Oscillations of the Knight Shift and Relaxation Time of Nuclei in Crossed Electric and Magnetic Fields

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The effect on an electric field on the Knight shift and relaxation time of nuclei in a strong magnetic field is studied. It is demonstrated that for determination of the amplitudes of the Knight shift and nuclear spin-lattice relaxation time of the oscillations which are similar to the Shubnikov-de Haas oscillations, the role of the cut-off factor in the theory may be played by the electric field perpendicular to the magnetic field.

THE question of the Knight-shift oscillations and of the spin-lattice relaxation time in a strong magnetic field has been considered in a number of papers.[1-3] To avoid the divergences that arise when the Landau levels cross the Fermi surface, it is customary, when estimating the oscillation amplitude, to introduce into the theory a cut-off parameter which is identified with the interaction that determines the broadening of the electric levels. Subsequently, Magrill and Savvinykh[4,5] have shown that a weak electric field can play the role of a cutoff factor in a semiconductor in the case of electron scattering by impurities, owing to the collision-induced change of the position of the center of the cyclotron orbit and the associated change in the "longitudinal" energy of the particle. In the present communication we show, for the case of metals and degenerate semiconductors, that the divergence of the density of states on the Fermi surface in crossed electric and magnetic fields is eliminated without broadening the levels if account is taken of the change of the state of the electron in the electric field.

The solution for the wave functions and the eigenvalues of the electron energy in a magnetic field **H** = (0, 0, H) and an electric field **E** = (E, 0, 0) is [6]

$$\psi_{nk_z\beta\sigma}(\mathbf{r}) = B\exp[i(k_z z + \ y)]\Phi_n(x/L_M - L_M(\beta+k_0))S_\sigma;$$

$$\varepsilon_{nk_z\beta\sigma} = \hbar^2k_z^2/2m + \hbar\omega(n+{}^1/_2) - k_0\beta\hbar^2/m - k_0^2\hbar^2/2m - \hbar\omega_\sigma/2: \quad (1)$$

$$\omega = \frac{eH}{mc},\ B = \frac{\sqrt{2}\,U_N(\mathbf{r})}{(L_yL_zL_M)^{1/2}},\ L_M^2 = \frac{c\hbar}{eH},\ k_0 = \frac{mcE}{\hbar H}$$

Here $u_N(\mathbf{r})$ is the Bloch wave function at the extremum of the conduction band, Φ_n is a Hermite function and S_σ and ω_σ are respectively the spin wave function and the splitting of the spin levels in frequency units. From the condition that the center of the cyclotron orbit be located inside the sample

$$0 < L_M^2(\beta+k_0) < L_x \quad (2)$$

we obtain the condition imposed on the wave vector β:

$$-k_0 < \beta < g - k_0, \quad g = m\omega L_x/\hbar. \quad (3)$$

The Knight-shift is calculated from the formula

$$\hbar\Delta\omega = {}^8/_3\pi\gamma_e\gamma_I\hbar^2SI\sum_{\mathbf{k}}|\psi_{\mathbf{k}}(0)|^2\{f_{\epsilon_-} - f_{\epsilon_+}\}, \quad (4)$$

where $\psi_{\mathbf{k}}(\mathbf{r})$ is the wave function of the electron, ϵ_- and ϵ_+ are the energy eigenvalues corresponding to the spin direction parallel and antiparallel to the field, and γ_e and γ_I are the electronic and nuclear gyromagnetic ratios. Substituting (1) and using the approximate equality for the Fermi distribution

$$f_{\varepsilon-} - f_{\varepsilon+} \approx {}^1/_2\hbar\omega_\sigma\delta(\varepsilon - \varepsilon_F), \quad (5)$$

we obtain

$$\hbar\Delta\omega = a\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}dk_z\int_{-k_0}^{g-k_0}\Phi_n^2[-L_M(\beta+k_0)]\delta\left[k_z^2 + \frac{2}{L_M^2}\left(n+\frac{1}{2}\right)\right.$$
$$\left. - 2k_0\beta - k_0^2 - k_F^2\right]d\beta, \quad a = \sqrt{2}\pi\gamma_e^2\hbar^3\omega_I N u_N^2(0)(\hbar\omega)^{1/2}\varepsilon_F^{-3/2}, \quad (6)$$

where ϵ_F is the Fermi energy and $\hbar\omega_I$ is the Zeeman energy of the nucleus.

Integration with respect to k_z with a δ-function yields

$$\hbar\Delta\omega = a\sum_n\int_0^g \Phi_n^2(L_M\beta)[k_F^2 - k_n^2 + 2k_0\beta - k_0^2]^{-1/2}d\beta \quad (7)$$

The limits of the integration and summation regions are given by the following equation:

$$k_F^2 + 2k_0\beta - k_0^2 = k_n^2 + \alpha_n^2, \quad k_n^2 = (2n+1)/L_M^2. \quad (8)$$

The parameter α_n has the dimension of a wave vector and characterizes the distance between the Fermi level and the nearest Landau level. The quantity α_n is defined by the inequality

$$0 \leqslant \alpha_n^2L_M^2/2 < 1;$$

and vanishes when the Landau level coincides with the Fermi level.

Breaking up the integration region into two parts, we can represent the sought expression in the form

$$\hbar\Delta\omega = a\sum_{n=0}^{n_0}\int_0^g\frac{\Phi_n^2(L_M\beta)d\beta}{(k_F^2 - k_n^2 + 2k_0\beta - k_0^2)^{1/2}}$$
$$+ a\sum_{n=n_0+1}^{n_0+n_g}\int_{\beta_n}^g\frac{\Phi_n^2(L_M\beta)d\beta}{(k_F^2 - k_n^2 + 2k_0\beta - k_0^2)^{1/2}} = J_1 + J_2; \quad (9)$$
$$n_0 = {}^1/_2L_M^2(k_F^2 - k_0^2 - \alpha_0^2), \quad n_g = L_M^2(k_0g - {}^1/_2\alpha_g^2),$$
$$\beta_n = (k_n^2 - k_F^2 + k_0^2 + \alpha_n^2)/2k_0.$$

α_0 and α_g have here the meaning of α_n defined above, with n = n_0 and n = n_g, respectively. The sum J_1 breaks up in turn into two parts: the first is the non-oscillating part

$$\hbar\Delta\omega_E = a\sum_{n=0}^{n_0-1}\int_0^g\frac{\Phi_n^2(L_M\beta)d\beta}{(k_F^2 - k_n^2 + 2k_0\beta - k_0^2)^{1/2}} \quad (10)$$

For the calculation it is possible, for example, to expand the denominator in a series in the electric field. We then obtain the value of the Knight shift that depends on the electric field:

$$\hbar\Delta\omega_E \approx a[(\tfrac{1}{2}n_F)^{1/2} - \tfrac{1}{2}n_F k_0 L_M], \quad n_F = \varepsilon_F/\hbar\omega. \tag{11}$$

The second part (the case $n = n_0$) gives the divergence at zero electric field, when the Landau level coincides with the Fermi level. However, when the field is not equal to zero, there is no divergence at $\alpha_0 = 0$. Thus, the electric field plays here the role of a cutoff factor:

$$\hbar\Delta\omega_{\rm osc} = a\int_0^{\varepsilon} \frac{\Phi_n^2(L_M\beta)\,d\beta}{(\alpha_0^2 + 2k_0\beta)^{1/2}}. \tag{12}$$

To calculate the integral, we recall that the Hermite function $\Phi_n(y)$ decreases exponentially beyond the point $y = 2n^{1/2}$, and up to this point, at large values of n, it is well described by the asymptotic formula

$$\Phi_n(y) \approx \frac{1}{\pi^{1/4}}\left[\frac{(n-1)!!}{n!!}\right]^{1/2} \cos\sqrt{2n+1}\,y. \tag{13}$$

Using this expression in (12), we obtain

$$\hbar\Delta\omega_{\rm osc.} \approx \frac{a\,(n_0-1)!!}{\pi^{1/2}n_0!!}\int_0^{\frac{2\sqrt{n_0}}{L_M}} \frac{\cos^2(\sqrt{2n_0+1}\,L_M\beta)\,d\beta}{(\alpha_0^2+2k_0\beta)^{1/2}} \approx \frac{a}{\pi}\left(\frac{2}{n_0}\right)^{1/2}\left\{\frac{(\alpha_0^2L_M^2+4k_0L_Mn_0^{1/2})^{1/2}}{2k_0L_M} - \frac{\alpha_0}{2k_0}\right\}. \tag{14}$$

It follows from (14) that in order for the oscillation amplitude to be determined by the electric field, it is necessary to satisfy the condition

$$\alpha_0^2L_M^2 < 4k_0L_Mn_0^{1/2}$$

(or $E > \alpha_0^2\hbar^2\omega/4e(\epsilon_F m)^{1/2}$ for the electric field). We then have

$$\hbar\Delta\omega_{\rm osc} \approx \frac{a}{\pi}\left(\frac{2}{k_0L_Mn_0^{1/2}}\right)^{1/2}.$$

If the field is small, $\alpha_0^2L_M^2 \gg 4k_0L_Mn_0^{1/2}$, we obtain

$$\hbar\Delta\omega_{\rm osc} \approx a2^{1/2}/\pi\alpha_0L_M. \tag{15}$$

This agrees with the result of Zvezdin and Zyryanov.[3] Let us note the value of the cutoff electric field, say, for the case of giant oscillations $\alpha_0L_M \sim 2/\pi n_F^{1/2}$, assuming $L_M = 10^{-6}$ cm ($H = 10^5$ Oe) and $\epsilon_F = 10^{-12}$ eV, we obtain $E \geqslant 10^{-2}$ V/cm. Such fields still do not cause heating of the conduction electrons, and therefore the effective temperature of the electrons can be regarded as equal to the lattice temperature. The sum J_2 also contributes to the oscillating part, but since the lower limit of the integral lies beyond the oscillating part of the Hermite function for all the terms, it is easy to show that the contribution from this entire sum is much smaller than J_1.

Analogous calculations can be made also for the rate of the spin-lattice relaxation of the nuclei by the conduction electrons. From the usual formula for the relaxation rate of nuclei with spin $1/2$[7] we obtain

$$1/T_1 = bF^2, \qquad b = 8\pi^3k_BTN^2\gamma_e^2\gamma_I^2\hbar^3u_N^4(0)\,\varepsilon_F^{-3}\hbar\omega, \tag{16}$$

where F denotes the sum in formula (7), which determines the Knight shift.

Using the same approximations as above, we arrive at the equation

$$\frac{1}{T_1} \approx b\left\{\frac{n_F}{2} - k_0L_Mn_F\left(\frac{n_F}{2}\right)^{1/2} + \frac{2}{\pi}\left(\frac{n_F^{1/2}}{k_0L_M}\right)^{1/2}\right\}. \tag{17}$$

The first term determines here the Korringa relaxation rate, and the second gives the monotonic dependence of the relaxation rate on the electric field, while the third gives the amplitude of the oscillations of the relaxation rate under the condition

$$E > \alpha_0^2\hbar^2\omega/4e(\varepsilon_Fm)^{1/2}.$$

The foregoing numerical estimates for the case of giant oscillations shows that the dependence of the oscillation amplitude of the Knight shift on the electric field can be observed experimentally.

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