

and the Poisson equation

$$\Delta\varphi=4\pi\sum_\alpha e_\alpha\int f_\alpha d\mathbf{v}_\alpha \qquad (3)$$

The index α designates electrons and ions.

Since the frequencies of the oscillations considered are much less than the electron cyclotron frequency ω_{He}, in order to find the corrections to the unperturbed electron distribution function we can use the drift approximation:

$$\frac{df_e^{(1)}}{dt}=i\frac{e\varphi^{(1)}}{T_e}(k_zv_z-\omega_*)f_{0e}, \qquad (4)$$

where $\omega_* = k_y\kappa cT_e/eH_0$; $1/\kappa$ is the characteristic size of the inhomogeneity; in derivation of Eq. (4) the function f_{0e} was assumed Maxwellian.

In what follows we will assume the inequalities $k_z \ll k_y$, $\Omega \ll k_z v_{Te}$, and $k_\perp v_{Te} \ll \omega_{He}$ to be satisfied. Expanding the correction to the equilibrium electron distribution function $f_e^{(1)}$ and the intrinsic potential of the wave in a Floquet series in harmonics of the frequency Ω of the external electric field, we obtain

$$f_e^{(1)}=e^{-i\omega t}\sum_{p=-\infty}^{\infty}f_p e^{ip\Omega t},\qquad \varphi^{(1)}=e^{-i\omega t}\sum_{p=-\infty}^{\infty}\varphi_p e^{ip\Omega t}. \qquad (5)$$

From the equation of motion in the drift approximation we find that under the action of an external electric field $E_x = E_0 \cos\Omega t$ the electrons move along the y axis with a velocity $v_y = u\cos\Omega t = cE_0H_0^{-1}\cos\Omega t$. Substituting the expansion (5) into Eq. (4), we obtain

$$f_e^{(1)}(v_z)=\sum_q i\frac{en_0\varphi_q}{mv_{Te}^3}\int_{-\infty}^{t}(k_zv_z-\omega_*)\exp\Big[i\frac{ku}{\Omega}(\sin\Omega\tau-\sin\Omega t) \qquad (6)$$

$$+iq\Omega\tau-i\omega\tau+ik_zv_z\tau-v_z^2/v_{Te}^2\Big]d\tau,$$

$$f_e^{(1)}(v_z)=\int f_e^{(1)}d\mathbf{v}_\perp.$$

To find the correction to the equilibrium electron density we will integrate Eq. (6) over v_z:

$$n^{(e)}=\sum_q i\frac{en_0\varphi_q}{mv_{Te}^3}\int_{-\infty}^{\infty}dv_z\int_{-\infty}^{t}(k_zv_z-\omega_*)\exp\Big[i\frac{ku}{\Omega}(\sin\Omega\tau-\sin\Omega t)$$

$$+iq\Omega\tau-i\omega\tau+ik_zv_z\tau-v_z^2/v_{Te}^2\Big]d\tau. \qquad (7)$$

Integration over v_z is carried out by elementary means and after substitution of $\tau = t - \tau'$ and expansion of $\sin\Omega(t-\tau')$ in the vicinity of the point $\tau' = 0$ ($k_zv_{Te} \gg \Omega$), we obtain

$$n^{(e)}=i\sqrt{\pi}\frac{en_0k_z}{mv_{Te}^3}\sum_q\int_0^\infty d\tau\left(i\frac{k_zv_{Te}}{2}\tau-\frac{\omega_*}{k_zv_{Te}}\right) \qquad (8)$$

$$\times\exp\left[\frac{k_z^2v_{Te}^2}{4}\tau^2+iku\tau\cos\Omega t-i\omega(t-\tau)+iq\Omega(t-\tau)\right]\varphi_q.$$

Integration of Eq. (8) over τ is easily performed by substitution of the variable

$$\tau_1=\frac{k_zv_{Te}}{2}\tau+i\frac{ku\cos\Omega t+\omega-q\Omega}{k_zv_{Te}}$$

and subsequent separate integration over the real and imaginary regions. Substituting also in the left-hand portion of Eq. (8) instead of $n^{(e)}$ its expansion in a Floquet series from (5), we obtain finally

$$n_q^{(e)}=\frac{en_0}{T_e}\varphi_q\left(1+i\sqrt{\pi}\frac{\omega-p\Omega-\omega_*}{k_zv_{Te}}\right)$$
$$-i\frac{\sqrt{\pi}}{2}\frac{en_0}{T_e}\frac{ku}{k_zv_{Te}}(\varphi_{q+1}+\varphi_{q-1}). \qquad (9)$$

The correction to the unperturbed ion density can be found similarly; it is simpler, however, to proceed as follows. Since $\Omega > \omega_{Hi}$, we can assume that the electric field of the magnetosonic wave does not act on the ions ($u_i \equiv 0$) and since, furthermore, the inequality $T_e > T_i$ is assumed satisfied, we can neglect the ionic Larmor current and in obtaining the expression for $n_q^{(i)}$ assume the plasma is homogeneous. Then integration of the kinetic equation for the ions gives

$$n_q^{(i)}=-\frac{en_0}{T_i}\varphi_q\left[1+\sum_{n=-\infty}^{\infty}i\sqrt{\pi}\frac{\omega-q\Omega}{k_zv_{Ti}}Y_n\left(\frac{\omega-n\omega_{Hi}-q\Omega}{k_zv_{Ti}}\right)\right.$$
$$\left.\times\Gamma_n\left(\frac{k^2v_{Ti}^2}{\omega_{Hi}^2}\right)\right], \qquad (10)$$

where

$$Y_n(x)=2e^{-x^2}\int_0^x e^{t^2}dt-i\sqrt{\pi}e^{-x^2},\qquad \Gamma_n(x)=e^{-x}I_n(x),$$

I_n is a Bessel function of imaginary argument of order n. Substituting the expressions for $n_q^{(e)}$ and $n_q^{(i)}$ respectively from (9) and (10) into Eq. (3), we obtain an infinite homogeneous system of algebraic equations for φ_p. Since, as we have already said above, we are discussing oscillations with $\omega \sim \Omega \gg \omega_{Hi}$, the frequency $\omega - \Omega$ can be close to one of the harmonics of the ion cyclotron frequency ω_{Hi} (just this case will be discussed below), and therefore the expressions for $n_p^{(i)}$ will be substantially different for the cases p = 1 and p ≠ 1, and the system of equations can conveniently be written separately for each of these cases:

$$\varphi_p=-\frac{\omega_{pi}^2}{k^2c_s^2}\varphi_p\left(1+i\sqrt{\pi}\frac{\omega-p\Omega-\omega_*}{k_zv_{Te}}\right)+i\frac{\sqrt{\pi}}{2}\frac{\omega_{pi}^2}{k^2c_s^2}$$
$$\times\frac{ku}{k_zv_{Te}}(\varphi_{p+1}+\varphi_{p-1})+\frac{\omega_{pi}^2}{(\omega-p\Omega)^2}\varphi_p \quad \text{for } p\neq 1, \qquad (11)$$

$$\varphi_1=\frac{\omega_{pi}^2}{k^2c_s^2}\varphi_1\left(1+i\sqrt{\pi}\frac{\omega-\Omega-\omega_*}{k_zv_{Te}}\right)-i\frac{\sqrt{\pi}}{2}\frac{\omega_{pi}^2}{k^2c_s^2}$$
$$\times\frac{ku}{k_zv_{Te}}(\varphi_2+\varphi_0)+\frac{\omega_{pi}^2}{k^2v_{Ti}^2}\left(1-\frac{\omega-\Omega}{\omega-\Omega-n\omega_{Hi}}\Gamma_n\right)\varphi_1 \quad \text{for } p=1. \qquad (12)$$

In derivation of Eq. (12) we have assumed that the inequality $|\omega-\Omega-n\omega_{Hi}| > k_zv_{Ti}$ is satisfied. (Note that in the second equation (11) the harmonic number n is fixed; it is chosen from a condition which will be obtained below.) The dispersion relation for the oscillations considered can be obtained by equating to zero the determinant of the system (11). It is simpler, however, to use again the smallness of Ω, $ku \ll k_zv_{Te}$ and to expand all quantities in a series in the parameter Ω/k_zv_{Te}:

$$\varphi_p=\overset{(0)}{\varphi_p}+\frac{\Omega}{k_zv_{Te}}\overset{(1)}{\varphi_p}+\ldots,\qquad \omega=\omega^{(0)}+i\gamma+\ldots \qquad (13)$$

(as will be shown below, $\gamma \sim \Omega/k_zv_{Te}$). Here it turns out that in the expansion (13) it is sufficient to limit ourselves to terms of first order. Since we are interested in oscillations with $\omega \sim \Omega$, in the zeroth approximation

*$[\mathbf{v}_\alpha\mathbf{H}] \equiv \mathbf{v}_\alpha \times \mathbf{H}$.

only the potentials with $p = 0, +1$ will be different from zero, and from (11) and (12) we obtain

$$\omega^{(0)} = \omega_{pi}(1 + \omega_{pi}^2 / k^2 c_s^2)^{-1/2}, \qquad (14)$$

$$\omega^{(0)} - \Omega = n\omega_{Hi}(1 + \Gamma_n) \qquad (15)$$

(in derivation of (15) it is important that the inequality $\Gamma_n \ll 1$ be satisfied).

From Eqs. (11) and (12), written down for $p = 0, +1$, we find in the first approximation

$$\left(\frac{2\gamma}{\omega^{(0)}} + \sqrt{\pi}\frac{\omega^{(0)} - \omega_*}{k_z v_{Te}}\right)\left(\frac{2\gamma T_e}{T_i}\frac{n\omega_{Hi}\Gamma_n}{(\omega^{(0)} - \Omega - n\omega_{Hi})^2} + \sqrt{\pi}\frac{n\omega_{Hi} - \omega_*}{k_z v_{Te}}\right) = \frac{\pi}{4}\frac{k^2 u^2}{k_z^2 v_{Te}^2}. \qquad (16)$$

From Eq. (16) it is easy to find the threshold buildup rate, when $\Gamma = 0$:

$$k^2 u_{\text{th}}^2 = 4(\omega^{(0)} - \omega_*)(n\omega_{Hi} - \omega_*). \qquad (17)$$

If $n\omega_{Hi} \gg \omega_*$, then it follows from (17) that

$$k^2 u_{\text{th}}^2 = 4\omega_{pi} n\omega_{Hi}(1 + \omega_{pi}^2 / k^2 c_s^2)^{-1/2}. \qquad (18)$$

We will consider first rather long-wave oscillations with $k \ll \omega_{pi}/c_s$, where $\omega^{(0)} = kc_s$. We will investigate first of all on what quantities the harmonic number n depends. Previously fulfillment was assumed of the condition

$$|\omega^{(0)} - \Omega - n\omega_{Hi}| \gg k_z v_{Ti}. \qquad (19)$$

It follows from (15) that in order that $\omega^{(0)} - \Omega$ be close to one of the cyclotron harmonics, it is necessary that the inequality $kv_{Ti} > \omega_{Hi}$ be satisfied. These two conditions with use of Eq. (18) give

$$n \gg \frac{\Omega^2}{\omega_{Hi}\sqrt{\omega_{Hi}\omega_{He}}}\frac{T_i}{T_e}. \qquad (20)$$

Then for the buildup threshold instead of (18) we can write

$$u_{\text{th}}^2 = 4c_s^2 \frac{\Omega}{\sqrt{\omega_{Hi}\omega_{He}}}\frac{T_i}{T_e}. \qquad (21)$$

From Eq. (21) it follows that the buildup threshold can be much smaller than the velocity of sound. Investigation of the case $k > \omega_{pi}/c_s$ is carried out in just the same way; it turns out that the buildup threshold in this case is still smaller than that calculated from Eq. (21), and for $k_{max} \sim \omega_{pi}/v_{Ti}$ (for $k > k_{max}$ the oscillations begin to be strongly absorbed by ions and the buildup threshold rises rapidly) it turns out to be

$$u_{\text{th}}^2 = 4v_{Ti}^2 \omega_{pi} / \sqrt{\omega_{Hi}\omega_{He}}.$$

However, in order that this case be realized it is necessary that the frequency of the magnetosonic wave be close to ω_{pi}, and in hydrogen plasma this is possible only if the inequality $\omega_{pi} < \sqrt{\omega_{Hi}\omega_{He}}$ is satisfied.

As can be seen from Eq. (17), the buildup threshold is strongly influenced by the inhomogeneity of the plasma: when the condition $\omega_* = n\omega_{Hi}$ is satisfied the buildup has no threshold, and if $\omega_* > n\omega_{Hi}$, then Eq. (20) loses its formal meaning. This is due to the fact that when the inequality $\omega_* > n\omega_{Hi}$ is satisfied the drift-cyclotron oscillations become unstable even in the absence of an external magnetosonic wave. The quasilinear relaxation of these oscillations leads in the last analysis to suppression of the instability[10] (this corresponds to the fact that even for $\omega_* > n\omega_{Hi}$ in the second parentheses of the left-hand portion of equality (16) the difference $n\omega_{Hi} - \omega_*$ must be equated to zero) and, thus, we can say that on fulfillment of the condition

$$\omega_* \geq n\omega_{Hi} \quad \left(\varkappa v_{Ti} \geq \left(\frac{T_i}{T_e}\right)^2 \frac{\Omega^2}{\sqrt{\omega_{Hi}\omega_{He}}}\right)$$

the buildup of the oscillations discussed has no threshold.

b. Inclusion of nonlinear effects. If the electron velocity in the field of the magnetosonic wave exceeds the threshold value determined by Eq. (25), ion-acoustic and ion-cyclotron oscillations will build up in the plasma with frequencies determined respectively by Eqs. (14) and (15). When the level of the oscillation energy becomes sufficiently large, nonlinear effects leading to limitation of the noise amplitude begin to play an important role. It is known that both for ion-acoustic[11] and ion-cyclotron oscillations[12] the principal effects leading to suppression of the instability is nonlinear scattering of waves by the ions. However, in the case being considered here of a parametric buildup of the oscillations, there is an additional nonlinear mechanism which suppresses instability. It turns out that the parametric instability being discussed is no different from decay in the plasma of an external monochromatic wave of large amplitude. Furthermore, as is well known,[13] this decay can be considered as an instability only in the initial stage of the process when the level of noise energy produced as the result of decay of the waves is small in comparison with the level of energy of the magnetosonic wave. When the energy densities of the unstable oscillations are comparable with the magnetosonic energy density, the inverse pumping process begins to play an important role.

It is well known[13] that oscillations in a finite frequency interval are produced as the result of decay of a monochromatic wave. For a sufficiently large level of energy in the packets (the value of which will be estimated below) the waves in them can be considered stochastic, so that the inverse pumping process will lead in the final analysis to establishment in the system of some quasistationary state in which the decrease in the number of quanta of ion-acoustic and ion-cyclotron waves as the result of their strong absorption by electrons and ions (the nonlinear effect!) is compensated by the increase of the number of quanta of these waves resulting from the decay of the magnetosonic wave (here it is assumed, of course, that the magnetosonic wave energy level does not change as the result of operation of the external source).

We will show now that in the case considered by us, nonlinear scattering of oscillations by ions is practically always small and stabilization of the instability can occur only as the result of the effects of inverse pumping of the waves.

An expression for the nonlinear damping decrement of ion-acoustic waves in ions has been obtained by Petviashvili[11]: in order of magnitude it is

$$\gamma_{n\text{-}l}^{*} \approx \omega_* \frac{T_i}{T_e}\frac{k}{\Delta k}\frac{W_*}{nT_e}, \qquad (22)$$

where W_s is the energy density of ion-acoustic noise.

The process of absorption of ion-cyclotron oscillations in nonlinear scattering by ions has been discussed by Petviashvili and by Karpman.[12] However, those papers discuss oscillations with $k_\perp v_{Ti} \sim \omega_{Hi}$, and in our case $k_\perp v_{Ti} \gg \omega_{Hi}$. Using the results of Petviashvili,[11] it is easy to show that when the strong inequality $k_\perp v_{Ti} \gg \omega_{Hi}$ is satisfied the nonlinear damping decdecrement is exponentially small ($\lambda_{n-1}^{c} \sim \exp[-k^2 v_{Ti}^2/\omega_{Hi}^2]$) and can be neglected in comparison with the linear damping decrement. When all that has been said above is taken into account it is easy to write the buildup condition, similar to Eq. (16):

$$\left(\frac{2\gamma}{\omega_s}+\sqrt{\pi}\frac{\omega_s}{k_z v_{Te}}+\sqrt{\pi}\frac{T_i}{T_e}\frac{k}{\Delta k}\frac{W_s}{nT}\right)\left(2\gamma\frac{T_e}{T_i}\frac{n\omega_{Hi}\Gamma_n}{(\omega-\Omega-n\omega_{Hi})^2}+\sqrt{\pi}\frac{n\omega_{Hi}-\omega_*}{k_z v_{Te}}\right)=\frac{\pi}{4}\frac{k^2u^2}{k_z^2 v_{Te}^2}.$$

Since $\omega_s/k_z v_{Te} \lesssim 1$ and $T_i/T_e < 1$, it follows from (22) that even for $kW_s/\Delta k nT \lesssim 1$ the nonlinear damping decrement of ion-acoustic waves is smaller than the linear decrement. (Usually in experiments on plasma heating by a magnetosonic wave $W_s/nT \ll 1$, and $k/\Delta k \lesssim 10$.) It is evident that if the electron velocity in the field of a magnetosonic wave differs by any appreciable amount from its threshold value determined by Eq. (21), then the nonlinear effects of scattering of waves by ions cannot suppress the instability of the oscillations discussed. It is clear that, in the case discussed, suppression of the instability can occur only as the result of inverse pumping of waves over the spectrum.

We will now show that the packets of ion-acoustic and ion-cyclotron waves formed as the result of decay can be discussed stochastically. According to Zaslavskiĭ[14] the condition of stochasticity for the decay processes has the following form:

$$\frac{\partial\Delta\omega_k}{\partial N_k}\Delta N_k \gg \Omega_k, \qquad (24)$$

where $\Delta\omega_k$ is the change in frequency as the result of resonance perturbation, ΔN_k is the maximal change in number of quanta produced by a resonance, and Ω_k is the characteristic distance between harmonics in the spectrum. Calculations similar to those carried out previously[15] show that the oscillations can be discussed stochastically on fulfillment of the condition

$$\frac{W_s W_c W_{\text{m.a.}}}{(nT)^3} \gg \frac{\sqrt{m/M}}{k^4 L_\perp^4} \qquad (25)$$

where $L_\perp$ is the transverse dimension of the system. Since the decay process and reverse fusion of the waves leads to a Rayleigh-Jeans distribution, we will finally have $W_s \sim W_c \sim W_{\text{m.a.}}$ and condition (25) can be rewritten in the following form:

$$\frac{W}{nT} \gg \left(\frac{m}{M}\right)^{1/6}(kL_\perp)^{-4/3} = \left(\frac{m}{M}\right)^{1/6}\left(\frac{c_s^2\, 4\pi nM}{H_0^2}\right)^{2/3} \ll 1,$$

which is satisfied in practically all experiments on plasma heating by a magnetosonic wave.

c. Plasma heating. Since wave packets formed as the result of decay can be considered stochastically, the inverse process of their fusion will lead to an effective broadening of the spectrum of the initially monochromatic magnetosonic wave and after a certain time three rather wide wave packets are formed in the plasma, which efficiently give up their energy to heating as the result of scattering of the waves by particles. Here, naturally, the energy of the packets should decrease. However, as we have already mentioned above, it is assumed that the power of the source of magnetosonic waves is sufficiently high to maintain the noise energy in the plasma at one level. When this fact is taken into account it is easy to estimate the rate of heating of the plasma particles; for this purpose it is sufficient to use the energy balance equation

$$\frac{dnT_\alpha}{dt} = -\int \gamma_k^\alpha W_k\, dk, \qquad (26)$$

where γ_k^α is the combined (linear and nonlinear) damping decrement of waves by particles of type α.

Since heating of the electron component occurs mainly as the result of the linear damping of waves by particles, and $\gamma_{k_s} \gg \gamma_{k_c}, \gamma_{k_{\text{m.a.}}}$, heating of the electrons is determined by damping of ion-acoustic noise and can be written

$$\frac{dnT_e}{dt} \approx -\int dk \frac{k_s^2 c_s^2}{k_{zs} v_{Te}} W_{k_s}. \qquad (27)$$

Heating of the ion component occurs as the result of nonlinear damping of waves by ions (the linear damping in all three packets is exponentially small). We have already shown above that the nonlinear damping of cyclotron waves by ions is also exponentially small and cannot play an appreciable role in the heating mechanism. Thus it remains for us to deduce whether or not it is necessary in the equation for ion heating to take into account the contribution from the magnetosonic wave (since as the result of fusion of the waves the packet of magnetosonic oscillations was broadened, it also can be nonlinearly scattered by ions). The value of the nonlinear damping decrement of magnetoacoustic waves by ions can be estimated in the following ways:[16]

$$\gamma_{\text{m.a.}}^{(i)} \sim \Omega\beta(kv_\sim/\Omega)^2,$$

where $\beta = nT/H_0^2$, $v_\sim \sim eE_{\text{m.s.}}/M\Omega$ is the velocity of the ion oscillations in the field of the wave. Since

$$W_{\text{m.a.}} = \frac{\omega_{pi}^2}{\Omega^2}|E_{\text{m.a.}}|^2,$$

we finally obtain

$$\gamma_{\text{m.a.}}^{(i)}/\gamma_s^{(i)} \sim \beta^2 T_e/T_i. \qquad (28)$$

The quantity on the left-hand side of Eq. (28) is, as a rule, much smaller than unity, i.e., the heating of the ion component is also determined by damping of ion-acoustic oscillations

$$\frac{dnT_i}{dt} \approx k_s c_s \frac{T_i}{T_e}\frac{W_s}{nT}. \qquad (29)$$

As has already been remarked above, $\gamma_s^{(e)} > \gamma^{(i)}$, and therefore the electron temperature should increase more rapidly than the ion temperature, which will lead to a still greater separation in the temperatures. Heating will continue until the condition of existence of short-wave ion-cyclotron waves $kv_{Ti} > \omega_{Hi}$ is destroyed; since $k \approx \Omega/c_s$, this condition reduces to the following: $T_i/T_e \gtrsim (\omega_{Hi}/\Omega)^2$. In the heating process, according to Eq. (20), the harmonic number n of the unstable ion-cyclotron waves will also decrease.

The Q of the system can be determined from the formula $Q = \Omega/2\gamma$, where γ is the decrement of the magnetosonic wave in the absence of an external source, i.e., that part of the wave energy which is dissipated per unit time in the plasma and goes into heating it. The size of this decrement is determined both by the intrinsic damping of the magnetosonic wave on resonance particles and by the amount of energy lost by it in excitation of short-wave potential oscillations and, in the last analysis, also in heating of the plasma. In order to find γ, we can make use of the law of conservation of energy, which follows from the equations for decay of waves with random phases (see, for example, Tsytovich[13]) and in the absence of an external source has the form

$$\frac{d}{dt}(W_{\text{m.a.}} + W_s + W_c) = \gamma_c W_c + \gamma_s W_s + \gamma_{\text{m.a.}} W_{\text{m.a.}}, \tag{30}$$

where γ_α is the decrement of a wave of type α in resonance particles. From this it follows that in a stationary state when $dW_s/dt = dW_c/dt = 0$ and $W_s \sim W_c \sim W_{\text{m.a.}}$,

$$\frac{dW_{\text{m.a.}}}{W_{\text{m.a.}}dt} = \gamma = \gamma_s + \gamma_c + \gamma_{\text{м.з.}}$$

(we recall that Eq. (30) was written in the absence of an external source and therefore for $dW_c/dt = dW_s/dt = 0$ $dW_{\text{m.a.}}/dt \neq 0$). Thus, the Q value is actually determined by the sum of the decrements of all types of waves, and not only by the decrement of the magnetosonic wave (which would determine the Q of the system in the absence of decay). Since $\gamma_s \gg \gamma_c, \gamma_{\text{m.a.}}$, we can finally write: $Q \approx \Omega/2\gamma_s = k_z v_{Te}/2\Omega$ and, since $kc_s \sim \Omega$, it follows from this formula that the Q does not exceed the order of 10.

2. INTERACTION WITH A PLASMA OF A MAGNETOSONIC WAVE WITH FREQUENCY $\Omega \sim \omega_{Hi}$

Interaction with a plasma of a magnetosonic wave with frequency $\Omega \sim \omega_{Hi}$ differs substantially from the case $\Omega \gg \omega_{Hi}$ discussed above. It turns out that decay of such a wave into short-wave potential oscillations (and for plasma heating just such decay is interesting) is possible only in an inhomogeneous plasma since, as will be shown below, one of the decaying waves is a drift wave with $kv_{Ti} > \omega_{Hi}$.

The approximations in which the dispersion law is obtained for unstable oscillations remain practically the same as in Sec. 1; specifically, we will assume below the following inequalities to be satisfied: $T_e > T_i$, $kv_{Ti} > \omega_{Hi}$, $k_z v_{Te} > \omega_{Hi}$. The magnitude of the wave vector of the magnetosonic wave is assumed much smaller than the wave vectors of the waves building up, one of which is an ion-cyclotron wave with $\omega_c \sim \omega_{Hi} \sim \Omega$, and the other a drift wave with frequency

$$|\omega_c - \Omega| \sim \frac{\omega_{*i}}{1-\Gamma_0}\Gamma_0 \ll \omega_{Hi}$$

and

$$|\Omega - \omega_c| > k_z v_{Ti} \quad (\omega_{*i} = \omega_* T_i / T_e).$$

The results obtained up to the present time on the interaction with a plasma of a magnetosonic wave with frequency $\Omega \sim \omega_{Hi}$[8] apply to experiments in apparatus of the Tokamak type with the following characteristic parameters: $T_e/T_i \sim 3$, $T_e \sim 50$–70 eV, $r \sim 7$ cm, $H_0 \sim 15$ kOe. For these parameters actually $\omega_* \ll \omega_{Hi}$ ($\omega_* \sim 10^{-2}\,\omega_{Hi}$) and the approximation chosen by us is valid.

An expression for perturbation of the electron and ion densities can be obtained by a means completely similar to the derivation of Eqs. (9) and (10), except that now the Larmor current must be taken into account also in the ion component, so that the perturbation of the ion density $n_p^{(i)}$ can be written in the following form:

$$n_p^{(i)} = \frac{en_0}{T_i}\varphi_p\left[1 + \sum_n \frac{\omega - p\Omega - \omega_{*i}}{k_z v_{Ti}} \times Y_n\left(\frac{\omega - n\omega_{Hi} - p\Omega}{k_z v_{Ti}}\right)\Gamma_n\left(\frac{k^2 v_{Ti}^2}{\omega_{Hi}^2}\right)\right]. \tag{31}$$

Substituting Eqs. (31) and (9) into the Poisson equation (2), we obtain

$$\varphi_p\left(1 - \frac{\omega - p\Omega - \omega_{*i}}{\omega - p\Omega}\Gamma_0 - \frac{\omega - p\Omega - \omega_{*i}}{\omega - \omega_{Hi} - p\Omega}\Gamma_1\right) + \frac{T_i}{T_e}\varphi_p\left(1 + i\sqrt{\pi}\frac{\omega - p\Omega - \omega_*}{k_z v_{Te}}\right) - i\frac{\sqrt{\pi}}{2}(\varphi_{p+1} + \varphi_{p-1})\frac{ku}{k_z v_{Te}}\frac{T_i}{T_e}. \tag{32}$$

Equation (32) is written down on the assumption of quasineutrality of the oscillations, $n_p^{(i)} = n_p^{(e)}$, and in addition there remain in the ion component of the density perturbation only the main terms of the expansion in harmonics of the cyclotron frequency ω_{Hi}. Investigation of Eq. (32) is carried out exactly in the same way as in the case $\Omega \gg \omega_{Hi}$, by expansion of the intrinsic potentials φ_p in the small parameter $\Omega/k_z v_{Te} < 1$. In the zeroth approximation we obtain

$$\omega = \omega_{Hi}\left(1 + \frac{\Gamma_1}{1 + T_e/T_i - \Gamma_0}\right), \quad \omega - \Omega = -\frac{\omega_{*i}\Gamma_0}{1 + T_i/T_e - \Gamma_0} \tag{33}$$

In the first approximation it follows from the system (32) that oscillations with frequencies determined by Eqs. (33) become unstable on fulfillment of the condition

$$u^2 = 4v_{Ti}^2\frac{\omega_*}{\omega_{Hi}}\frac{\omega_{Hi}^2}{k^2 v_{Ti}^2}. \tag{34}$$

Since in the approximation considered by us $\omega_* \ll \omega_{Hi}$ and $kv_{Ti} > \omega_{Hi}$, it follows from (34) that the threshold buildup rate can be much lower than the thermal velocity of the electrons and for the experimental data given above amounts to $u \sim 10^{-1}\,v_{Ti}$, which corresponds to an intensity of the variable magnetic field of the magnetosonic wave $\tilde{H} \sim 10$ Oe. The further discussions on establishment in the system of a quasistationary state as the result of the effects of broadening of the packets and their reverse fusion, given in the first part of the article, remain valid also for the case being discussed in the limit $W_{\text{m.s.}}/nT \ll 1$ (which corresponds to the experimental conditions of Vdovin et al.[8]). The Q of the system, evaluated from the formula $Q = \Omega/2\gamma$, has a value $Q \leq 10$ and is in good agreement with the experimental value obtained by Vdovin et al.[8]

CONCLUSIONS

We have discussed the interaction with a plasma of an external magnetosonic wave of large amplitude with frequency $\omega_{Hi} \ll \Omega \ll \omega_{He}$ and $\Omega \sim \omega_{Hi}$. It is shown that when the condition $\omega_{Hi} \ll \Omega \ll \omega_{He}$ is satisfied the magnetosonic wave can, as the result of a decay process, excite in a plasma with $T_i < T_e$ ion-acoustic oscillations (with frequency $\omega_s \sim \Omega$) and ion-cyclotron oscillations (with frequency $\omega_c \ll \Omega$), in which the threshold buildup rate can be much less than the velocity of sound, and when the condition $\omega_* > \omega_c$ is satisfied the buildup has no threshold. It turns out that suppression of the instability occurs not as the result of nonlinear scattering of waves by ions, but as the result of the effective inverse pumping of waves over the spectrum. Since wave packets produced as the result of decay can be considered stochastically, the inverse pumping process leads to the fact that a quasistationary state is formed in the system, for which the decrease in energy of the wave packets as a result of scattering of waves by plasma particles is compensated by the influx of energy from an external source of magnetosonic waves of sufficient power. Here the electron component of the plasma is heated more rapidly than the ion component, which should lead to still greater separation in the temperatures. It is shown that the Q of such a system is determined by the value of the decrement of a potential wave with frequency $\omega \sim \Omega$ and does not exceed a value of the order of 10.

In the second part of the article we have discussed the interaction with a plasma of magnetosonic waves with frequency $\Omega \sim \omega_{Hi}$. In this case decay of such a wave is possible into two short-wave potential oscillations: ion-cyclotron oscillations with frequency $\omega_c \sim \omega_{Hi} \sim \Omega$ and drift oscillations with frequency $\omega_{dr} \ll \omega_{Hi}$, where on fulfillment of the inequality $\omega_* \ll \omega_{Hi}$ (and just this case is realized in the presently known experimental apparatus for heating of a plasma by a wave with frequency $\Omega \sim \omega_{Hi}$[8]) the threshold buildup rate is much smaller than the thermal velocity of the ions. The Q of such a system also does not exceed a value of the order of 10.

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