Theory of Magnetic Reversal Nuclei

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The energy barrier R_{min} defining the probability of formation of magnetic reversal nuclei in ferromagnets is calculated for the case of weak metastability. It is demonstrated that in the case of weak anisotropy the barrier R_{min} is much smaller than that previously assumed[5,6]. The shape of the so-called critical nuclei is found. Surface nuclei are investigated. It is shown that at a reasonably close distance to the Curie temperature thermal activation of the nuclei may be observed.

1. INTRODUCTION

As is well known, the cause of magnetic hysteresis is the possible existence of metastable states. In the simplest model of a uniaxial ferromagnet, which is described, for example, in the book of Landau and Lifshitz[1], metastable states can be realized in the case when

$$H_z^{2/3} + (H_x^2 + H_y^2)^{1/3} < (\beta M)^{2/3}.$$

Here z is the easy magnetization axis, **H** is the magnetic field, **M** is the magnetization, and β is the anisotropy constant (the anisotropy energy is

$$U_{\text{an}} = {}^1/_2\beta(M_x^2 + M_y^2); \quad M_x^2 + M_y^2 + M_z^2 = M^2 = \text{const}).$$

If the field **H** is directed along the easy axis ($H_x = H_y = 0$) and is smaller in absolute magnitude than βM, then the metastable state is the one with magnetization antiparallel to the field.

A metastable state can be destroyed, for example, if a nucleus of magnetization reversal is produced as a result of thermal fluctuations, and its dimensions are large enough to permit the growth of the nucleus to lead to a decrease of the free energy $\tilde{\mathcal{F}}$ of the body[2] 1). We recall that a metastable state is stable against infinitesimally small perturbations (such perturbations increase the free energy of the body), and consequently, to produce a nucleus capable of growing it is necessary to overcome a finite energy barrier. The probability of formation of such nuclei as a result of thermal fluctuations is proportional to $\exp(-R_{min}/T)$, where T is the temperature, and the energy barrier R_{min} is the work necessary to produce the so-called critical nucleus, which is in unstable equilibrium with the medium, namely, its shape is such that at a specified nucleus thickness the free energy $\tilde{\mathcal{F}}$ of the body is minimal, and the thickness corresponds to the maximum free energy $\tilde{\mathcal{F}}$. Thus, the critical nucleus corresponds to a saddle point of the functional $\tilde{\mathcal{F}}\{z(x, y)\}$, where the function z(x, y) describes the shape of the nucleus. We assume here that the dimensions of the nucleus are large with a domain-wall thickness δ; this assumption, as will be shown below, is valid only in the case of weak metastability ($H \ll \beta M$). It is precisely in this case that the nuclear can be considered within the framework of the macroscopic theory; in the absolute case, the parameters of the critical nucleus should be obtained by solving the static equation $\mathbf{M} \times \mathbf{H}_{\text{eff}} = 0$ of Landau and Lifshitz [4].

Doring[5] (see also[6]) considered only nuclei of ellipsoidal form and did not take into account the possibility of inclination of the magnetization away from the easy axis. In the case of weak anisotropy, this inclination leads to an appreciable decrease of the sum of the magnetostatic energy and the magnetic-anisotropy energy[7,8]. The exact shapes of critical nuclei are obtained below for a number of limiting cases. It is shown that the energy barrier R_{min} is much smaller in the case of weak anisotropy than that obtained by Doring. In some cases, apparently, the time of thermal activation of the nuclei can be an observable quantity2). Surface nuclei are also considered below. It will be shown that under certain conditions the formation of surface nuclei can be more probable. The obtained estimate of R_{min} is also valid for cubic ferromagnets. Our formulas also determine the so-called starting field H_S[5,6], i.e., the field above which nuclei of specified thickness can grow.

2. THERMODYNAMIC RELATIONS

The work R which we must calculate is equal to the change, due to the presence of the nucleus, of the free energy $\tilde{\mathcal{F}}$ of the body. The free energy $\tilde{\mathcal{F}}$ is defined in the book of Landau and Lifshitz[1]:

$$\delta\tilde{\mathcal{F}} / \delta\mathbf{H}(\mathbf{x}) = -\mathbf{B}(\mathbf{x}) / 4\pi. \tag{1}$$

Here **H** and **B** are respectively the field and the induction. The free energy $\tilde{\mathcal{F}}$ can be represented in the form of the sum of the magnetic anisotropy energy $\tilde{\mathcal{F}}_{\text{an}}$, the magnetostatic energy $\tilde{\mathcal{F}}_{\text{m}}$, and the surface-tension energy $\tilde{\mathcal{F}}_{\text{ten}}$:

1)In recent papers by Lifshitz and Kagan and by Iordanskiĭ and Finkel'shteĭn[3], they investigated the formation of nuclei as a result of quantum tunneling. This mechanism of nucleus formation will not be considered here. It can be shown that the corresponding probability is exceedingly small.

2)The thermodynamic analysis yields only the exponential factor in the formula $w = A_{\exp}(-R_{min}/T)$ for the probability w of formation of a supercritical nucleus per unit time. The calculation of the preexponential factor is a problem of kinetic theory, and is extremely complicated even in the case of an isotropic system, such as a liquid-vapor system (see[9,10]). We limit ourselves in this article only to the calculation of R_{min}. For rough estimates we can assume apparently that $A \sim V/\delta^3\tau$, where V is the volume of the system, δ is the thickness of the domain boundary, and τ is the microscopic relaxation time. In typical ferromagnets, this quantity can vary in a very wide range; at $V = 1$ cm^3 we obtain $A \sim 10^{21}$–10^{30} sec^{-1}.

$$\tilde{\mathscr{F}}_{\rm an} = \frac{\beta}{2}\int d^3\mathbf{x}(M_x^2+M_y^2), \tag{2}$$

$$\tilde{\mathscr{F}}_m = -\int d^3\mathbf{x}[(\mathbf{M}+\delta\mathbf{M})(\mathbf{H}+\delta\mathbf{H})+(\mathbf{H}+\delta\mathbf{H})^2/8\pi], \tag{3}$$

$$\tilde{\mathscr{F}}_{\rm ten} = \int \Delta\, dS. \tag{4}$$

In (3), **M** and **H** are the values of the magnetization and the field in the homogeneous metastable state (i.e., far from the nucleus), and $\delta\mathbf{M}$ and $\delta\mathbf{H}$ are the changes of these quantities due to the formation of the nucleus. In (4), Δ is the surface tension on the phase separation boundary, and the integration is carried out over the surface of the nucleus.

The term

$$-\int d^3\mathbf{x}\left(\mathbf{M}\delta\mathbf{H}+\frac{1}{4\pi}\mathbf{H}\delta\mathbf{H}\right) = -\frac{1}{4\pi}\int d^3\mathbf{x}\mathbf{B}\delta\mathbf{H}$$

in formula (3) for $\tilde{\mathscr{F}}_m$ is equal to zero, since $\mathbf{B} = \text{curl}\,\mathbf{A}$, $\delta\mathbf{H} = -\nabla\delta\varphi$, and

$$\int d^3\mathbf{x}\mathbf{B}\delta\mathbf{H} = -\int d^3\mathbf{x}\,\text{div}(\varphi\,\text{rot}\,\mathbf{A}) = 0.$$

Similar reasoning shows that

$$-\int d^3\mathbf{x}\left[\delta\mathbf{M}\delta\mathbf{H}+\frac{(\delta\mathbf{H})^2}{8\pi}\right] = -\int d^3\mathbf{x}\left[\frac{\delta\mathbf{B}-\delta\mathbf{H}}{4\pi}\delta\mathbf{H}+\frac{(\delta\mathbf{H})^2}{8\pi}\right] = \int d^3\mathbf{x}\frac{(\delta\mathbf{H})^2}{8\pi}.$$

Thus, the change of the magnetostatic energy is

$$\tilde{\mathscr{F}}_m - \tilde{\mathscr{F}}_m^{(0)} = -\int d^3\mathbf{x}\mathbf{H}\delta\mathbf{M} + \int d^3\mathbf{x}\frac{(\delta\mathbf{H})^2}{8\pi} \tag{5}$$

Formula (5) was used by Doring[5].

Let us now discuss the question of the surface tension (see formula (4)). It is shown in the author's earlier papers[7,8] that for boundaries inclined to the easy-magnetization axis, the concept of the surface tension, generally speaking, becomes meaningless, since the free-energy density

$$\tilde{F} = -\frac{1}{4\pi}\int_0^H \mathbf{B}d\mathbf{H} = U_{\rm an} - \mathbf{M}\mathbf{H} - \frac{H^2}{8\pi}$$

changes on going through such a boundary. The surface tension Δ can be defined only accurate to $\delta(\tilde{F}_1 - \tilde{F}_2)$, where δ is the thickness of the domain wall. If the phase separation boundary is parallel to the easy axis, then $\tilde{F}_1 = \tilde{F}_2$, and the surface tension has a rigorously defined meaning. In the absence of the field (H = 0), the surface tension Δ on a wall parallel to the easy axis in a uniaxial ferromagnet is equal to

$$\Delta = 2\beta\delta M^2. \tag{6}$$

Let us consider now the case of strong anisotropy ($\beta/4\pi \gg 1$), assuming that the fields on the phase separation boundary are small compared with βM (it is precisely this case which is of interest to us). The surface tension should be defined as the contribution of the domain wall to the free energy

$$\tilde{\mathscr{F}} = \int d\zeta\left[U_{\rm an} - \mathbf{M}\mathbf{H} - \frac{H^2}{8\pi} + \frac{\alpha}{2}\left(\frac{\partial\mathbf{M}}{\partial\zeta}\right)^2\right]. \tag{7}$$

Here the ζ axis is perpendicular to the separation boundary (all the quantities in the transition layer vary only in the direction of this axis), and the last term in the integrand is the so-called inhomogeneity energy. It is obvious that at any inclinations of the boundary relative to the easy axis, the uncertainty in the surface tension does not exceed in order of magnitude

$$\delta[MH + (4\pi M)^2] \ll \beta\delta M^2,$$

i.e., the surface tension can be defined with good accuracy. Discarding the constant part of the integrand (i.e., those terms which do not change on going through the separation boundary), and neglecting the terms that are small compared with βM^2, we obtain for Δ the formula

$$\Delta = \int_{-\infty}^{\infty} d\zeta\left\{\frac{\beta}{2}M^2\sin^2\theta + \frac{\alpha}{2}M^2\left[\left(\frac{d\theta}{d\zeta}\right)^2 + \sin^2\theta\left(\frac{d\varphi}{d\zeta}\right)^2\right]\right\}$$

$$M_z = M\cos\theta,\quad M_x = M\sin\theta\cos\varphi,\quad M_y = M\sin\theta\sin\varphi.$$

It is obvious that the minimum of this expression is reached at φ = const (the value of φ in our approximation remains indeterminate), and we obtain for $\cos\theta$ the same formula as in the case of H = 0 for a boundary parallel to the easy axis:

$$\cos\theta = -\text{th}(\zeta/\delta), \quad \delta^2 = \alpha/\beta. \tag{8}$$

The surface tension Δ is determined as before by formula (6), i.e., it is independent of the inclination of the boundary relative to the easy axis.

In concluding this section, it is to be noted that the thermodynamic boundary condition obtained by the present author[8,11] does not hold on the boundaries of the nucleus[8,11], for the same reasons that the condition that the pressures be equal is not satisfied in the problem of nucleation in a liquid-vapor system.

3. ELONGATED NUCLEI (WEAK FIELDS)

In the case considered here (uniaxial ferromagnet, H ∥ z), the critical nucleus is symmetrical relative to the easy axis (the z axis). The axial cross section of the nucleus is shown schematically in Fig. 1. The shape of the critical nucleus is described by the function $\rho_0(z)$. In a field that is small in comparison with βM, the "equation of state" of a uniaxial ferromagnet is

$$\delta H_\rho = \beta M_\rho, \tag{9}$$

i.e., the magnetic permeability μ in the direction perpendicular to the easy axis is equal to

$$\mu = 1 + 4\pi/\beta. \tag{10}$$

It will be shown below that in a weak field ($\mu H \ll 4\pi M$) the nucleus is elongated along the easy axis ($l \gg \rho_0(0)$, see Fig. 1), and the deviation of the magnetization from the easy axis is small ($M_\rho \ll M$). In this case the equation div **B** = 0 can be linearized:

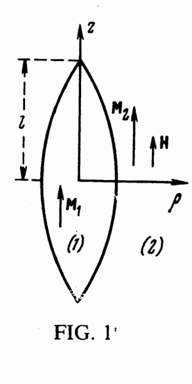

FIG. 1

$$\operatorname{div}\mathbf{B}=\frac{\mu}{\rho}\frac{\partial}{\partial\rho}(\rho\delta H_\rho)+\frac{\partial\delta H_z}{\partial z}=0. \quad (11)$$

The field δH and the polarization M_ρ are produced by fictitious "magnetic charges" concentrated on the surface of the nucleus. Since the nucleus is elongated, the density of the "charges" varies slowly and not too far from the nucleus we have $\delta H_z \ll \mu\delta H_\rho$ (δH_z would be zero in the case of an infinite homogeneously-charged cylinder). Inside the nucleus the field δH_ρ is also small (much smaller than outside the nucleus). Therefore, in the first approximation in $\rho_0(0)/l$, the magnetization inside the nucleus is not deflected away from the easy axis:

$$M_\rho^{(1)}=\delta H_\rho^{(1)}=0. \quad (12)$$

On the phase separation boundary (at $\rho=\rho_0(z)$) it is necessary to satisfy the condition B_n = const:

$$(13)$$

i.e.,
$$-4\pi M\dot\rho_0=4\pi M\dot\rho_0+\mu\delta H_\rho^{(2)},\qquad \dot\rho_0=d\rho_0/dz,$$
$$\mu\delta H_\rho^{(2)}=-8\pi M\dot\rho_0,\quad \rho=\rho_0(z).$$

At not too large distances, the field $\delta H_\rho^{(2)}$ is equal to

$$\delta H_\rho^{(2)}=-8\pi M\rho_0\dot\rho_0/\mu\rho. \quad (14)$$

To determine the region of applicability of this formula, we make the following substitutions in the magnetostatic equations

$$\delta H_\rho=Mh_\rho,\qquad \delta H_z=\mu^{1/2}Mh_z,\qquad \rho=\mu^{1/2}r.$$

Then the magnetostatic equations take the form

$$\operatorname{div}\mathbf{h}=0,\qquad \operatorname{rot}\mathbf{h}=0,$$

with the field $\mathbf{h}$ produced by the surface "charge"

$$\sigma(z)=-2\dot\rho_0(z)/\mu.$$

At $r\gg l$, the field $\mathbf{h}$ has the form of a dipole field, i.e., it decreases like $(r^2+z^2)^{-3/2}$. It is obvious that formula (14) is valid only if

$$\rho\ll L\sim\mu^{1/2}l. \quad (15)$$

In this region, the sum of the anisotropy energy and the energy of the demagnetizing fields δH is equal to

$$\tfrac{1}{2}\beta M_\rho^2+(\delta H)^2/8\pi\approx\mu(\delta H_\rho)^2/8\pi\gg(\delta H_z)^2/8\pi.$$

Integrating, we obtain

$$\int d^3x\frac{\mu(\delta H_\rho)^2}{8\pi}=\frac{8\pi M^2}{\mu}\int_{-l}^{l}dz\int_{\rho_0}^{l\mu^{1/2}}\rho_0^2\dot\rho_0^2\frac{2\pi\rho\,d\rho}{\rho^2} \quad (16)$$
$$=\frac{(4\pi M)^2}{\mu}\int_{-l}^{l}dz\,\rho_0^2\dot\rho_0^2\ln\frac{\mu^{1/2}l}{\rho_0}\approx\frac{(4\pi M)^2}{\mu}\ln\frac{\mu^{1/2}l}{\rho_0(0)}\int_{-l}^{l}dz\,\rho_0^2\dot\rho_0^2.$$

Since the integral with respect to ρ diverges logarithmically on the upper limit, we have set this limit equal to $\mu^{1/2}l$. For an ellipsoid, this part of the energy is equal to

$$\frac{32}{3\mu}\pi^2M^2\frac{\rho_0^4(0)}{l}\ln\frac{\mu^{1/2}l}{\rho_0(0)}.$$

At $\mu=1$ (i.e., at $\beta=\infty$), this expression coincides with that obtained by Doring by another method[5,6]. At large μ (i.e., at $\beta/4\pi\ll1$), Doring's estimate is incorrect.

The change of the magnetostatic energy (see formula (15)) contains also a term $-2MHV$ proportional to the volume of the nucleus:

$$V=\pi\int_{-l}^{l}\rho_0^2\,dz.$$

The surface-tension energy is proportional to the area S of the separation boundary

$$\tilde{\mathcal{F}}_{\text{ten}}=\Delta S=2\pi\Delta\int_{-l}^{l}\rho_0\,dz.$$

Thus, the work R necessary to produce the nucleus can be represented in the form

$$R=\int_{-l}dz\left[\frac{1}{\mu}(4\pi M)^2\rho_0^2\dot\rho_0^2\ln\frac{\mu^{1/2}l}{\rho_0(0)}+2\pi MH\rho_0(\rho_m-\rho_0)\right]. \quad (17)$$

We have introduced here the notation

$$\rho_m=\Delta/MH=2\beta\delta M/H\gg\delta. \quad (18)$$

It is easy to write down the first integral of the Euler-Lagrange equation (the "energy" integral) for the functional R:

$$\ln\frac{\mu^{1/2}l}{\rho_0(0)}\rho_0^2\dot\rho_0^2=\frac{\mu H}{8\pi M}(\rho_0+\rho_1)(\rho_m+\rho_1-\rho_0). \quad (19)$$

Here ρ_1 is the integration constant. Since the right-hand side in (13) should be positive at $\rho_0=0$, the quantities ρ_1 and $\rho_m+\rho_1$ should be of the same sign. The right-hand side is invariant against the substitution $\rho_m+\rho_1\rightleftarrows-\rho_1$. We can therefore assume that $\rho_1\geq0$. From (19) we obtain

$$z=\pm\left(\frac{8\pi M}{\mu H}\ln\frac{\mu^{1/2}l}{\rho_0(0)}\right)^{1/2}\int_{\rho_0}^{\rho_0(0)}\frac{\rho\,d\rho}{[(\rho+\rho_1)(\rho_m+\rho_1-\rho)]^{1/2}}. \quad (20)$$

If $\rho_0(0)<\rho_m+\rho_1$, then the function $\rho_0(z)$ has, in the first approximation in $\rho_0(0)/l$, a kink at $z=0$; this kink becomes smoothed out at distances $\delta z\ll l$ in the succeeding approximations. The kink is missing if $\rho_0(0)=\rho_m+\rho_1$ (the case $\rho_0(0)>\rho_m+\rho_1$ is impossible). The length of the nucleus is determined from the equation

$$z(\rho_0=0)=l. \quad (21)$$

Using formulas (17), (19), and (20), we obtain

$$R=(4\pi M)^2\left[\frac{H}{2\pi\mu M}\ln\frac{\mu^{1/2}l}{\rho_0(0)}\right]^{1/2} \quad (22)$$
$$\times\int_0^{\rho_0(0)}\rho\,d\rho\frac{[\rho(\rho_m-\rho)+(\rho+\rho_1)(\rho_m+\rho_1-\rho)]}{[(\rho+\rho_1)(\rho_m+\rho_1-\rho)]^{1/2}}.$$

This integral can be calculated analytically, but it is more convenient to investigate it without performing the integration. Making the substitution $\rho=\rho_m x$ and putting $\rho_0(0)=\eta\rho_m$, we represent R in the form

$$R=(4\pi M)^2\left(\frac{H}{2\pi\mu M}\ln\frac{\mu^{1/2}l}{\eta\rho_m}\right)^{1/2}\rho_m^3 f(\alpha,\eta), \quad (23)$$

$$f(\alpha,\eta)=\int_0^\eta dx\left\{2x[(x+\alpha)(1+\alpha-x)]^{1/2}-\frac{\alpha(1+\alpha)x}{[(x+\alpha)(1+\alpha-x)]^{1/2}}\right\} \quad (24)$$

Let us consider first a nucleus with $\rho_0(0)=\rho_m+\rho_1$, i.e., $\eta=1+\alpha$. In this case formula (23) should contain the function

$$\tilde{f}(\alpha)=f(\alpha,1+\alpha)=\int_0^{1+\alpha}dx\Big[2x\sqrt{(x+\alpha)(1+\alpha-x)}-\frac{\alpha(1+\alpha)x}{\sqrt{(x+\alpha)(1+\alpha-x)}}\Big]. \quad (25)$$

It is obvious that in the calculation of the derivative $dR/d\alpha$ we can neglect the derivative of the logarithmic factor. The error incurred thereby is small and of the order of

$$\ln^{-1}[l/\rho_0(0)]dR/d\alpha.$$

We therefore calculate the derivative $d\tilde{f}(\alpha)/d\alpha$. The calculation of the derivative of the first term in (20) is trivial. In the calculation of the derivatives of the second term it is necessary prior to the differentiation to make the change of variable $1+\alpha-x=t$, and then, after the differentiation, it is convenient to go back to the original variable. After simple transformations we obtain

$$\frac{d\tilde{f}}{d\alpha}=-\alpha^2(1+\alpha)\int_0\frac{dx}{[(x+\alpha)^3(1+\alpha-x)]^{1/2}}<0. \quad (26)$$

We recall that $\alpha>0$. Consequently, $\tilde{f}(\alpha)$ has its largest value at $\alpha=0$:

$$\tilde{f}(0)=\pi/8. \quad (27)$$

We calculate analogously the derivative $\partial f(\alpha,\eta)/\partial\alpha$:

$$\frac{\partial f(\alpha,\eta)}{\partial\alpha}=\frac{\alpha(1+\alpha)\eta}{[(\eta+\alpha)(1+\alpha-\eta)]^{1/2}}-\alpha^2(1+\alpha)\int_0^\eta\frac{dx}{[(1+\alpha-x)(x+\alpha)^3]^{1/2}}. \quad (28)$$

Let us show that this derivative is positive. Replacing in the integrand $1+\alpha-x$ by $1+\alpha-\eta$, we increase the value of the integral. Consequently,

$$\frac{\partial f(\alpha,\eta)}{\partial\alpha}\geqslant\frac{\alpha(1+\alpha)}{(1+\alpha-\eta)^{1/2}}\Big[\frac{\eta}{(\eta+\alpha)^{1/2}}-\alpha\int_0^\eta\frac{dx}{(x+\alpha)^{3/2}}\Big]=\frac{\alpha(1+\alpha)}{(1+\alpha-\eta)^{1/2}}\Big[(\eta+\alpha)^{1/2}+\frac{\alpha}{(\eta+\alpha)^{1/2}}-2\alpha^{1/2}\Big].$$

The function $(\eta+\alpha)^{1/2}+\alpha/(\eta+\alpha)^{1/2}$ increases with increasing η. Its smallest value is $2\sqrt{\alpha}$, i.e.,

$$\partial f(\alpha,\eta)/\partial\alpha\geqslant 0. \quad (29)$$

This means that at a given η (i.e., at a given thickness of the nucleus), the minimum of R corresponds either to $\alpha=0$ (at $\eta<1$) or $\alpha=\eta-1$ (at $\eta>1$). In the former case it is easy to show that

$$(\partial f/\partial\eta)_{\alpha=0}>0,\quad \eta<1. \quad (30)$$

In the latter case

$$\frac{df(\eta,\alpha=\eta-1)}{d\eta}=\frac{d\tilde{f}(\alpha)}{d\alpha}<0,\ \eta>1. \quad (31)$$

(see formula (26)). Consequently the work $R(\eta)$ has a maximum at $\eta=1$.

The point $\eta=1$, $\alpha=0$ is a saddle point of the functional R. Thus,

$$R_{min}=2\pi^3M^2\Big(\frac{H}{2\pi\mu M}\ln\frac{\mu^{1/2}l}{\rho_m}\Big)^{1/2}\rho_m^3. \quad (32)$$

Using (21), we obtain

$$\frac{\mu^{1/2}l}{\rho_m}=\Big(\frac{2\pi^3M}{H}\ln\frac{\mu^{1/2}l}{\rho_m}\Big)^{1/2},$$

i.e.,

$$\frac{\mu^{1/2}l}{\rho_m}=\Big(\frac{\pi^3M}{H}\ln\frac{2\pi^3M}{H}\Big)^{1/2}\gg\mu^{1/2};\ \ln\frac{\mu^{1/2}l}{\rho_m}=\frac{1}{2}\ln\frac{\pi^3M}{H}. \quad (33)$$

Substituting the obtained value of $\ln(\mu^{1/2}l/\rho_m)$ into (32), we get

$$R_{min}=\pi^3M^2\Big(\frac{H}{\pi\mu M}\ln\frac{\pi^3M}{H}\Big)^{1/2}\rho_m^3=\frac{\mu^2}{4}\beta^3M^2\delta^3\Big(\frac{4\pi M}{\mu H}\Big)^{5/2}\Big(\ln\frac{\pi^3M}{H}\Big)^{1/2}. \quad (34)$$

The equation for the shape of the critical nucleus is ($\rho_1=0$; $\rho_0(0)=\rho_m$):

$$z=\pm\Big(\frac{8\pi M}{\mu H}\ln\frac{\mu^{1/2}l}{\rho_m}\Big)^{1/2}\int_{\rho_0}^{\rho_m}\Big(\frac{\rho}{\rho_m-\rho}\Big)^{1/2}d\rho,$$

i.e.,

$$\frac{z}{\rho_m}=\pm\Big(\frac{4\pi M}{\mu H}\ln\frac{\pi^3M}{H}\Big)^{1/2}\Big\{\arcsin\Big(1-\frac{\rho_0}{\rho_m}\Big)+\frac{1}{\rho_m}[\rho_0(\rho_m-\rho_0)]^{1/2}\Big\}. \quad (35)$$

This equation does not hold at values of ρ_0 such that $|\dot{\rho}_0|\sim 1$, i.e., when

$$\rho\leqslant\frac{\mu H\rho_m}{4\pi M\ln(\pi^3M/H)}\sim\frac{\mu\beta\delta}{4\pi\ln(\pi^3M/H)}$$

This region can be broader than the domain wall only in the case of strong anisotropy ($\beta/4\pi\gg 1$, $\mu\approx 1$).

In concluding this section, let us discuss the question of nuclei in cubic ferromagnets (the latter, as a rule, have low anisotropy). At a small deviation of the magnetization M from the easy axis, the anisotropy of cubic ferromagnets differs little from uniaxial: $U_{an}\approx\frac{1}{2}\beta(M_x^2+M_y^2)$. At the same time, the energy of the surface tension Δ on a boundary parallel to the easy axis (the z axis) depends on the orientation of the boundary in the xy plane[12,13]:

$$\Delta=\Delta_0\Big(1+\frac{\sin^2 2\varphi}{2(4-\sin^2 2\varphi)^{1/2}}\operatorname{arcch}\frac{2}{|\sin 2\varphi|}\Big).$$

Here $\Delta_0=(\alpha\beta)^{1/2}$ is the surface tension on the boundary parallel to the crystallographic plane (100) (the parameter α was defined in the preceding section; see formula (7)), and φ is the angle of rotation of the boundary in the xy plane. The function $\Delta(\varphi)$ has a minimum at $\varphi=0$. The critical nucleus is in this case not a figure of revolution (since $\Delta(\varphi)\neq$ const), and the preceding analysis is valid only qualitatively. Nonetheless, simple reasoning yields the upper and lower limits of R_{min}.

The upper limit of R_{min} can easily be obtained by assuming the nucleus to be a figure of revolution. In this case the expression for R_{min} contains $\rho_m=\bar{\Delta}/MH$, where

$$\bar{\Delta}=\frac{4}{\pi}\int_0^{\pi/4}\Delta(\varphi)d\varphi=1.28\Delta_0. \quad (36)$$

If it is assumed that $\Delta=\Delta_0=$ const, then we obtain the lower limit of R_{min}. Thus, R_{min} can be estimated from the formula

$$R_{min}=\pi^3M^2\Big(\frac{H}{\pi\mu M}\ln\frac{\pi^3M}{H}\Big)^{1/2}\rho_m^3, \quad (37)$$

where

$$\Delta_0/MH<\rho_m<\bar{\Delta}/MH. \quad (38)$$

The corresponding values of R_{min} differ by a factor 2.1.

The following remark must be made concerning the applicability of the obtained estimates in the case of cubic ferromagnets. In cubic ferromagnets, the thickness of a plane 180° domain wall depends strongly on the angle φ[12,13]. In particular, at $\varphi = 0$ the thickness of the transition region is determined by the magnetostriction

$$\delta(\varphi = 0) \sim (\Delta / \beta M^2) \ln (\beta / k),$$

where k is the magnetostriction constant $(k \ll \beta)$. In the case $\varphi \sim \pi/2 - \varphi \sim 1$ the thickness of the transition region is $\delta(\varphi) \sim \Delta/\beta M^2$. If the thickness of the transition region separating the embryo from the metastable phase at $\varphi = 0$ were to be the same as in the case of a plane boundary, our estimates would be valid only for $\Delta M/H \gg \ln(\beta/k) \gg 1$. Apparently, however, the thickness of a curved boundary decreases with increasing curvature and at $\varphi = 0$ it is always smaller than $\rho_m = \Delta/MH$, provided only $H \ll \beta M$. In this case the inequality $H \ll \beta M$ determines, as before, the region of applicability of the obtained estimates.

4. SPHERICAL NUCLEI IN THE CASE OF STRONG ANISOTROPY. SURFACE NUCLEI

A. Spherical nuclei. In the case $\beta/4\pi \lesssim 1$, the region $\mu H \ll 4\pi M$ coincides with the region of weak metastability $H \ll \beta M$. In the case of strong anisotropy $(\beta/4\pi \gg 1)$, the case $4\pi M \ll H \ll \beta M$ is also possible, and this case will be investigated below. If the anisotropy is large, then the deviation of the magnetization from the easy axis is not favored energywise ($U_{an} = 0$). On the boundary of the nucleus, the normal component of the magnetization experiences a jump $\sigma = 2M_{in}$, which can be regarded as the surface density of fictitious magnetic charges producing a demagnetization field δH. In this case l is of the order of $\rho_0(0)$, and the demagnetization fields play a negligible role:

$$(\delta H)^2 / 8\pi \sim 4\pi M^2 \ll MH.$$

The work R can be represented in the form

$$R = -2MHV + \Delta S = MH(-2V + \rho_m S), \tag{39}$$

where V and S are the volume and area of the nucleus. This expression differs from the corresponding expression for the isotropic liquid-vapor system only by the factor preceding V. It is therefore obvious that the critical nucleus should be spherical:

$$R = 4\pi MH(-{}^2/_3 r^3 + \rho_m r^2),$$

where r is the radius of the nucleus. The maximum of the function R(r) corresponds to

$$r = \rho_m, \tag{40}$$

$$R_{min} = {}^4/_3 \pi MH \rho_m{}^3 = {}^{32}/_3 \pi \beta M^2 \delta^3 (\beta M / H)^2. \tag{41}$$

B. Surface nuclei. In some cases, the formation of surface nuclei can be more probable, since the corresponding energy barrier R^S_{min} is smaller than the barrier that determines the probability of formation of nuclei in the volume of the sample (the latter will be denoted here by R^V_{min})[3)].

The simplest example is the case $4\pi M \ll H \ll \beta M$. Let us explain here that the field $H = \{0, 0, H\}$ is the internal field, which does not coincide with the field $H^{(e)}$ in vacuum. The latter can be determined from the conditions of the continuity of H_t and B_n on the boundary of the sample. The work R is determined by formula (39), where V is the volume of the nucleus and S is the area of the separation boundary. Putting $V = V'/2$ and $S = S'/2$, where V' and S' are the volume and surface area of the doubled figure, we can easily verify that the critical nucleus has the form of a hemisphere and

$$R^S_{min} = {}^1/_2 R^V_{min} = {}^{16}/_3 \pi \beta M^2 \delta^3 (\beta M/H)^2. \tag{42}$$

We emphasize that this result does not depend on the orientation of the easy axis relative to the surface of the sample.

In the case $H \ll 4\pi M \ll \beta M$, the barrier R^S_{min} differs little from R^V_{min} (it is assumed here the easy magnetization axis is perpendicular to the surface of the sample):

$$\frac{R^V_{min} - R^S_{min}}{R^V_{min}} = \frac{\pi\gamma^5}{120}; \quad \gamma = 4\left[\frac{H}{\pi M} \ln \frac{\pi^3 M}{H}\right]^{1/2} \ll 1. \tag{43}$$

We shall not discuss this case in detail here. We indicate only that the critical nucleus has the shape shown in Fig. 2, the boundary of the nucleus is described by Eq. (35), in which one must put $\mu = 1$, and the radius of the base of the nucleus a is equal to

$$a = \rho_m \gamma^2 = \frac{32}{\pi} \beta\delta \ln \frac{\pi^3 M}{H} \gg \delta. \tag{44}$$

Thus, the radius of the critical nucleus is $\rho_m = \Delta/MH$ in all the cases considered here (for arbitrary $\beta/4\pi$). This relation determines the so-called starting field[5,6]:

$$H_s = \Delta / rM, \tag{45}$$

at values above which the growth of nuclei of radius r becomes possible. If the presence of defects makes motion of the domain wall difficult, then it is necessary to replace in all the formulas H by $H - H_0$, where H_0 is the critical field[5,6] in which the wall motion becomes possible.

5. CONCLUSION

For typical ferromagnets (Fe, Co), the characteristic value R_{min}/T at room temperatures is of the order of 10^3, so that under ordinary conditions thermal activation of the nuclei is impossible. However, at a reasonable proximity to the Curie temperature, the barrier R_{min} can be greatly reduced by decreasing the magnetization (the values of β and δ change little on approaching the Curie point). The obtained formulas

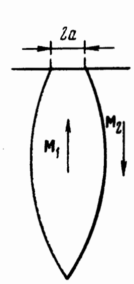

FIG. 2

[3)]It should be borne in mind that the probability of formation of surface nuclei is proportional to the surface of the sample and not to its volume.

are also suitable near the Curie temperature, where the thermal activation of the nuclei should become observable. Our estimates allow us to suppose that the barrier R_{min} is small in the observed region near the metastability limit ($R_{min} = 0$ at $H = \beta M$).

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