Scattering and Transformation of Waves by Fluctuations in a Nonequilibrium Plasma in a Semiconductor with an Anisotropic Energy Band

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It is shown that in a nonequilibrium plasma in a semiconductor with a complex dispersion law for the energy carriers, the scattering (transformation) cross section of the waves depends strongly on the shape of the energy bands and the field orientation. For certain directions of the fields (external, incident and scattered wave) the scattering cross section may assume very high values; this may be ascribed to the appearance of anomalously large fluctuations in the nonequilibrium plasma for these field orientations. Moreover, account of divergence of the energy dispersion law from an isotropic parabolic law and the presence of a heating field result in the appearance of a specific current which induces additional scattering (transformation of the waves). An estimate shows that for experimentally easily attainable field strengths, this additional contribution to the scattering (transformation) coefficient of the waves may appreciably exceed the usually considered contribution, which is related only to fluctuations of the carrier concentration in the plasma. Expressions for the scattering and transformation coefficients of the incident transverse wave by fluctuation waves in semiconductors with weak anisotropic bands of the heavy hole type in p-Ge are obtained for field orientations for which the fluctuation waves may be divided strictly into longitudinal and transverse waves.

1. INTRODUCTION

INVESTIGATION of the scattering and transformation of electromagnetic waves by fluctuation oscillations has been used successfully for a long time for the diagnostics of gas-discharge plasmas. The availability of powerful lasers in the infrared band has made it possible to perform similar experiments[1,2] also for semiconductor plasma diagnostics. The intensity of the scattered waves is determined both by the fluctuation level and by the intensity of the incident wave. In the case of a nonequilibrium plasma with a high level of fluctuations, the cross sections for the scattering and transformation of the waves can become quite large.

In strong electric fields, a solid-state plasma can deviate strongly from equilibrium (especially in materials with high carrier mobility). In semiconductors in which the carrier energy dispersion is not isotropic and parabolic, both the frequencies and the decrements of the natural oscillations become dependent on the magnitude and orientation of the external electric field[3,4]. It has turned out, in particular, that in semiconductors with anisotropic dispersion, such as p-Ge, the damping decrement of the plasma waves may even reverse sign in relatively easily attainable fields (at definite orientations of the heating field). This results in anomalous fluctuations, strongly anisotropic in magnitude, at frequencies close to the natural frequency[5]. Since in the equilibrium state the magnitude and the character of the fluctuations in a semiconductor plasma depend strongly on the scattering mechanisms and on the shapes of the energy bands, an investigation of the scattering and transformation of waves by these fluctuations can yield valuable information both concerning the mechanism of scattering of hot carriers and concerning the details of the shapes of the energy bands.

The influence of the law of carrier-energy dispersion on the scattering of light in an equilibrium semiconductor plasma was investigated in detail by Wolff for a nonparabolic but isotropic band[6]. It was shown that the nonparabolicity of the band affects strongly the single-particle scattering of light (due to the interaction between the light wave and individual carriers). In particular, the light-scattering coefficient acquires an additional term (due to the nonparabolicity of the band), whose magnitude can greatly exceed the coefficient of single-particle scattering in the case of a simple parabolic band. As to the scattering of light by collective (plasma) oscillations, in the absence of an external magnetic field the influence of the band nonparabolicity reduces to a slight correction connected with the change of the carrier effective mass. In a magnetic field, the nonparabolicity of the band leads to an additional scattering of light, due to collective plasma oscillations (Bernstein oscillations).

The present paper is devoted to an investigation of the influence of the law of energy dispersion on the scattering of light by collective oscillations of a nonequilibrium semiconductor plasma. It is shown that the deviation of the energy dispersion law from isotropic and parabolic, in conjunction with the action of an external electric field (which heats the carriers), leads to the appearance of qualitatively new effects (not previously considered). Thus, in the case considered by us the cross section for the scattering of light is essentially anisotropic and can become anomalously large for a definite value and orientation of the external field. Changes in the value and orientation of the heating field also cause shifts in the positions of the Raman satellites. Furthermore, the scattering coefficient acquires an additional term due to the joint action of the external field and the deviation of the energy dispersion law from isotropic and parabolic. The aforementioned effects do not occur in an equilibrium plasma.

2. FORMULATION OF PROBLEM AND INITIAL EQUATIONS

We consider a homogeneous semiconductor with one type of carrier, whose energy ϵ has a dispersion law different from the isotropic parabolic law $\epsilon = p^2/2m$ (p is the carrier momentum and m is its effective mass). Assume that an external carrier-heating constant electric field $\mathbf{F}$ is applied to such a semiconductor. The carrier momentum distribution function $f(\mathbf{p})$ in the constant field $\mathbf{F}$ is assumed known (the choice of the explicit form of $f(\mathbf{p})$ will be discussed later on). We consider the scattering and transformation of an external incident wave, the field of which we represent in the form

$$\mathbf{E}^0(\mathbf{r}, t) = \mathbf{E}^0 \cos(\omega^0 t - \mathbf{k}^0\mathbf{r}), \tag{1}$$

by the fluctuations in the nonequilibrium plasma. In (1), ω^0 is the frequency and $\mathbf{k}^0$ the wave vector of the incident wave. The fields $\mathbf{E}^0$, $\delta\mathbf{E}$, and $\mathbf{E}'$ of the incident, fluctuating, and scattered waves are assumed to be small compared with the constant field $\mathbf{F}$. The field determining the total carrier distribution function can be written in the form of the sum

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{F} + \mathbf{E}^0(\mathbf{r}, t) + \delta\mathbf{E}(\mathbf{r}, t) + \mathbf{E}'(\mathbf{r}, t). \tag{2}$$

The total carrier momentum distribution function $\chi(\mathbf{p})$ can be written in the form of an analogous sum:

$$\chi(\mathbf{p}, \mathbf{r}, t) = f(\mathbf{p}) + \varphi^0(\mathbf{p}, \mathbf{r}, t) + \delta\varphi(\mathbf{p}, \mathbf{r}, t) + \varphi'(\mathbf{p}, \mathbf{r}, t). \tag{3}$$

Here φ^0, $\delta\varphi$ and φ' are increments to the distribution function in the constant field $f(\mathbf{p})$, and are due to $\mathbf{E}^0$, $\delta\mathbf{E}$, and $\mathbf{E}'$, respectively. Assuming $\delta\mathbf{E}$ and $\mathbf{E}^0$ to be small quantities, we determine φ^0, $\delta\varphi$, and φ', as usual[7,8], by the method of successive approximations. The field of the scattered wave (which is proportional to the product of $\mathbf{E}^0$ and $\delta\mathbf{E}$) will then be a quantity of second order of smallness, i.e., $|\mathbf{E}'| \ll |\mathbf{E}^0|, |\delta\mathbf{E}|$. In addition, it is obvious that $|\varphi'| \ll |\varphi^0|, |\delta\varphi|$.

Substituting the distribution function in the form (3) and the field in the form (2) in the kinetic equation, and gathering terms of equal order of smallness, we obtain the following system of equations, in the Fourier representation, for the determination of φ^0, $\delta\varphi$, and φ':

$$\hat{L}_{\mathbf{k},\omega}\varphi^0(\mathbf{k}, \omega) = -e_0\mathbf{E}^0(\mathbf{k}, \omega)\hat{\mathbf{K}}(\mathbf{k}, \omega)f(\mathbf{p}), \tag{4}$$

$$\hat{L}_{\mathbf{k},\omega}\delta\varphi(\mathbf{k}, \omega) = -e_0\delta\mathbf{E}(\mathbf{k}, \omega)\hat{\mathbf{K}}(\mathbf{k}, \omega)f(\mathbf{p}) + y(\mathbf{p}, \mathbf{k}, \omega), \tag{5}$$

$$\hat{L}_{\mathbf{k},\omega}\varphi'(\mathbf{k}, \omega) = -e_0\mathbf{E}'(\mathbf{k}, \omega)\hat{\mathbf{K}}(\mathbf{k}, \omega)f(\mathbf{p}) - \frac{e_0}{(2\pi)^4}\int d\mathbf{k}_1 d\omega_1 \times\{\mathbf{E}^0(\mathbf{k}_1, \omega_1)\hat{\mathbf{K}}(\mathbf{k}_1, \omega_1)\delta\varphi(\mathbf{k}-\mathbf{k}_1, \omega-\omega_1) + \delta\mathbf{E}(\mathbf{k}_1, \omega_1)\hat{\mathbf{K}}(\mathbf{k}_1, \omega_1)\varphi^0(\mathbf{k}-\mathbf{k}_1, \omega-\omega_1)\}. \tag{6}$$

Here

$$\hat{L}_{\mathbf{k},\omega} \equiv i(\omega - \mathbf{k}\mathbf{v}) + e_0\mathbf{F}\frac{\partial}{\partial\mathbf{p}} - \hat{\nu}(\mathbf{p}), \tag{7}$$

$$\hat{\mathbf{K}}(\mathbf{k}, \omega) \equiv \left(1 - \frac{\mathbf{k}\mathbf{v}}{\omega}\right)\frac{\partial}{\partial\mathbf{p}} + \mathbf{v}\left(\frac{\mathbf{k}}{\omega}\frac{\partial}{\partial\mathbf{p}}\right), \tag{8}$$

e_0 is the elementary charge, $\hat{\nu}(\mathbf{p})$ is the collision operator, $y(\mathbf{P}, \mathbf{k}, \omega)$ are random forces usually introduced into the right-hand side of the kinetic equation when fluctuations are investigated, and $\mathbf{v} = \partial\epsilon/\partial\mathbf{p}$ is the carrier velocity. On going over to the Fourier representation, we have eliminated the magnetic fields by using their connection with the electric fields via the Maxwell equations. The functions φ^0, $\delta\varphi$, and φ' (and also $\hat{L}$ and $\hat{\mathbf{K}}$) depend, of course, not only on $\mathbf{k}$ and ω, but also on $\mathbf{p}$, but for brevity we do not write this argument out. This should not lead to misunderstandings.

The integral in the right-hand side of (6) can be calculated easily if it is recognized that the quantities $\mathbf{E}^0(\mathbf{k}, \omega)$ can be represented, using (1) and (4), in the form

$$\mathbf{E}^0(\mathbf{k}, \omega) = \tfrac{1}{2}(2\pi)^4\{\delta(\omega+\omega^0)\delta(\mathbf{k}+\mathbf{k}^0) + \delta(\omega-\omega^0)\delta(\mathbf{k}-\mathbf{k}^0)\}\mathbf{E}^0, \tag{9}$$

$$\varphi^0(\mathbf{k}, \omega) = \tfrac{1}{2}(2\pi)^4\{\delta(\omega+\omega^0)\delta(\mathbf{k}+\mathbf{k}^0) + \delta(\omega-\omega^0)\delta(\mathbf{k}-\mathbf{k}^0)\}\tilde{\varphi}^0(\mathbf{k}, \omega). \tag{10}$$

The function $\tilde{\varphi}^0(\mathbf{k}, \omega)$ satisfies in this case the equation

$$\hat{L}_{\mathbf{k},\omega}\tilde{\varphi}^0(\mathbf{k}, \omega) = -e_0\mathbf{E}^0\hat{\mathbf{K}}(\mathbf{k}, \omega)f(\mathbf{p}). \tag{11}$$

After evaluating the integral in the right-hand side of (6), the latter takes the form

$$\hat{L}_{\mathbf{k},\omega}\varphi'(\mathbf{k}, \omega) = -e_0\mathbf{E}'(\mathbf{k}, \omega)\hat{\mathbf{K}}(\mathbf{k}, \omega)f(\mathbf{p}) - \tfrac{1}{2}\{\Psi(\mathbf{k}, \omega; \mathbf{k}^0, \omega^0) + \Psi(\mathbf{k}, \omega; -\mathbf{k}^0, -\omega^0)\}, \tag{12}$$

where

$$\Psi(\mathbf{k}, \omega, \mathbf{k}^0, \omega^0) \equiv e_0\mathbf{E}^0\hat{\mathbf{K}}(\mathbf{k}^0, \omega^0)\delta\varphi(\mathbf{k}+\mathbf{k}^0, \omega+\omega^0) + e_0\delta\mathbf{E}(\mathbf{k}+\mathbf{k}^0, \omega+\omega^0)\hat{\mathbf{K}}(\mathbf{k}+\mathbf{k}^0, \omega+\omega^0)\tilde{\varphi}^0(-\mathbf{k}^0, -\omega^0). \tag{13}$$

We have used real quantities throughout, both in the description of the fields and in the description of the distribution function. Therefore the Fourier components of all the quantities satisfy a relation of the type $\delta\mathbf{E}^*(\mathbf{k}, \omega) = \delta\mathbf{E}(-\mathbf{k}, -\omega)$ (the asterisk denotes the complex conjugate). If it is recognized furthermore that $\hat{L}^*_{-\mathbf{k},-\omega} = \hat{L}_{\mathbf{k},\omega}$ and $\hat{\mathbf{K}}(\mathbf{k}, \omega) = \hat{\mathbf{K}}(-\mathbf{k}, -\omega)$, then (13) leads to the relations

$$\Psi(-\mathbf{k}, -\omega; \mathbf{k}^0, \omega^0) = \Psi^*(\mathbf{k}, \omega; -\mathbf{k}^0, -\omega^0),$$
$$\Psi(-\mathbf{k}, -\omega; -\mathbf{k}^0, -\omega^0) = \Psi^*(\mathbf{k}, \omega; \mathbf{k}^0, \omega^0), \tag{14}$$

which we shall need later on.

The vector components of the current connected with the function $\varphi'(\mathbf{k}, \omega)$ can be represented in the form

$$j_\alpha'(\mathbf{k}, \omega) \equiv e_0\int\frac{\partial\varepsilon}{\partial p_\alpha}\varphi'(\mathbf{k}, \omega)d\mathbf{p} = \sum_\beta \sigma_{\alpha\beta}(\mathbf{k}, \omega)E_\beta'(\mathbf{k}, \omega) + J_\alpha'(\mathbf{k}, \omega), \tag{15}$$

where

$$\sigma_{\alpha\beta}(\mathbf{k}, \omega) = -e_0^2\int\frac{\partial\varepsilon}{\partial p_\alpha}\hat{L}^{-1}_{\mathbf{k},\omega}\hat{K}_\beta(\mathbf{k}, \omega)f(\mathbf{p})d\mathbf{p} \tag{16}$$

is the electric-conductivity tensor, which makes the usual contribution to the dielectric tensor, and the current

$$J_\alpha'(\mathbf{k}, \omega) = -\tfrac{1}{2}e_0\int\frac{\partial\varepsilon}{\partial p_\alpha}\hat{L}^{-1}_{\mathbf{k},\omega}\{\Psi(\mathbf{k}, \omega; \mathbf{k}^0, \omega^0) + \Psi(\mathbf{k}, \omega; -\mathbf{k}^0, -\omega^0)\}d\mathbf{p} \tag{17}$$

causes scattering (transformation) of the incident wave. Substituting (15) in Maxwell's equations, we obtain the connection between $\mathbf{J}'(\mathbf{k}, \omega)$ and the field of the scattered wave $\mathbf{E}'(\mathbf{k}, \omega)$:

$$\hat{\Lambda}\mathbf{E}'(\mathbf{k}, \omega) = \frac{4\pi i}{\omega}\mathbf{J}'(\mathbf{k}, \omega). \tag{18}$$

Here $\hat{\Lambda}$ is a tensor whose matrix elements are

$$\Lambda_{\alpha\beta}(\mathbf{k}, \omega) = \left(\frac{ck}{\omega}\right)^2\left(\frac{k_\alpha k_\beta}{k^2} - \delta_{\alpha\beta}\right) + \varkappa_{\alpha\beta}(\mathbf{k}, \omega), \tag{19}$$

$\kappa_{\alpha\beta}$ is the dielectric tensor given by

$$\varkappa_{\alpha\beta}(\mathbf{k},\omega)=\varkappa_0\delta_{\alpha\beta}+\frac{4\pi}{i\omega}\sigma_{\alpha\beta}(\mathbf{k},\omega) \tag{20}$$

and κ_0 is the dielectric constant of the crystal without allowance for the contribution made to it by the free carriers.

If the fluctuations are calculated, then Eqs. (18) with allowance for (17) describe all the wave scattering and transformation processes in the presence of a constant field and for an arbitrary carrier energy dispersion law.

3. CALCULATION OF THE CURRENT PRODUCING THE SCATTERED WAVES

To investigate further the wave scattering and transformation processes it is necessary to have an explicit expression for the current $\mathbf{J}'(\mathbf{k},\omega)$ that causes the wave scattering. To obtain this expression it is necessary to determine $\delta\varphi(\mathbf{k},\omega)$ and $\widetilde{\varphi}^0(\mathbf{k},\omega)$ from (5) and (11), respectively, and substitute them in (12), after which the last equation is solved. The formal solution of (5) (and also of (11) can easily be written in the form

$$\delta\varphi(\mathbf{k},\omega)=\hat{L}_{\mathbf{k},\omega}^{-1}\{-e_0\delta\mathbf{E}(\mathbf{k},\omega)\hat{\mathbf{K}}(\mathbf{k},\omega)f(\mathbf{p})+y(\mathbf{p},\mathbf{k},\omega)\}. \tag{21}$$

In analogy with (15), we obtain for the current connected with the fluctuation wave

$$\delta\mathbf{j}(\mathbf{k},\omega)=\hat{\sigma}(\mathbf{k},\omega)\delta\mathbf{E}(\mathbf{k},\omega)+\mathbf{j}^{(0)}(\mathbf{k},\omega), \tag{22}$$

where $\hat{\sigma}(\mathbf{k},\omega)$ is the conductivity tensor whose components are defined by (16), and

$$\mathbf{j}^{(0)}(\mathbf{k},\omega)\equiv e_0\int\frac{\partial\varepsilon}{\partial\mathbf{p}}\hat{L}_{\mathbf{k},\omega}^{-1}y(\mathbf{p},\mathbf{k},\omega)\,d\mathbf{p} \tag{23}$$

is the random-current vector. We shall need these expressions later on.

Now we proceed to the actual rather than formal determination of the solutions of (5) and (11), i.e., we determine the explicit form of $\hat{L}_{\mathbf{k},\omega}^{-1}$. To this end, we impose certain limitations on the region of the investigated frequencies and employed fields. We assume the following inequalities to be satisfied:

$$\frac{e_0F}{\bar{p}\omega}\ll 1,\quad \frac{|\hat{\nu}f(\mathbf{p})|}{\omega f(\mathbf{p})}\approx\frac{\nu}{\omega}\ll 1. \tag{24}$$

Here $\bar{p}$ is the average value of the momentum and ν is the effective collision frequency. We note that when the second inequality is satisfied, the limitation imposed on the field $\mathbf{F}$ by the first inequality is quite mild. In particular, if this inequality is satisfied the field $\mathbf{F}$ can still be so strong that the diffusion approximation can no longer be used to determine $f(\mathbf{p})$.

In the present paper we consider low-resistance semiconductors, when the frequency of the Langmuir oscillations is $\omega_p\gg\nu$. The second condition is therefore quite natural for the electromagnetic wave scattering (transformation) problem considered here (in addition to making the calculations easier), since only waves with $\omega>\omega_p$ penetrate into the plasma. In this approximation, accurate to terms of first order of smallness, we have

$$\hat{L}_{\mathbf{k},\omega}^{-1}\approx -i\frac{1}{\omega-\mathbf{k}\mathbf{v}}+\frac{1}{\omega-\mathbf{k}\mathbf{v}}\left[e_0\mathbf{F}\frac{\partial}{\partial\mathbf{p}}-\hat{\nu}\right]\frac{1}{\omega-\mathbf{k}\mathbf{v}}. \tag{25}$$

With the aid of (25) we can easily obtain $\delta\varphi$ from (21) and determine in smaller fashion $\widetilde{\varphi}^0$ from (11).

Knowing these quantities, we obtain from (13) and (17) the Fourier components of the current $\mathbf{J}'(\mathbf{k},\omega)$. In addition to assuming that the parameters (24) are small, we also assume that the phase velocities of the incident, fluctuation, and scattered (transformed) waves are much larger than the average electron velocity, i.e., that for all these waves we have

$$k\bar{v}/\omega\ll 1. \tag{26}$$

For the waves considered here, this inequality is well satisfied almost always.

We write the carrier energy dispersion law in the form

$$\varepsilon(\mathbf{p})=p^2/2m+\Delta\varepsilon(\mathbf{p})\equiv\varepsilon_0(p)+\Delta\varepsilon(\mathbf{p}). \tag{27}$$

The first term here describes an averaged isotropic parabolic band with a certain effective mass m (for details see the next section), and $\Delta\epsilon(\mathbf{p})$ characterizes the deviation from this band, which we assume to be small. p-Ge and p-Si are examples of such a situation. We now substitute in (17) the expression (25) as well as $\delta\varphi$ and $\widetilde{\varphi}^0$ (determined in the manner indicated above), and then expand in (17) in terms of the parameter (26). We retain only the linear terms in the parameters (26) and (24), and neglect terms of higher orders. In addition, when integrating with respect to $\mathbf{p}$ in the terms that are linear in the small parameters (26) and (24), we shall assume that $\epsilon(\mathbf{p})=p^2/2m$. As a result we obtain

$$J_\alpha'(\mathbf{k},\omega)=-{}^1/_2\{J_\alpha^{(+)}(\mathbf{k},\omega)+J_\alpha^{(-)}(\mathbf{k},\omega)\}, \tag{28}$$

$$\begin{aligned}J_\alpha^{(-)}(\mathbf{k},\omega)=&\frac{e_0^3}{\Delta\omega^{(-)}\omega^0}\int d\mathbf{p}f(\mathbf{p})\left(\mathbf{E}^0\frac{\partial}{\partial\mathbf{p}}\right)\cdot\left(\delta\mathbf{E}(\mathbf{q}^{(-)},\Delta\omega^{(-)})\frac{\partial}{\partial\mathbf{p}}\right)v_\alpha(\mathbf{p})\\&+\frac{e_0^3n}{m^2\omega^0(\Delta\omega^{(-)})^2}\Big\{E_\alpha^0(\delta\mathbf{E}(\mathbf{q}^{(-)},\Delta\omega^{(-)}),\mathbf{q}^{(-)})\\&+\frac{\Delta\omega^{(-)}}{\omega}k_\alpha(\delta\mathbf{E}(\mathbf{q}^{(-)},\Delta\omega^{(-)})\mathbf{E}^0)\\&+\frac{\Delta\omega^{(-)}}{\omega^0}\delta E_\alpha(\mathbf{q}^{(-)},\Delta\omega^{(-)})(\mathbf{k}^0\mathbf{E}^0)\Big\};\\&\Delta\omega^{(\pm)}\equiv\omega\pm\omega^0,\quad \mathbf{q}^{(\pm)}\equiv\mathbf{k}\pm\mathbf{k}^0.\end{aligned} \tag{29}$$

We note that (29) was derived without making any assumptions at all concerning the form of the function $f(\mathbf{p})$, except for the usual assumption that it be normalized to the total carrier density n:

$$\int f(\mathbf{p})\,d\mathbf{p}=n. \tag{30}$$

The terms in (17) which are linear in the operator $e_0\mathbf{F}\partial/\partial\mathbf{p}-\hat{\nu}$ are exactly equal to zero for the average band.

When substituting (21) in (17) we have left out, as usual[7], the term containing $y(\mathbf{p},\mathbf{k},\omega)$. The validity of such an approximation will be discussed in somewhat greater detail later.

The expressions obtained by us for the Fourier components of the current producing the wave scattering differ from the analogous expression for a gas-discharge plasma[8] in that (29) contains the first term, which vanishes for the simple dispersion law $\epsilon=p^2/2m$. As will be shown below, it vanishes also when the constant field $\mathbf{F}$ tends to zero. Thus, the appearance of this term (the additional term in (29)) is due entirely to the deviation of the shape of the energy band from isotropic and parabolic in the nonequilibrium plasma[1)] (due to the presence of the field $\mathbf{F}$).

Since we are interested here in the Raman scattering of waves (with change of frequency of the scattered wave by an amount equal to the frequency of the natural oscillations of the plasma) by the fluctuations in a nonequilibrium plasma in a semiconductor, we shall stop briefly at the end of this section to discuss the character of these fluctuations near the natural frequencies. This question was considered in greater detail earlier[5].

The fluctuating fields $\delta\mathbf{E}(\mathbf{r}, t)$ and $\delta\mathbf{H}(\mathbf{r}, t)$ and the current $\delta\mathbf{j}(\mathbf{r}, t)$ are of statistical origin, so that we are interested not in the exact details of these quantities but in the statistical mean values of the products of the fluctuating quantities taken at different points of space and time (correlators). In particular, such important characteristics as the energy density and flux of the fluctuating fields are expressed in the form of products of fluctuating quantities.

In the calculation of the correlators by the kinetic-equation method, the statistical-averaging operation is determined completely specifying the correlator of the random forces $y(\mathbf{p}, \mathbf{r}, t)$. In a nonequilibrium system and for quasiclassical fluctuations, the correlator of the random forces can be represented in the form[9]

$$\langle y(\mathbf{p}, \mathbf{r}, t) y(\mathbf{p}', \mathbf{r}', t')\rangle = \delta(\mathbf{r}-\mathbf{r}')\delta(t-t')g(\mathbf{p}, \mathbf{p}'). \qquad (31)$$

Here $\langle \ldots \rangle$ denotes the operation of statistical averaging.

In the high-frequency case of interest to us ($\omega \gg \nu$), when the correlations connected with the pair interaction are negligible, the function $g(\mathbf{p}, \mathbf{p}')$ is given by

$$g(\mathbf{p},\mathbf{p}') = \delta(\mathbf{p}-\mathbf{p}')\left\{\int w(\mathbf{p},\mathbf{p}'')f(\mathbf{p})\,d\mathbf{p}'' + \int w(\mathbf{p}'',\mathbf{p})f(\mathbf{p}'')\,d\mathbf{p}''\right\} - w(\mathbf{p},\mathbf{p}')f(\mathbf{p}) - w(\mathbf{p}',\mathbf{p})f(\mathbf{p}'), \qquad (32)$$

where $w(\mathbf{p}, \mathbf{p}')$ is the probability of transition from the state $\mathbf{p}$ to the state $\mathbf{p}'$ per unit time.

Using (23) and (31), we can readily obtain the correlator of the Fourier component of the random-current vector

$$\langle j_\alpha^{(0)*}(\mathbf{k},\omega) j_\beta^{(0)}(\mathbf{k}',\omega')\rangle = (2\pi)^4\delta(\omega-\omega')\delta(\mathbf{k}-\mathbf{k}')\langle j_\alpha^{(0)} j_\beta^{(0)}\rangle_{\mathbf{k},\omega}. \qquad (33)$$

The spectral distribution of the random currents $\langle j_\alpha^{(0)} j_\beta^{(0)}\rangle_{\mathbf{k},\omega}$, neglecting small quantities of the order of (24) and (26), is of the form

$$\langle j_\alpha^{(0)} j_\beta^{(0)}\rangle_{\mathbf{k},\omega} \approx \frac{e_0^2}{\omega^2}\iint \frac{\partial\varepsilon}{\partial p_\alpha}\frac{\partial\varepsilon}{\partial p_\beta'} g(\mathbf{p},\mathbf{p}')\,d\mathbf{p}\,d\mathbf{p}'. \qquad (34)$$

If the fluctuation current (22) is substituted in Maxwell's equations, then we obtain for $\delta\mathbf{E}(\mathbf{k}, \omega)$ an equation that differs from (18) only in that $\mathbf{J}'(\mathbf{k}'\omega)$ is replaced by $\mathbf{j}^{(0)}(\mathbf{k}, \omega)$. The correlator of the Fourier component of the field $\delta\mathbf{E}$ can therefore also be represented in a form analogous to (33) with a field-fluctuation spectral distribution given by

$$\langle \delta E_\alpha \delta E_\beta\rangle_{\mathbf{k},\omega} = \frac{16\pi^2}{\omega^2}\sum_{\gamma,\delta=1}^{3}(\Lambda_{\alpha\delta}^{-1})^*\Lambda_{\beta\gamma}^{-1}\langle j_\gamma^{(0)} j_\delta^{(0)}\rangle_{\mathbf{k},\omega}. \qquad (35)$$

[1)]Additional terms of similar nature, as shown in[3,4], cause additional amplification (or quenching) of the natural oscillations of a nonequilibrium semiconductor plasma.

Here $\Lambda_{\alpha\beta}^{-1}$ are the matrix elements of the inverse of the tensor $\hat{\Lambda}$.

It follows from (35), in particular, that near the natural frequencies the spectral distribution of the fluctuations of the longitudinal field takes the form[5]

$$\langle \delta E_\parallel^2\rangle_{\mathbf{k},\omega} = \frac{8\pi^2\omega}{\varkappa_0\gamma}\langle j_\parallel^{(0)2}\rangle_{\mathbf{k},\omega}\delta(\omega^2-\omega_l^2(k)), \qquad (36)$$

where $\omega_l(\mathbf{k})$ is the natural frequency of the longitudinal oscillations and γ is the damping decrement.

We see from this that the spectral density has a peak near the natural frequencies, and that the intensity of the peak increases with decreasing damping decrement of the oscillations.

We have already mentioned that when we substituted (21) in (17) when deriving the expression for the current that causes the wave scattering, we have omitted from (21) the term containing $y(\mathbf{p}, \mathbf{k}, \omega)$. It is easy to see that if we retain this term, then an additional term, equal to

$$-\frac{e_0^2}{\omega\Delta\omega^{(\pm)}}\int\frac{\partial\varepsilon_0}{\partial p_\alpha}\left(\mathbf{E}^0\frac{\partial}{\partial\mathbf{p}}\right)y(\mathbf{p},\mathbf{q}^{(\pm)},\Delta\omega^{(\pm)})\,d\mathbf{p} = \frac{e_0^2E_\alpha^0}{m\omega\Delta\omega^{(\pm)}}\int y(\mathbf{p},\mathbf{q}^{(\pm)},\Delta\omega^{(\pm)})\,d\mathbf{p},$$

appears in the current $J_\alpha^{(\pm)}(\mathbf{k}, \omega)$, as well as terms proportional to the small parameters (such as (26)). Using the explicit form of $g(\mathbf{p}, \mathbf{p}')$ from (32), we can easily show that the correlators of this term with all the terms of (29) and with itself vanish identically, since $\int g(\mathbf{p}, \mathbf{p}')d\mathbf{p}' = \int g(\mathbf{p}', \mathbf{p})d\mathbf{p} = 0$ (conservation of the number of particles in the collisions). Neglect of the terms which have not been written out and which are proportional to the small parameters does not introduce a noticeable error, since their correlators near the natural frequencies are small in comparison with the correlators of the fluctuating fields having a δ-like singularity in this frequency region (see, for example, (36)).

4. CROSS SECTION FOR THE SCATTERING AND TRANSFORMATION OF WAVES

The intensity of wave scattering (transformation) is characterized by the scattering (transformation) cross section, defined by the equation

$$\Sigma = \bar{\mathscr{P}}(VS_0)^{-1}. \qquad (37)$$

Here $\bar{\mathscr{P}}$ is the average energy increment of the scattered (transformed) waves per unit time, V is the value of the scattering volume, and S_0 is the energy flux density of the incident wave. The average scattered (transformed) power is

$$\bar{\mathscr{P}} = -\int d\mathbf{r}\langle \mathbf{J}'(\mathbf{r},t)\mathbf{E}'(\mathbf{r},t)\rangle = -(2\pi)^{-5}\,\mathrm{Re}\int d\mathbf{k}\,d\omega\,d\omega'\,\langle \mathbf{J}'(\mathbf{k},\omega')\mathbf{E}'^*(\mathbf{k},\omega)\rangle\exp[-i(\omega'-\omega)t]. \qquad (38)$$

Let us write out (38) in greater detail for the case when the scattered wave is transverse. In this case it follows from (18) that

$$\mathbf{E}_\perp'(\mathbf{k},\omega) = \frac{4\pi i}{\omega}\mathbf{J}_\perp'(\mathbf{k},\omega)\left[\varkappa_\perp(\mathbf{k},\omega) - \left(\frac{ck}{\omega}\right)^2\right]^{-1}. \qquad (39)$$

Substituting (39) in (38), we obtain

$$\overline{\mathscr{P}}_{tr} = -\frac{2}{(2\pi)^4}\,\mathrm{Im}\int \frac{d\mathbf{k}\,d\omega\,d\omega'}{\omega}\,\frac{\langle \mathbf{J}_\perp'(\mathbf{k},\omega)\mathbf{J}_\perp'(\mathbf{k},\omega')\rangle}{\varkappa_\perp^*(\mathbf{k},\omega)-(ck/\omega)^2}\exp[-i(\omega'-\omega)t]. \tag{40}$$

We have noted earlier that the correlators of the Fourier components of the fluctuating field can be represented in a form analogous to (33), with a spectral density (35). Taking this circumstance into account, it follows from (29) that we can write

$$\langle \mathbf{J}_\perp^{(-)}(\mathbf{k},\omega)\mathbf{J}_\perp^{(-)}(\mathbf{k}',\omega')\rangle = (2\pi)^4\delta(\omega-\omega')\delta(\mathbf{k}-\mathbf{k}')\langle|J_\perp^{(-)}|^2\rangle_{\mathbf{k},\omega}. \tag{41}$$

Analogous expressions are obtained for the correlator of the current $\mathbf{J}_\perp^{(+)}$. As to the crossing correlators of $\mathbf{J}_\perp^{(-)}$ and $\mathbf{J}_\perp^{(+)}$, they contain the factors $\delta(\omega' - \omega \pm 2\omega^0)$.

After substituting (28) in (40), it is necessary to average over the period of the oscillations of the incident wave, since the change occurs within a time much longer than $(\omega_0)^{-1}$, so that we are interested in the power averaged over the period of the oscillations. Assuming that such an averaging has been carried out, and recognizing that $\delta(\mathbf{k} - \mathbf{k}') = V/(2\pi)^3$, we obtain from (40) after integrating with respect to ω'

$$\overline{\mathscr{P}}_{tr} = -\frac{V}{(2\pi)^3}\,\mathrm{Im}\int\frac{d\mathbf{k}\,d\omega}{\omega}\langle|J_\perp^{(-)}|^2\rangle_{\mathbf{k},\omega}\left[\varkappa_\perp^*(\mathbf{k},\omega)-\left(\frac{ck}{\omega}\right)^2\right]^{-1} \tag{42}$$

In the derivation of (42) we took into account the fact that the contribution from the correlator of $\mathbf{J}_\perp^{(+)}$ coincides fully with contribution from the correlator of $\mathbf{J}_\perp^{(-)}$ (this can easily be verified by replacing $\mathbf{k}$, ω, and ω') in (40) by $-\mathbf{k}$, $-\omega$ and $-\omega'$). Assuming that the waves in question are weakly damped, we can write

$$\mathrm{Im}\left[\varkappa_\perp^*(\mathbf{k},\omega)-\left(\frac{ck}{\omega}\right)^2\right]^{-1} \approx \pi\delta\left\{\mathrm{Re}\,\varkappa_\perp(\mathbf{k},\omega)-\left(\frac{ck}{\omega}\right)^2\right\}. \tag{43}$$

Taking (43) into account, we can easily carry out the integration modulo $\mathbf{k}$ in (42) by writing $d\mathbf{k} = k^2dkdo$. As a result we obtain

$$\overline{\mathscr{P}}_{tr} = \frac{V}{16\pi^2c^3}\int do\,d\omega\,\omega^2[\mathrm{Re}\,\varkappa_\perp(\mathbf{k},\omega)]^{1/2}\langle|J_\perp^{(-)}|^2\rangle_{\mathbf{k},\omega}\big|_{k=k(\omega)},$$
$$k(\omega) = \omega[\mathrm{Re}\,\varkappa_\perp(\omega)]^{1/2}/c. \tag{44}$$

Having (44), we obtain an expression for the differential cross section (scattering coefficient) of the scattering

$$d\Sigma_{tr} = d\overline{\mathscr{P}}_{tr}(VS_0)^{-1} = \frac{S_0^{-1}\omega^2}{16\pi^2c^3}[\mathrm{Re}\,\varkappa_\perp(\mathbf{k},\omega)]^{1/2}\langle|J_\perp^{(-)}|^2\rangle_{\mathbf{k},\omega}\big|_{k=k_0}\,d\omega\,do, \tag{45}$$

$k_0 = k(\omega)$. We obtain in similar fashion an expression for the scattering coefficient in the case when the scattered wave is longitudinal (wave transformation):

$$d\Sigma_l = d\overline{\mathscr{P}}_l(VS_0)^{-1} = \frac{S_0^{-1}}{8\pi^2}\sum_s\langle|J_l^{(-)}|^2\rangle_{\mathbf{k},\omega}\frac{k_s^2}{\omega}\left|\frac{d\varkappa_l(k,\omega)}{dk}\right|^{-1}_{k=k_s}d\omega\,do, \tag{46}$$

where $k_s = k_s(\omega)$ are the roots of the dispersion equation of $\kappa_l(\mathbf{k}, \omega)$. The summation over s is carried out over all the roots of this equation.

We now have all the formulas needed to analyze concrete situations. We confine ourselves here to p-type germanium. The law of energy dispersion in p-Ge can be represented for the heavy holes in the form[2)]

$$\varepsilon(\mathbf{p}) = \frac{p^2}{2m}+\frac{C^2}{8m_0B'p^2}\sum_\alpha\left(p_\alpha^4-\frac{1}{5}p^4\right) \equiv \frac{p^2}{2m}+\Delta\varepsilon(\mathbf{p}), \tag{47}$$

where $m^{-1} = (A - B')m_0^{-1}$, $B' = (B^2 + \frac{1}{5}C^2)^{1/2}$, m_0 is the mass of the free electron, and A, B, and C are known constants.

To proceed with the calculations we must specify $f(\mathbf{p})$. We assume that the kinetic equation in a constant field $\mathbf{F}$ has been solved, and $f(\mathbf{p})$ is known. In the literature, extensive use is made of an approximation of $f(\mathbf{p})$ in the form of a momentum-shifted Maxwellian function with temperature T. Such an approximation agrees with the direct measurement of the hole distribution function in p-Ge[10,11]. It also agrees with the theoretical considerations [12], if one deals with atomic semiconductors in which the lattice mechanisms predominate over the impurity mechanisms of carrier scattering.

We thus write $f(\mathbf{p})$ in the form

$$f(\mathbf{p}) \approx D_n\exp\{-\varepsilon(\mathbf{p}-\mathbf{p}^0)/T\}, \tag{48}$$

where D_n is a normalization constant and T and $\mathbf{p}^0$ are obtained from the energy and momentum conservation laws (T is in energy units). We now substitute (47) and (48) in (29) and obtain the explicit form of the first term in (29), accurate to the terms linear in $\Delta\epsilon(\mathbf{p})$:

$$\int d\mathbf{p}\,f(\mathbf{p})\left(\mathbf{E}^0\frac{\partial}{\partial\mathbf{p}}\right)\left(\delta\mathbf{E}\frac{\partial}{\partial\mathbf{p}}\right)v_\alpha(\mathbf{p}) \approx$$
$$\approx \frac{nC^2}{8m_0B'(2mT)^{1/2}}\left(\mathbf{E}^0\frac{\partial}{\partial\rho}\right)\left(\delta\mathbf{E}\frac{\partial}{\partial\rho}\right)\frac{\partial}{\partial\rho_\alpha}\Phi(\rho); \tag{49}$$

$$\Phi(\rho) = \left\{\frac{35}{4\rho^2}\left[1-\frac{3}{2\rho^2}\left(1-\frac{e^{-\rho^2}}{\rho}\int_0^\rho e^{x^2}dx\right)+\rho^2-\frac{7}{2}\right]\right\}\sum_{\alpha=1}^{3}\left(\frac{\rho_\alpha^4}{\rho^4}-\frac{1}{5}\right)$$
$$= \frac{105}{8}\sum_{k=4}^{\infty}\frac{(-2)^k\rho^{2(k-2)}}{(2k+1)!!}\sum_{\alpha=1}^{3}\left(\frac{\rho_\alpha^4}{\rho^4}-\frac{1}{5}\right) \equiv M(\rho)\sum_{\alpha=1}^{3}\left(\rho_\alpha^4-\frac{1}{5}\rho^4\right),$$
$$\rho = p^0/(2mT)^{1/2}. \tag{50}$$

In a nonequilibrium plasma of a semiconductor with an anisotropic carrier-energy dispersion law, the natural oscillations, as shown in[3,4], do not separate, strictly speaking, into longitudinal and transverse oscillations. This makes the investigation of the wave-scattering processes very complicated. To illustrate the influence of the band shape on the wave scattering and transformation processes we confine ourselves only to wave incidence and scattering directions for which the fluctuation waves causing the given scattered wave can be separated strictly into longitudinal and transverse ones. We thus consider the following situations.

1) The direction of the incident wave, specified by the vector $\mathbf{k}^0$, coincides with the [100] direction of the crystal. The vectors $\mathbf{F} \parallel \mathbf{E}^0$ are directed along [001]. We then obtain from (45) for the coefficient of scattering along the $\mathbf{k}^0$ direction

$$d\Sigma_{tr}^{[001]} = \frac{3\sigma_0}{(4\pi)^4}\left(\frac{m_0}{m}\right)^2\left(\frac{\omega}{\omega^0}\right)^2\left(\frac{\omega_p}{\Delta\omega}\right)^4$$
$$\times\left[\frac{\mathrm{Re}\,\varkappa_\perp(\omega)}{\mathrm{Re}\,\varkappa_\perp(\omega^0)}\right]^{1/2}\left(\frac{\varkappa_0 q}{e_0}\right)^2\langle\delta E_\parallel^2\rangle_{\mathbf{q},\Delta\omega}\,d\omega\,do. \tag{51}$$

The superscript of Σ indicates the direction of polarization of the scattered wave for which the scattering coefficient has been written out. In (51), $\sigma_0 = \frac{8}{3}\pi(e_0^2/m_0c^2)^2$ is the Thomson cross section for scattering of the electromagnetic waves by the free electrons, and $\omega_p^2 = 4\pi ne_0^2/\kappa_0 m$. In addition, we took

2)We neglect the contribution of the light holes.

into account the fact that $S_0 = (c/4\pi)[\mathrm{Re}\,\kappa_\perp(\omega^0)]^{1/2}E^{02}$. For the given concrete case, the formula for the scattering coefficient coincides formally with the analogous expression in a plasma with the standard energy dispersion law $\epsilon = p^2/2m$[13]. The only difference is that the fluctuations $\langle \delta E_\parallel^2\rangle_{q,\Delta\omega}$ have a different character (the details will be discussed later.

2. $E^0 \parallel [100]$; $k^0 \parallel F \parallel [001]$. For wave scattering along the [001] direction (the direction of k) we have

$$d\Sigma_{tr}^{[100]} = \frac{3\sigma_0}{16\pi^2}\left(\frac{\omega}{\omega^0}\right)^2 \left\{\frac{e_0 n C^2 \rho R(\rho)}{20B'(2mT)^{1/2}\Delta\omega} + \frac{e_0 n m_0 q}{m^2(\Delta\omega)^2}\right\}^2 \langle\delta E_\parallel^2\rangle_{q,\Delta\omega}\left[\frac{\mathrm{Re}\,\varkappa_\perp(\omega)}{\mathrm{Re}\,\varkappa_\perp(\omega^0)}\right]^{1/2} d\omega\, do, \qquad (52)$$

$$R(\rho) = -12M(\rho) + 3\rho\frac{dM}{d\rho} + \rho^2\frac{d^2M}{d\rho^2} \approx -\frac{8}{3}\left(1 - \frac{2}{33}\rho^2 - \ldots\right) \quad \text{for} \quad \rho < 1. \qquad (53)$$

Retention of only the first term in (53) corresponds to the diffusion approximation in the determination of f(p).

Let us also obtain from (46) the coefficient of transformation of an transverse incident wave into a longitudinal one in the case when $k^0 \parallel [001]$ and $k \parallel k^0$ for different orientations of the heating field F and for different polarizations of the incident wave E^0

$$d\Sigma_l^{[100]} = \frac{3\sigma_0}{8\pi^2}\left\{\frac{e_0 n C^2 \rho R(\rho)}{20B'(2mT)^{1/2}\Delta\omega} + \frac{e_0 n m_0 k}{m^2\omega\Delta\omega}\right\}^2 \left(\frac{\omega}{\omega^0}\right)^2 \times \frac{\langle\delta E_\perp^2\rangle_{q,\Delta\omega}}{[\mathrm{Re}\,\varkappa_\perp(\omega^0)]^{1/2}} \sum_s \frac{c^3 k_s^2}{\omega^3}\left|\frac{d\varkappa_l(k,\omega)}{dk}\right|^{-1}_{k=k_s(\omega)} d\omega\, do. \qquad (54)$$

2) $F \parallel [100]$, $E^0 \parallel F$.

$$d\Sigma_l^{[001]} = \frac{3\sigma_0}{8\pi^2}\left\{\left(\frac{e_0 C^2 n \rho R(\rho)}{20B'(2mT)^{1/2}\Delta\omega}\right)^2 \langle\delta E_\parallel^2\rangle_{q,\Delta\omega} + \left(\frac{e_0 n m_0 k}{m^2\omega\Delta\omega}\right)^2 \langle\delta E_\perp^2\rangle_{q,\Delta\omega}\right\}\left(\frac{\omega}{\omega^0}\right)^2 \frac{d\omega\, do}{[\mathrm{Re}\,\varkappa_\perp(\omega^0)]^{1/2}} \times \sum_s \frac{k_s^2 c^3}{\omega^3}\left|\frac{d\varkappa_l(k,\omega)}{dk}\right|^{-1}_{k=k_s(\omega)}. \qquad (55)$$

It is seen from (52)–(55) that in a nonequilibrium semiconductor plasma with a nonstandard energy dispersion law, an additional contribution is made to the wave scattering (transformation) coefficient by the presence of a heating field and by the deviation of the energy dispersion law from the parabolic one $\epsilon = p^2/2m$. The value of this contribution plays the principal role in the scattering of waves by fluctuations in the long-wave region ($q, k \to 0$).

5. CONCLUSION

The foregoing investigations have shown that a number of new regularities, which do not take place in a plasma with the standard dispersion law $\epsilon = p^2/2m$, appear in the processes of scattering and transformation of waves in a nonequilibrium semiconductor plasma with a complicated dispersion law $\epsilon(p)$. The deviation of the dispersion law $\epsilon(p)$ from the standard one becomes manifest in two respects in the scattering (transformation) of the waves.

1. In the presence of a constant electric field (that heats the carriers), the deviation of the dispersion law from the standard one leads to the occurrence of a field dependence in the frequencies and damping decrements of the natural oscillations of the plasma, and the dispersion law $\epsilon(p)$ affects the wave scattering phenomena via these characteristics. For this reason, particularly in strong fields in semiconductors such as p-Ge at liquid-nitrogen temperatures (when the carrier distribution function is noticeably elongated along the field), the scattering and transformation coefficients are strongly anisotropic, and may assume anomalously large values for individual directions. As seen, for example, from (36) (see[5] for details), the anomalies in the fluctuations are connected with the anomalies in the behavior of the decrements γ. The quantity γ has been investigated in detail in[3,4].

2. In a nonequilibrium plasma with a nonstandard dispersion law $\epsilon(p)$, the current causing the wave scattering acquires an additional term and this, as seen from (52), (54), and (55), leads to the appearance of additional terms in the scattering and transformation coefficients. It is interesting to note that the dependence on the heating field F, and also on the momentum q transferred in the scattering, differs in the additional terms for the wave scattering coefficients from the analogous dependences of these coefficients in a plasma with the standard dispersion law. The ratio of the first term in the curly brackets of (52) to the second at $\rho \le 1$, with allowance for (53), is approximately equal to

$$-\frac{2}{15}\frac{C^2}{B'}\frac{m}{m_0}\rho\frac{\Delta\omega}{q\bar{v}}.$$

Substituting the values of the constants C, B′, and m for p-Ge, we obtain $-0.55\rho\Delta\omega/q\bar{v}$. This estimate shows that the additional term in (52) can easily exceed the main term that is due to allowance for the concentration fluctuation. For example, it is easy to realize experimentally a situation wherein $\rho \lesssim 1$ and $q\bar{v}/\Delta\omega \approx 0.05$. Their approximate ratio is in this case equal to 10. We see thus that the processes of scattering and transformation of the electromagnetic waves in a nonequilibrium plasma of a semiconductor are very sensitive to the details of the band structure. In particular, the appearance of anomalously large scattering cross sections (with allowance for their sharp anisotropy) can be used to transform the frequencies into the intervals $\omega^0 \pm \omega p$.

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