Investigation of Plasma Fluctuations in a Stellarator

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Oscillations of the density and electric field in a plasma in the L-1 stellarator have been investigated. A correlation analysis method has been used to determine the longitudinal and transverse wavelengths and frequencies of the oscillations. A relation has been established between the mode of oscillation and the structure and magnitude of the magnetic field intensity. Stabilization of the oscillations by ionic collisions has been observed. It is shown that the observed fluctuations have the nature of drift-dissipative waves excited as the result of a Cerenkov mechanism and of electron collisions. Other mechanisms for pumping energy into the oscillations are discussed.

1. INTRODUCTION

STUDY of the instabilities of plasma in magnetic traps presents substantial interest for a program of controlled thermonuclear fusion, since they can substantially affect transport processes. At the present time many experimental studies have been published (see, for example, Refs. 1–3) in which the relation of the observed fluctuations and the plasma lifetime has been investigated. Summarizing the results of these studies, we can conclude that in different apparatus under identical experimental conditions, and even in the same apparatus but under different experimental conditions, the effect of instabilities on transport processes is substantially different. The difficulties in bringing to light the consequences of oscillations present in the plasma are associated with possible different mechanisms of their contribution to diffusion and thermal conduction. In addition to reduction of the plasma containment time as the result of appearance of a drift flux which is the average of the product of the oscillations of the density $\tilde{n}$ and the electric field $\tilde{E}$, i.e., $\sim\langle\tilde{n}\tilde{E}\rangle/H$, the fluctuations result in increased diffusion, which is due, for example to the change in particle trajectories in the variable electric field $\tilde{E}$.[4,5] Inclusion of such effects, in the current state of theory and experiment, is extraordinarily complicated. On the other hand, by learning only the nature and type of excited instabilities we can, using the predictions of the theory, find means of suppressing them and in this way increasing the efficiency for containment of particles and energy in the trap.

The purpose of the present work was to study the spectral and space-time characteristics of the density and electric-field oscillations, and also the dependence of these oscillations on the magnetic field strength H_0 and the rotational transform angle i of the lines of force. Determination of these characteristics of the oscillations, together with knowledge of the main parameters of the plasma,[3,6] has permitted, on the basis of existing theoretical concepts, identification of the type of oscillations excited, and indication of possible buildup mechanisms and effects leading to stabilization.

The experiments were carried out in the L-1 stellarator,[7] which is a closed magnetic trap with a helical field with multipolarity $l = 2$. The large radius of the toroidal vacuum chamber is $R = 60$ cm, and the small radius $a = 5$ cm. The number of steps of the helical coil is $N = 7$. The rotational transform angle of the magnetic lines of force is $i < 2\pi$. The range of magnetic field strengths in which the experiments were carried out was $H_0 = 2$–10 kOe. The pressure of neutral gas in the chamber could be controlled over the range $p = 5 \times 10^{6}$–10^{-4} torr. The study of fluctuations was made in a decaying plasma produced by the external injection method.[8] The quasistatic plasma potential and the fluctuations of the density and electric field were studied by means of single and double Langmuir probes. The first results of these studies have been published previously.[3]

In the present work we have made a more complete study of long-wave oscillations by means of a correlation technique.[9]

2. METHOD AND EXPERIMENTAL ARRANGEMENT

In the experiments being described, large-scale long-wave oscillations were investigated. From general considerations it is clear that in a closed system with a rotational transform, as a result of the periodicity conditions, development of long-wave oscillations occurs in the density gradient preferentially near magnetic surfaces with a rational angle of rotational transform of the lines of force

$$i = 2\pi l_0 / m_0, \tag{1}$$

where l_0 and m_0 are integers. Therefore the experiments were carried out under those conditions in which the magnetic surfaces with rational values $i = \pi\epsilon^2 N(1 + 2\alpha^2 r^2)$ were located in the density gradient ($\epsilon = H_\varphi/H_0$, H_φ is the amplitude of the fundamental harmonic of the helical field, $\alpha = N/R$).

The oscillations observed in most plasma experiments are a complex fluctuation process. Therefore, for analysis of the experimental data, it is necessary to use the method of correlation analysis. Use of the correlation method permits determination of the degree of turbulence of the plasma and the space-time characteristics of the fluctuations.[10]

In the present work in analysis of the experimental results we calculated a normalized correlation function[1)]

[1)]When a constant component was present in the signals, corresponding formulas[9] were used which avoided its effect on the final result.

$$R_{12}(\tau) = K_{12}(\tau) / \sqrt{K_{11}(0) K_{22}(0)}, \qquad (2)$$

where $K_{12}(\tau)$ is the mutual correlation function of two signals $\xi(t)$ and $\eta(t)$ which are, for example, the readings of two probes measuring some kind of fluctuations. In the calculations we made use of an approximate expression for $K_{12}(\tau)$:

$$K_{12}(\tau) \approx \frac{1}{N'} \sum_{i=1}^{N'} \xi(i\delta)\eta(i\delta + \tau),$$

where N' is the number of intervals into which the time period studied is broken up, δ is the length of an interval, and $\tau = n\delta$ is the time shift of one signal with respect to the other. In the case when $\xi(t)$ and $\eta(t)$ are the same signal, Eq. (2) permits calculation of the autocorrelation function $R_{11}(\tau)$, which is the time characteristic of the process being studied.

The period of time T was chosen, on the one hand, sufficiently large that increasing it by a factor of two made practically no change in $R_{12}(\tau)$ and, on the other hand, much smaller than the time of variation of the plasma parameters ($T = 150-200$ μsec). Since the characteristic frequencies of the oscillations studied were of the order of tens of kilohertz, and the lifetime of the plasma was $\approx 1-2$ msec, the choice $T = 150-200$ μsec satisfies the requirements enumerated above. The number of intervals into which the time period T was broken down was varied over the range $N' = 108-240$, which provided the necessary accuracy of the calculations.

In the present work a computer was used to calculate the correlation functions. This permitted us to calculate, in addition to $R_{12}(\tau)$, the autocorrelation functions $R_{11}(\tau)$, $R_{22}(\tau)$ and $K_{12}(\tau)$, $\sqrt{K_{11}(0)}$, and $\sqrt{K_{22}(0)}$ for a pair of signals obtained in one operating pulse of the apparatus. The last two quantities are time-average effective amplitudes of the fluctuations.

In order to obtain a complete picture of the properties of the oscillations existing in the plasma, it was necessary to investigate also the spatial correlation of the fluctuations, i.e., to determine $R_{12}(\tau, \rho)$, where ρ is the distance between the two points of observation. In a magnetic field a plasma, like the oscillations developed in it, is anisotropic and therefore R_{12} is a function of four variables $R_{12}(\tau, \rho_1, \rho_2, \rho_3)$. In the particular case of a toroidal magnetic trap $\rho_1 \equiv \Delta z$ is the distance along the magnetic field lines, $\rho_2 \equiv \Delta l$ is the distance along the azimuth of the section of the magnetic surface, and $\rho_3 \equiv \Delta r$ is the distance along the plasma density gradient, which coincides with the normal to the magnetic surface.

In the experiments on measurement of the correlation function $R_{12}(\tau, \Delta z)$, the probes were exposed to the same magnetic field line by means of an electron beam.[9,11] In measurement of $R_{12}(\tau, \Delta l)$ the probes had to be located on the same magnetic surface, and the same method was used to establish this placement.

The probes used in all measurements were Langmuir probes. Density fluctuation measurements were made by single probes under the condition of saturation ion current. The quasistatic floating potential U_f was also measured by single probes. The quasistatic electric field E and its oscillations $\tilde{E}$ were recorded[2)] by double probes. The probe dimensions (length 2 mm, diameter 0.1–0.6 mm) assured the necessary localization of the measurements. The characteristics of the measuring equipment (bandwidth 2–300 kHz) permitted the experimentally observed signals to be reproduced without appreciable phase and amplitude distortion.[3)]

[2)]The space-time correlation functions of $\tilde{E}$ agree rather well with the corresponding characteristics of $\tilde{n}$. Therefore we will limit ourselves below to description of the results of detailed investigation of the density fluctuations. The characteristic value of the amplitude is $\tilde{E} \lesssim 1$ V/cm.

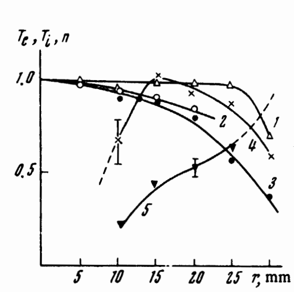

FIG. 1

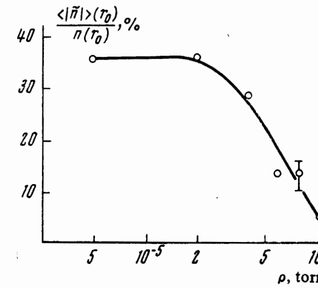

FIG. 2

FIG. 1. Distribution in radius in relative units of electron temperature T_e (curve 1), ion temperature T_i (curve 2), plasma density n (curve 3), fluctuation amplitude $\langle\tilde{n}\rangle$ (curve 4), and relative fluctuation amplitude $\langle\tilde{n}\rangle/n$ (curve 5). $H_0 = 3$ kOe, $i = 4\pi/3$.

FIG. 2. Relative amplitude of oscillations as a function of gas pressure, $i = 4\pi/3$, $H_0 = 3$ kOe, $r_0 = 20$ mm.

3. EXPERIMENTAL STUDY OF PLASMA DENSITY OSCILLATIONS IN A STELLARATOR

In the first studies of low-pressure plasma confinement in the L-1 stellarator it was already established[3] that density and electric field oscillations build up in a plasma, with spectra characterized by several separated frequencies.

Detailed measurements showed that, immediately after injection, oscillations are excited over the entire volume of the plasma. On establishment of a quasistationary density distribution (200–300 μsec) the fluctuations in the central region of the plasma are damped. At this same time a quasistationary distribution of the ion temperature T_i and electron temperature T_e is established. Normalized distributions of n, T_i, and T_e as a function of radius are shown in Fig. 1 (curves 3, 2, and 1). Oscillations in the region of the density gradient exist during the entire decay time of the plasma. The autocorrelation functions $R_{11}(\tau)$ show that the correlation time of the signals is substantially greater than their period of oscillation.

1. Amplitude-time Characteristics of Fluctuations

Measured values of the spatial distribution of density-oscillation amplitude are shown in Fig. 1 (curve 4). Also shown are values of $\langle|\tilde{n}|\rangle_{rel}$ in relative units as a function of radius for $H_0 = 3$ kOe and $i = 4\pi/3$. Since the oscillation amplitude varies randomly both with time and from one measurement to another, two probes were used in investigation of the radial distribution of oscillation amplitudes, for normalization. One—the measurement probe—was moved along a radius, and the second—a fixed probe set at radius r_0—served as a monitor, so that $\langle|\tilde{n}|\rangle_{rel} = \langle|\tilde{n}|\rangle(r)/\langle|\tilde{n}|\rangle(r_0)$. As can be seen from Fig. 1, the oscillations encompass practically the en-

[3)]Specially made measurements with a bandwidth $f = 10^6$ Hz showed that frequencies above 200 kHz are absent in the spectrum.

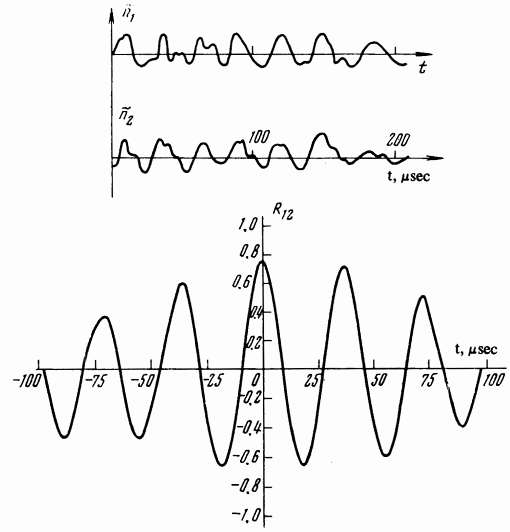

FIG. 3. Oscillograms of oscillations of the saturation ion current to probes located on the same line of force (above), and the mutual correlation function of these signals (below), $i = 4\pi/3$, $H_0 = 3$ kOe, $\Delta z = 570$ cm.

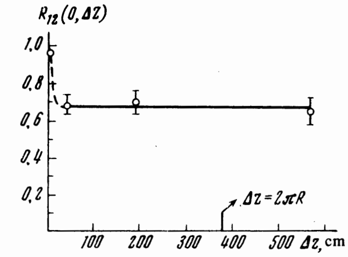

FIG. 4. Correlation function as a function of distance Δz along the magnetic force line. $i = 4\pi/3$, $H_0 = 3$ kOe.

tire region of nonuniformity of the plasma density. Figure 1 (curve 5) shows the dependence on radius, measured under the same conditions, of the relative value of density fluctuation $\langle|\tilde{n}|\rangle(r)/n(r)$, where $n(r)$ is the quasistationary density value at the point of measurement of the fluctuations. It is evident that this value increases along the direction to the boundary surface, reaching almost 100% at the surface ($r_{bound.} = 30$ mm), and in the center of the plasma in the region of low density gradients the relative amplitude of the oscillations falls off rapidly. Similar measurements made for $i = \pi/2$ and π showed that the relative size of the density fluctuations $\langle|\tilde{n}|\rangle(r)/n(r)$ in the region of greatest oscillation amplitudes $\langle|\tilde{n}|\rangle$ amounts to 10–30% and remains practically constant as the field varies over the range 2.5–5 kOe.

The level of the oscillations decreases substantially with an increase of gas pressure. The gas pressure usually does not exceed $p = 5 \times 10^{-6}$ torr [12] in the plasma-decay process. In experiments with admission of gas it was established that the relative amplitude of the fluctuations begins to fall for a pressure increase of approximately an order of magnitude. As can be seen from Fig. 2, the relative amplitude of fluctuations begins to decrease at $2\text{–}4 \times 10^{-5}$ torr, reaching several percent at $p \approx 10^{-4}$ torr.

2. Measurement of the Longitudinal Length of Oscillations

In clarifying the nature of the oscillations which arise, it is necessary to determine the wave-vector components longitudinal ($k_\parallel$) and transverse ($k_\perp$) with respect to the magnetic field vector H_0.

In order to find $k_\parallel$ it is necessary to investigate the space-time correlation function of the two signals along the direction of the lines of force. From the time shift $\Delta\tau$ of the maximal value $R_{12}(\Delta\tau, \Delta z) = R_{12}^{max}(\Delta\tau, \Delta z)$ relative to the moment of time $\tau = 0$, the distance between the probes Δz, and the frequency of the fundamental mode determined from the mutual correlation function, we can calculate $k_\parallel = \omega/v_{phase\,\parallel}$, where $v_{phase\,\parallel} = \Delta z/\Delta\tau$.

Detailed measurements of the longitudinal correlation function $R_{12}(\tau, \Delta z)$ of the density of oscillations were carried out near the magnetic surface with transform angle $i = 4\pi/3$. Three probes were placed at different cross sections of the torus on the same magnetic line of force. The distances between them were respectively 0.5, 1, and 1.5 revolutions of the line of force about the principal axis of the torus, i.e., Δz was 190, 380, and 570 cm. In addition, measurements were made at closer distances $\Delta z \approx 0.5$ and 40 cm. These measurements were made for magnetic surfaces with average radii r equal to 15, 20, and 25 mm. Figure 3 shows for illustration signals from the oscillations of density from two probes located on the magnetic surface $r = 20$ mm, $\Delta z = 570$ cm, and the mutual correlation function of these signals. As can be seen, the correlation coefficient is rather large ($R_{12}^{max} = 0.8$), the fundamental mode of oscillations with frequency $f = 28$ kHz is clearly distinguished in the correlation function, and the correlation time of the signals is considerably greater than the period of oscillations. Some isolated frequency whose value could change as a function of the experimental conditions was always present in the spectrum of oscillations. Sometimes harmonics were also observed in addition to the fundamental frequency. It can be seen from Fig. 3 that the mutual correlation function is symmetric relative to the time $\tau = 0$ (i.e., $R_{12}^{max}(\tau, \Delta z) = R_{12}(0, \Delta z)$). This indicates the absence of a phase shift of the fundamental mode along the length of the line of force in distances greater than the length of the apparatus ($\Delta z = 570$ cm $> 2\pi R$). A similar picture is observed at the magnetic surface with mean radius $r = 15$ mm. Figure 4 shows the correlation coefficient $R_{12}(0, \Delta z)$ as a function of the distance between probes. Beginning with $\Delta z = 40$ cm, the correlation coefficient is practically unchanged, remaining at a level $R_{12}(0, \Delta z) = 0.7$. The correlation coefficient measured by probes placed at a distance 0.5 cm from each other turned out to be 0.95. The drop in the correlation coefficient in the range of values $\Delta z = 0.5\text{–}40$ cm can be assigned to the presence of short-wave oscillations (0.5 cm $\le \lambda_\parallel \le 40$ cm). Their contribution to the amplitude of the fluctuations is relatively small.

From the measurements presented it follows that the fundamental mode of oscillation consists of density perturbations drawn out along the magnetic lines of force, with a wave vector $k_\parallel = 0$. This structure is observed in the radial region located near a magnetic surface with a rational transform angle. In view of the closed nature of the lines of force, similar long-wave perturbations close on themselves, and the azimuthal

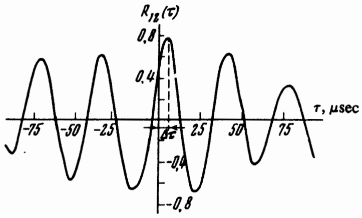

FIG. 5. Correlation function of two signals from probes displaced along a radius by $\Delta r = 5$ mm. $i = 4\pi/3$, $H_0 = 3$ kOe.

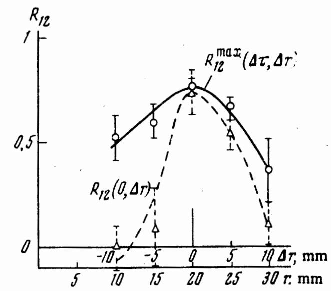

FIG. 6. Correlation function as a function of distance along the radius r, $i = 4\pi/3$, $H_0 = 3$ kOe, solid line—$R_{12}^{max}(\Delta\tau, \Delta r)$, dashed line—$R_{12}(0, \Delta r)$.

mode of oscillations in the cross section of the torus should be equal to or equal to a multiple of the number m_0 of the resonance of the magnetic lines of force.[3] When the substantial extent of the radial region of existence of oscillations is considered (see Fig. 1), it is necessary to take into account that the presence of crossing of magnetic lines of force (shear)

$$\theta = \frac{r^2}{2\pi R}\frac{di}{dr}$$

leads to the fact that, on deviation from the resonant magnetic surface by Δr, a certain $k_{\parallel}^{*}$ should appear. Actually, since at the resonant surface $k_{\parallel} = 0$, it follows from geometrical considerations that $k_{\parallel}^{*} = k_{\varphi}\theta\Delta r/r$, where k_{φ} is the azimuthal wave number and r is the radius of the resonant surface. This relation permits evaluation for $i = 4\pi/3$ of the expected value of phase velocity of the wave along the lines of force of the magnetic field at a certain distance (≈ 1 cm) from the resonant surface, $v_{\text{phase}\,\parallel}^{*} = \omega/k_{\parallel}^{*} \approx 5 \times 10^7$ cm/sec. Correlation measurements of oscillations carried out for these same conditions gave the close value $v_{\text{phase}\,\parallel} \approx 2 \times 10^7$ cm/sec. We note that the measured value of phase velocity of the wave lies in the interval between the ion (v_{Ti}) and electron (v_{Te}) thermal velocities for the entire range of particle temperatures observed in our experiment.

3. Radial Correlation Measurements

An important characteristic of the oscillations of a nonuniform plasma is the structure of the wave along the direction of variation of the plasma density, i.e., along the normal to the magnetic surfaces.

Correlation measurements along a radius were made by means of two probes. A fixed probe was placed successively at several fixed locations along the radius (15, 20, and 25 mm) and for each of these values the second probe was moved from the center of the plasma filament to the periphery in 5-mm steps. In each series of measurements, the two probes were given a preliminary exposure on the same line of force. Figure 5 shows the mutual correlation function $R_{12}(\tau)$ obtained for $H_0 = 3$ kOe, $i = 4\pi/3$ and a radial distance between the probes $\Delta r = 5$ mm. The maximum of the function R_{12}^{max} was displaced by $\Delta\tau \approx +5$ μsec. On increasing the radial distance between the probes, together with an increase in the phase shift, i.e., with increasing $\Delta\tau$, a decrease occurs in the maximum absolute values of R_{12}^{max}. The results of these measurements are shown in Fig. 6, which shows the decrease in R_{12}^{max} on both sides of the point $r = 20$ mm at which the fixed probe was located. The curve shown in Fig. 6 characterizes the region of localization of the fundamental mode of oscillations, which roughly coincides with the region of existence of density fluctuations (see Fig. 1) and with the region of plasma with different density and temperature gradients.

From the mutual space correlation function of the signals of the two probes displaced along the radius, plotted with inclusion of the phase shift, we can estimate the radial wavelength λ_r of the fundamental mode of oscillation. Figure 6 (the dashed curve) shows the function $R_{12}(0, \Delta r)$ for $\tau = 0$. It follows from consideration of the curve in Fig. 6 that $\lambda_r \gtrsim 2$ cm, and high radial modes of oscillation are not observed.

In principle, in propagation of a wave in a region of variable plasma parameters (n, T) and their gradients, the mutual space correlation function should depend not only on the relative distance Δr between the probes but also on the coordinate of the fixed probe. Values were measured for r equal to 15, 20, and 25 mm, which showed that although $R_{12}^{max}(\Delta\tau, \Delta r, r)$, like $R_{12}(0, \Delta r, r)$, differs somewhat, the region of existence of the fundamental mode of oscillation and the radial wavelength nevertheless change insignificantly.

4. Coupling of Azimuthal Modes and Frequencies of Oscillations with the Magnetic Field

In determination of the azimuthal phase velocity $v_{\text{phase}\,\varphi}$, the mode of oscillation m, and the frequency ω_d in the coordinate system moving with the plasma, it is necessary to take into account the Doppler shift due to presence of a quasistatic radial electric field,[3] $\omega_d = \omega_L - k_{\varphi}u_E$, where ω_L is the oscillation frequency in the laboratory system, k_{φ} is the wave number, and $u_E = cE_r/H$. According to our earlier measurements,[13] the radial dependence of the plasma potential has the nature of a quadratic parabola, so that $k_{\varphi}u_E$ does not depend on radius. This permitted determination of E_r at one point, but not over the entire region of oscillation.

The measurements were made in the following way. In one of the cross sections of the vacuum chamber several single probes measuring density oscillations were placed on the same magnetic surface. The distance between the probes along the magnetic surface was Δl. From the mutual correlation function of the signals, we determined the fundamental oscillation frequency ω_L and from the time shift of the correlation function maximum $\Delta\tau$ we determined the azimuthal phase velocity in the laboratory system $v_{\text{phase}} = \Delta l/\Delta\tau$ and the azimuthal mode $m = \omega_L\Delta\tau/\Delta\varphi$, where $\Delta\varphi$ is the angular distance between the measuring probes. Near one of the measurement points was located a group of two

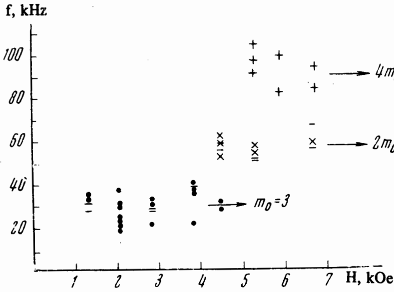

FIG. 7. Frequency and oscillation mode as a function of magnetic field strength H_0. $i = 4\pi/3$; O–$m_0 = 3$; X–$2m_0 = 6$; +–$4m_0 = 12$; the bars denote cases for which determination of the mode was difficult.

pairs of mutually perpendicular probes which measured the components of the electric field normal to the lines of force of the magnetic field. The value of E_r was measured in the same operating pulse of the apparatus as the density oscillations. The value of E_r determined in this way permits calculation of u_E and the plasma filament rotation frequency $f_E = u_E/2\pi r$.

Detailed measurements have confirmed our earlier representations[3] of the azimuthal structure of the wave. The lowest azimuthal mode number of the oscillations, determined for rotational transform angles $i = \pi/2$, π, and $4\pi/3$, turned out to be equal to the number m_0 of the resonance of the magnetic surfaces (see Eq. (1)). For these values of i this corresponds to m_0 equal to 4, 2, and 3, which agrees with the conclusions of measurements of the longitudinal correlation of the signals.

Figure 7 shows measured values of the frequency $f_L = \omega_L/2\pi$ of the oscillations as a function of magnetic field value H_0 for $i = 4\pi/3$. Each measurement, made in one operating pulse of the apparatus, is represented by a point on the plot. The spread in the results obtained under the same conditions is apparently due to definite instability of the mode of operation of the spark gun. It is evident from the figure that for small magnetic fields, as a rule, only the lowest oscillation mode $m_0 = 3$ arises (the points; the lines denote cases for which determination of the mode was difficult). Near the magnetic field value $H_0 = H_{cr} \approx 5$ kOe, in addition to the fundamental mode, there begins to appear also its multiple mode $2m_0 = 6$, the two modes appearing now one, now the other, or simultaneously. With further increase in the magnetic field, the fundamental mode disappears and, in addition to the mode $2m_0 = 6$, the mode $4m_0 = 12$ also begins to appear.[4)] Similar results were obtained for $i = \pi$. In this case for a field $H_0 \approx 2.5$ kOe only the fundamental mode of oscillation $m_0 = 2$ exists. For $H_0 \gtrsim 5$ kOe the mode $2m_0 = 4$ is excited, as a rule. From the data presented it can be seen that the number m_0 of the fundamental mode is determined by the structure of the magnetic field, and its multiplicity depends on the value of H_0. The experiments permit us to find from the known frequencies and modes of oscillation the dependence of the oscillation frequency of the first mode $f_{L1} = f_L/m$ on the value of the magnetic field in the laboratory system of coordinates.

4)The error in determination of Δl makes it difficult to determine accurately the number of the higher modes, so that appearance also of $3m_0=9$ is not excluded.

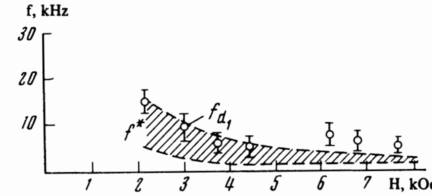

FIG. 8. Oscillation frequency of the first mode f_d in the coordinate system moving with the plasma, and electron drift frequency f^* (cross-hatched region), as a function of magnetic field. The value of f^* was calculated for $T_e = 2$–4 eV and $\partial \ln n/\partial r = 0.5$–$1$ cm^{-1}.

In order to determine the frequency f_d of the wave and its direction of propagation, it is necessary to take into account the Doppler effect due to the presence of a radial electric field. For this reason we measured the dependence of E_r on the field strength over the range $H_0 = 2$–8 kOe, which permitted calculation of $f_E(H_0)$. As a result we obtained the dependence of the frequency f_{d_1} of the first mode of oscillation on the magnetic field in the coordinate system moving with the plasma. Figure 8 shows as an illustration the function $f_{d_1}(H_0)$ for $i = 4\pi/3$. The cross-hatched region in Fig. 8 shows the values of the drift-electron frequency $f^*(H)$ calculated for the range of plasma parameters $T_e = 2$–4 eV and $\partial \ln n/\partial r = 0.5$–$1$. As can be seen, rather good agreement is observed between the experimentally determined and calculated values of the frequencies. A certain discrepancy in the region of high magnetic fields may be due, as we have indicated above, to inaccuracy in determination of the numbers of the higher modes.

The direction of propagation of the wave in the plasma coordinate system depends on the value of the transform angle i, but is constant over the entire range of H_0. Thus, for $i = \pi$ the direction of the wave coincides with the direction of diamagnetic drift of the electrons, and for $i = 4\pi/3$ the wave is propagated in the opposite direction.

4. DISCUSSION OF EXPERIMENTAL RESULTS

In turning to discussion of the results obtained, it is necessary to consider the question of whether the theory of instability of a nonuniform plasma is applicable to the experiment being described.

At the present time we can base the discussion mainly on the linear theory developed for an unbounded plasma and satisfied in the approximation of geometrical optics. In a real experiment there is a plasma of finite size with a rather large established amplitude of oscillations, $\langle |\tilde{n}| \rangle (r)/n(r) \approx 30\%$, i.e., with a substantially nonlinear mode of oscillation. The geometrical-optics approximation is valid for wavelengths substantially smaller than the size of the nonuniformity, a condition which, as experiment has shown, is not satisfied in our case (see Fig. 6). In the theory there are a number of additional approximations which are not consistent with the conditions existing in the present experiment. Thus, in the theoretical discussion of the problem of excitation of instability, a solution is sought in the form of a series of plane waves, which cannot describe the perturbation in a closed toroidal system. Furthermore, for example, the spectra of long-wave drift oscillations found for the frequency region $k_{\parallel}v_{Ti} \ll \omega \ll k_{\parallel}v_{Te}$ for $\nu_e \ll \omega$, $k_{\parallel}v_{Te}$ or $\nu_e \gg \omega$, $k_{\parallel}v_{Te}$ are well known; solutions are known also for $\omega \gg k_{\parallel}v_{Te}$. In our experiment the values of ω, $k_{\parallel}v_{Te}$, and ν_e are extremely close together and there are no exact solutions for this case. From all this it follows that in comparison of experiment with theory it is reasonable to compare the functional dependences and to discuss agreement of numerical values in order of magnitude, while not pretending to any accurate quantitative agreement.

The stability of a plasma in a stellarator can be affected by a number of factors: curvature of the magnetic field lines, nonuniformity of the plasma density and the temperature of its components, different dissipation mechanisms, the presence of an electric field, which is frequently nonuniform over radius, existence of trapped and passing particles, and so forth. At the present time there is no theory which takes into account the influence of all these factors as a set, and therefore, knowing the frequency of the oscillations and the structure of the wave, we must consider a wide circle of mechanisms capable of contributing to excitation of the instability being discussed.

Analysis of the types of long-wave low-frequency oscillations capable of existing in a plasma in the L-1 stellarator has shown that the most likely is excitation of drift-dissipative instabilities. As is well known,[14-16] this instability is stabilized by a shear of the magnetic field lines $\theta_T > \sqrt{1 + T_e/T_i}\ \rho_i/L_0$ where ρ_i is the Larmor radius of the ions and L_0 is the characteristic size of the nonuniformity in density. According to an estimate, under the experimental conditions the necessary value of magnetic field shear ($\theta_T > 5 \times 10^{-2}$) is much greater than that existing in the L-1 stellarator ($\theta \approx 10^{-3}$).

In order to excite drift oscillations, it is necessary to satisfy the condition $k_{\parallel}v_{Ti} \ll \omega \ll k_{\parallel}v_{Te}$. The left portion of the inequality under our conditions (T_i = 10–20 eV) is satisfied over the entire cross section of the plasma filament. The right-hand part is violated near the resonant magnetic surface, where $k_{\parallel} = 0$. However, as experiment has shown, closer to the plasma surface a region exists in which this condition is satisfied. ($v^*_{\text{phase}\,\parallel} = 2\text{–}5 \times 10^7$ cm/sec, $v_{Te} \approx 10^8$ cm/sec). The idea of efficient buildup of drift oscillations outside the resonant magnetic surface, for R = 25–30 mm, is illustrated in Fig. 1 (curve 4). A similar region would have to exist also on the other side of the resonant surface, closer to the center of the plasma, but the density gradient is very small there and, evidently, for this reason the oscillation amplitude drops substantially.

From the point of view of the necessity of satisfying the condition $v^*_{\text{phase}\,\parallel} < v_{Te}$ it is possible to understand also the dependence of the mode of the excited oscillations on magnetic field strength. For drift-dissipative oscillations the region of localization $X = r_1 - r_2$ is determined by the expression[14]

$$X < \frac{L_0}{\theta}\sqrt{\frac{\rho_i}{m\lambda_e}}\left(\frac{m_e}{m_i}\right)^{1/4}[\text{cm}], \tag{3}$$

where λ_e is the electron mean free path. Using Eq. (3), we will estimate the phase velocity of the wave at the boundary of the localization region:

$$v^*_{\text{phase}\,\parallel} = \frac{\omega}{k_{\parallel}^*} = \omega^*/\frac{m}{r_c}\theta\frac{X}{L_s} > \frac{T_e}{L_0}\sqrt{\frac{c\lambda_e}{em_e^{1/2}T_i^{1/2}}}\sqrt{\frac{m}{H_0}}\left[\frac{\text{cm}}{\text{sec}}\right]$$

where

$$\omega^* = \frac{k_\varphi cT_e}{eH}\frac{\partial \ln n}{\partial r} \approx \frac{m}{r_0}\frac{cT_e}{eH_0L_0} \tag{4}$$

is the electron drift frequency. Using the plasma-parameter values of the L-1 stellarator and expressing H_0 in kiloOersteds, we obtain a simple relation between the phase velocity and the ratio of the oscillation mode and the magnetic field:

$$v^*_{\text{phase}\,\parallel} > 4 \cdot 10^7\sqrt{m/H_0}\ [\text{cm/sec}]. \tag{4'}$$

Taking into account that Eq. (4) determines the value of $v^*_{\text{phase}\,\parallel}$ at the boundary of the localization region and that the phase velocity of the wave increases as the resonant surface is approached, we can assume that those modes of oscillation are excited for which $v^*_{\text{phase}\,\parallel}$ is minimal, i.e., when $\sqrt{m/H_0} \le 1$ (for $\sqrt{m/H_0} = 1$ the value of $v^*_{\text{phase}\,\parallel}$ is only 2–3 times smaller than v_{Te}). On the other hand, the increment in the oscillations increases with the number of the mode and from this point of view it is natural to expect excitation of higher modes. As a result of these factors, with increase of the magnetic field, origin is possible of higher modes which do not violate the condition $\sqrt{m/H_0} \le 1$. The structure of the magnetic field determines the fundamental mode of oscillation and its harmonics. Therefore the transition to excitation of higher modes which are multiples of the fundamental can occur for definite values of magnetic field and, consequently, appearance of these modes has a threshold nature. This threshold nature of the excitation of higher modes is well confirmed in experiment, which can be seen from analysis of Fig. 7.

Thus, the series of factors listed above, as well as the coincidence of oscillation frequencies with electron drift frequencies, and the inverse proportionality of the oscillation frequency to magnetic field strength (Fig. 8) confirm the suggestion which has been made that drift-dissipative instability is excited in the experiments being analyzed.

Under conditions in which the electron collision frequency ν_e ($\approx 10^5$ sec^{-1} for a plasma density n = 10 $= 10^{10}$ cm^{-3}) is roughly equal to ω and $k_{\parallel}v_{Te}$, it is possible to excite both a class of oscillations with a Cerenkov buildup mechanism and a class due to electron collisions. The characteristic frequency for both classes is the electron drift frequency. The increments in the oscillations are respectively[15]

$$\gamma_1 = \gamma_r - \gamma_i, \qquad \gamma_2 = \frac{\nu_e}{|k_{\parallel}|v_{Te}}\gamma_r - \gamma_i, \tag{5}$$

where

$$\gamma_r = \sqrt{\frac{\pi}{2}}\frac{\omega}{|k_{\parallel}|v_{Te}}\left\{\frac{k_\perp^2\rho_i^2}{T_i/T_e}\left(1 + \frac{T_i}{T_e}\frac{\partial \ln T_i}{\partial \ln n}\right) - \frac{1}{2}\frac{\partial \ln T_e}{\partial \ln n} - \frac{g}{v_S^2\partial \ln n/\partial r}\right\} \tag{6}$$

$$\gamma_i = \frac{7}{10}\nu_{ii}k_\perp^4\rho_i^4\left(1 + \frac{T_e}{T_i} - \frac{3}{28}\frac{\partial \ln nT_i}{\partial \ln n}\right),$$

$$g = v_{Ti}^2/R_{\text{eff}}, \quad v_S = \sqrt{\frac{T_e}{m_i}}.$$

Under the conditions of the experiment $\nu_e/k_{\parallel}v_{Te}$ is close to unity and, consequently, $\gamma_1 \approx \gamma_2$. According to an estimate, $\gamma_r \approx 4 \times 10^4$ sec^{-1}. The second part of the increment $\gamma_i > 0$ since, according to Fig. 1, $^3/_{28}\, \partial \ln T_i / \partial \ln n < 1$, and, consequently (see Eq. (7)), ion collisions are a stabilizing factor. Since in our case the main contribution to ion collisions is from neutral particles, ν_{ii} is replaced[17] by a quantity $\approx \nu_{i0}\omega^2/k_{\parallel}^2 v_{Ti}^2$. For a working pressure of 5×10^{-6} torr, $\gamma_i \approx 2 \times 10^3\ \text{sec}^{-1}$. From comparison of this value with the theoretical value $\gamma_r \approx 4 \times 10^4\ \text{sec}^{-1}$ it is evident that increasing ν_{i0} by 10–100 times should lead to stabilization of the oscillations. These estimates are confirmed by experiment. As shown in Fig. 2, a noticeable decrease in the relative level of oscillations $\langle |\tilde{n}| \rangle (r_0)/n(r_0)$ occurs on change of the pressure by a factor of 10, i.e., at $p \approx 5 \times 10^{-5}$ torr, and for $p > 10^{-4}$ torr it is not possible to detect oscillations. According to our estimates $\gamma_{12} < 0$ at this pressure.

A number of other classes of drift-dissipative instabilities with $v_{\text{phase}\,\parallel} \ll v_{Te}$ have been considered; however, for our plasma parameters and value of θ their excitation is unlikely.

As can be seen from Figs. 1 and 6, oscillations exist over a rather wide radial region in which $v_{\text{phase}\,\parallel} \gg v_{Te}$. Although oscillations can be maintained in it as the result of the instabilities discussed above, the existence also of other mechanisms providing their own energy contribution is not excluded.

In our case, in a system with a periodically varying curvature of the lines of force and, consequently, a variable longitudinal velocity of the particles, kinetic buildup is possible of oscillations associated with resonant interaction of moving ions with the wave.[18] Since the increment of this instability is large, and the condition for its stabilization by magnetic field shear are barely satisfied, we can expect that it will provide an additional contribution of energy to the oscillations.

Another cause of excitation of instabilities in the region $k_{\parallel} \approx 0$ may be curvature of the lines of force, leading to excitation of flute or gravitational-dissipative modes.[14] These instabilities have apparently been stabilized in experiment, so that we can assume that they do not exert appreciable influence on the buildup of the oscillations observed.

5. CONCLUSION

As the result of an investigation of long-wave plasma oscillations in the L-1 stellarator it has been established that azimuthal modes of oscillation are determined by the structure of the vacuum magnetic field and depend on its intensity. Thus, for rational values of the rotational transform angle $\iota = 2\pi l_0/m_0$, the modes are equal to or multiples of m_0. Near resonant magnetic surfaces the wave vector $k_{\parallel} \approx 0$, and the density perturbations are flutes winding around the toroidal plasma column and closing on themselves. The oscillations encompass the entire region of plasma nonuniformity, so that $k_r \approx 1/r$. The measured oscillation frequency (in the coordinate system moving with the plasma) is close to the electron drift frequency and is inversely proportional to the magnetic field intensity. The stabilization of oscillations on increasing the ion-neutral-particle collision frequency has been established.

Analysis of the experimental data and comparison with theory provide a basis for suggesting that a drift-dissipative instability is responsible for excitation of the oscillations. The main mechanisms for transfer of energy to the wave may be Cerenkov absorption or dissipation in electron collisions. The possibility is not excluded of an additional energy contribution to the oscillations as the result of interaction with the wave of traveling ions periodically changing their longitudinal velocity in the nonuniformities of the magnetic field.

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