Neutral Excitations in He II

D. M. Chernikova

L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences

The possibility of the existence of stable optical excitations, such as excitons, in HeII is discussed. The decay spectrum of such an exciton is investigated for the model of an imperfect Bose gas. Some peculiarities of the dielectric constant of liquid helium in the ultraviolet region are noted. Estimates of the spectral line widths are made in connection with the recent appearance of some new experimental data.

IT has been observed in a number of recent experimental studies[1-3] that neutral excitations, excitons of sorts, are produced in superfluid helium bombarded by α particles. These excitations are the analog of excited helium atoms or molecules, but move in a dense medium consisting of the same atoms in the ground state. Excitons of the type of the metastable molecule $(2^3\Sigma)\,He_2^*$, which were observed in the spectrum of dense He gas, were investigated in [1,2]. It was observed in [3] that the scintillations in He II are delayed in time, and this circumstance was explained by assuming that the excited states of the He atoms produced by α-particle bombardment decay more slowly in the superfluid helium than in the normal phase. It was already suggested in [1] that at low temperatures, when the number of normal excitations in He II is small, the exciton could move with practically no scattering, i.e., behave like a new type of quasiparticle in He II. Thus, neutral excitations in He II, with energy on the order of the characteristic energy of the helium spectrum, should decay only at the boundary of the He II volume.

In the first part of the paper we shall deal with exciton states of the type of an excited He* atom, coupled to the ground state by a dipole transition. This type of excitation should play an important role in the optical properties of helium. In all the processes connected with such an excitation of He*, the decisive fact is that this excitation is transferred in resonant fashion from one atom to the other. Radiative decay is excluded, owing to the high density $n\lambda^3 \gg 1$. The amplitude of the resonant transfer of the excitation in dipole-dipole interaction between the atoms

$$V = \frac{\hat{d}_1\hat{d}_2}{r^3} - \frac{3(\hat{d}_1\hat{r})(\hat{d}_2\hat{r})}{r^5} \qquad (1)$$

(d_1 and d_2 are the dipole-moment operators: $|d| \sim ea_0$) is determined at low densities by the so-called Weisskopf radium

$$\rho_0 \sim \frac{d}{v^{1/2}} \sim \frac{a_0^{1/2}}{p^{1/2}}\left(\frac{M}{m}\right)^{1/2} \quad (\rho_0 p \gg 1) \qquad (2)$$

(M is the mass of the He atom, m is the electron mass, a_0 is a quantity on the order of the dimension of the atom) and is large. The effects connected with resonant transfer of excitation energy are decisive for the absorption line shape in the vicinity of the natural frequencies, and were investigated[4,5] under the conditions $n\rho_0^3$

$\ll 1$ and $n\lambda^3 \gg 1$. In real helium the condition for paired collisions ($n\rho_0^3 \ll 1$) is, of course, not satisfied, the interaction forces (1) cease to be paired, and consequently a quantitative calculation is impossible. The production of the exciton and its properties will be considered first qualitatively on the basis of the model of a nonideal Bose gas.[6] In the second part of the paper we attempt to analyze the experimental situation.

Let us dwell in greater detail on the choice of the model. It was proposed in [6] that the atoms of a Bose gas in the ground state interact via short-range forces characterized by a scattering amplitude $f_0 \sim a_0$, with $n_0 f_0^3 \ll 1$ (n_0 is the density of the number of particles in the condensate). The excitation-transfer amplitude (2) goes over at small collision momenta ρ_0, $p \sim 1$, into

$$\mathcal{F}_0 = \rho_0 \sim a_0 M / m, \qquad (2')$$

i.e., it is large compared with f_0. (Scattering by the potential (1) becomes significant in this energy region.) We see that $\mathcal{F}_0 \gg f_0$, and therefore the condition for paired collisions imposes the much stronger limitation $n_0\mathcal{F}_0^3 \ll 1$. We shall henceforth assume absolute zero temperature throughout. By virtue of the foregoing, the particle interaction in the ground state can be neglected in the first approximation. The energy of the excited atom is

$$\bar{\varepsilon}_0(p) = \omega_0 + p^2/2M,$$

where ω_0 is the frequency corresponding to the excitation energy. The frequency ω_0 is very high. In the first approximation, the interaction (1) produces a frequency shift $\Delta\omega \propto n_0\mathcal{F}_0/M$; the next effect is the damping of the excitation by scattering from the condensate particles. The characteristic relaxation frequency

$$1/\tau \sim n_0 v\mathcal{F}_0^2 \sim n_0 p\mathcal{F}_0^2/M$$

is compared with the kinetic energy of the excitation at

$$p \sim n_0\mathcal{F}_0^2. \qquad (3)$$

In a nonideal Bose gas, the effects of the interaction are significant at momenta $p \sim (n_0 f_0)^{1/2}$. Thus, in order for
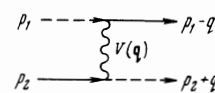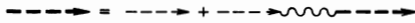

FIG. 1

FIG. 2

the singularities of the spectrum of the nonideal Bose gas to affect the relaxation of the excited atom (exciton), it is necessary to have

$$n_0 \mathcal{F}_0^2 \lesssim (n_0 f_0)^{1/2}. \tag{4}$$

We have referred above, for concreteness, only to excitations connected with the ground state by a dipole transition. The foregoing holds true for any situation in which the "exciton" atom is characterized by a large amplitude for scattering by the surrounding atoms, for example, for quadrupole interaction. Bearing in mind that all the results obtained here pertain to a certain model, we shall henceforth neglect the degeneracy of the excited state and the tensor character of the amplitudes corresponding to the interaction (1). Thus, to take into account the interaction with resonant transfer, we shall include henceforth in all the processes the matrix element shown in Fig. 1, where the continuous line corresponds to the ground state and the dashed line to the excited state of the atom.

EXCITON IN AN IDEAL BOSE GAS

In accordance with the statements made at the end of the preceding section, let us consider one excited particle moving in the presence of a background of Bose particles in the ground state. Neglecting the interaction, the Green's functions of the excited and unexcited particles are respectively $\mathcal{G}_0(p)$ and $G_0(p)$:

$$\mathcal{G}_0(p) = \frac{1}{\omega - \tilde{\varepsilon}_0(\mathbf{p}) + i\delta}, \; G_0(p) = \frac{1}{\omega - \varepsilon_0(\mathbf{p}) + i\delta} \tag{5}$$

$(\epsilon_0(\mathbf{p}) = \mathbf{p}^2/2M)$. In the first approximation $\mathcal{G}$ (thick dashed line) is expressed by the equation of Fig. 2:

$$\mathcal{G}(p) = 1 / (\omega - \tilde{\varepsilon}_0(\mathbf{p}) - n_0 V(\mathbf{p}) + i\delta), \tag{5'}$$

where $V(\mathbf{p})$ is the Fourier component of the interaction with the resonant transfer.

Strictly speaking, $V(\mathbf{p})$ should be express in the known fashion[5,6] in terms of the forward scattering amplitude, but in the small-momentum region (3) we have $p\mathcal{F}_0 \sim n_0 \mathcal{F}_0^3 \ll 1$, as a result of which this operation reduces to simple renormalizations, as is usually done in the Born approximation.[7] Indeed, let us consider the self-energy part shown in Fig. 3a, which is of the first order in n_0. Substituting in it the expressions (5) and (5') for the functions G_0 and $\mathcal{G}$ (we shall henceforth assume everywhere that the external frequency ω is close to the shifted natural frequency $\tilde{\omega}_0 = \omega_0 - 4\pi n_0 \mathcal{F}_0/M$) and integrating with respect to the internal frequency, we obtain an integral of the form

$$\frac{n_0 \mathcal{F}^2}{M} \int \frac{d\mathbf{q}}{(2\pi)^3} \frac{1}{\tilde{p}_0 - \mathbf{p}^2/4M - \mathbf{q}^2/M + i\delta},$$

$\tilde{p}_0 = p_0 - \tilde{\omega}_0$, and p_0 is the fourth component of the momentum p. Subtracting from it the principal value

$$\frac{n_0 \mathcal{F}^2}{M} \int \frac{d\mathbf{q}}{(2\pi)^3} \left(\frac{\mathbf{p}^2}{4M} - \frac{\mathbf{q}^2}{M} \right)^{-1}$$

which can be included in the definition of $V(\mathbf{p})$, we obtain

in accordance with (3) the term

$$\frac{n_0 \mathcal{F}^2}{M} \left(M\tilde{p}_0 - \frac{\mathbf{p}^2}{4} \right)^{1/2},$$

whose magnitude in comparison with the kinetic energy is of the order of

$$n_0 \mathcal{F}_0^2 / p. \tag{6}$$

Figure 3b shows a more complicated diagram of the self-energy, where the thick wavy line is connected in accordance with Fig. 3c with the function $\mathcal{G}(p)$ (5'). The integration with respect to the frequencies ω_1 and ω_2 can easily be performed and corresponds to the substitutions $\omega_1, \omega_2 \to \epsilon_0(\mathbf{q}_1), \epsilon_0(\mathbf{q}_2)$ in the denominators of the dashed line. The remaining integrals with respect to $d\mathbf{q}_1$ and $d\mathbf{q}_2$ converge at $q_1, q_2 \sim p$, from which we readily obtain the order of magnitude $n_0^2 \mathcal{F}_0^4/M$ of the integral corresponding to the diagram of Fig. 3b. Thus, the ratio of the contributions of the diagrams 3b and 3a is of the order of $n_0 \mathcal{F}_0^2/p$. We see readily that in the region where $n_0 \mathcal{F}_0^2/p \sim 1$ it is necessary to sum an infinite sequence of diagrams of the type of Fig. 3b. Figure 4 shows a diagram of sufficiently general form, a study of which explains the general principles of constructing the existing diagrams. In Fig. 4, the dashed line is the sum of the diagrams of Fig. 2, i.e., the expression (5'): analogously, the thick wavy line is given by the sum of Fig. 3c. The line of particles in the ground state remains unchanged so long as we do not take the interaction between them into account. Consequently, there are no particles above the condensate, i.e., the dashed and continuous lines are always directed to the same side. Physically, diagrams of this type describe a process in which the excited particle, scattered by the interaction of Fig. 1, knocks out the main particles from the condensate (in order not to clutter the figures, we have left out throughout the ends corresponding to the condensate particles). Owing to the ordered nature of all processes (when viewed, say, from left to right), we retain the ability, noted above for the diagram of Fig. 3b, to integrate with respect to all the frequencies of the continuous lines, which reduces to the substitution $\omega - \Sigma\omega_i \to \omega - \Sigma\epsilon_0(\mathbf{q}_i)$ of the corresponding frequencies in the arguments of the dashed lines.

Unfortunately, the presence of intersections does not
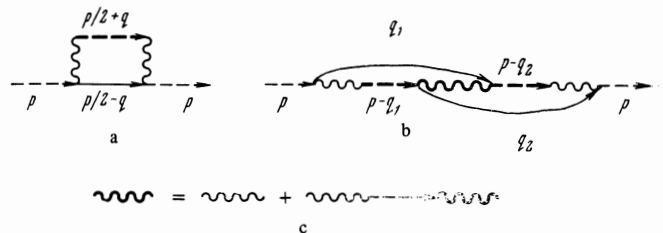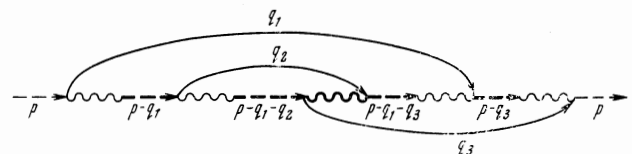

FIG. 3

FIG. 4

make it possible to sum explicitly the entire aggregate of diagrams of the type of Fig. 4. A qualitative result can be obtained by using scaling considerations. In fact, the spectrum of the free excitons $\omega - \omega_0 = p^2/2M$ corresponds to a packet-propagation velocity proportional to the momentum. One can expect scattering to cause this velocity to be smaller at sufficiently small momenta (more accurately at $p \ll n_0\mathscr{F}_0^2$). Assuming that $\omega - \omega_0 \propto p^\alpha$, the condition that all the diagrams of type 3b and 4 be of the same order yields $\alpha = \frac{3}{2}$. Together with the condition that the dispersion law obtained at $p \sim n_0\mathscr{F}_0^2$ be a continuation of the usual quadratic law, we get

$$\omega - \omega_0 = A (n_0\mathscr{F}_0^2)^{1/2} p^{3/2}/M, \quad p \ll n_0\mathscr{F}_0^2, \tag{7}$$

where A is a coefficient which generally speaking need not be real.

We return now to the case when the Bose-gas particles interact with one another. At sufficiently small momenta, this interaction determines the effects of scattering of an exciton by the main particles. The characteristic momentum in the problem of the non-ideal Bose gas is $p_c = (4\pi n_0 f_0)^{1/2}$ [6], and its value determines the limit at which the linear section of the excitation spectrum, which is characteristic of the superfluid state, goes over into the free-particle spectrum. If $p_c \ll n_0\mathscr{F}_0^2$, then in accordance with the foregoing it is necessary to know the solution of the problem considered above for the Green's functions of an exciton interacting only with the condensate. On the other hand, if $p_c \gg n_0\mathscr{F}_0^2$, then the Green's functions $G(p)$ in the diagrams of Figs. 3b and 4 should be the functions of the nonideal Bose gas. The corresponding integrals will be cut off at momenta $\sim p_c$. Thus, the necessary approximation is given by the diagram of Fig. 3a, allowance for which leads to a correct expression for the damping and to corrections to the exciton spectrum. It is obvious from general considerations that the exciton spectrum remains quadratic.

EXCITATION IN A NONIDEAL BOSE GAS

As mentioned above, we shall assume that

$$(\mathscr{F}_0/f_0)^{1/2}(n_0\mathscr{F}_0^3)^{1/2} \ll 1. \tag{8}$$

In an interacting Bose gas, the functions $G(p)$ (5) should be replaced by the Green's functions[6]

$$G(p) = \frac{\omega + \varepsilon_0(\mathbf{p}) + 4\pi n_0 f_0/M}{\omega^2 - \varepsilon^2(\mathbf{p}) + i\delta}, \quad \hat{G}(p) = \check{G}(p) = -\frac{4\pi n_0 f_0/M}{\omega^2 - \varepsilon^2(\mathbf{p}) + i\delta},$$
$$\varepsilon(\mathbf{p}) = [\varepsilon_0^2(\mathbf{p}) + 8\pi n_0 f_0\varepsilon_0(\mathbf{p})/M]^{1/2}. \tag{9}$$

The diagrams determining the self-energy part $\Sigma(p)$ of the Green's function $\mathscr{G}$ are shown in Fig. 5. We present here an expression for $\Sigma(p)$:

$$\Sigma(p) = \frac{4\pi}{M} n_0\mathscr{F}_0 - n_0\left(\frac{4\pi}{M}\mathscr{F}_0\right)^2 \oint \frac{dq}{(2\pi)^3}\left(\frac{p^2}{4M} - \frac{q^2}{M}\right)^{-1}$$
$$+ i\frac{4\pi}{M}\mathscr{F}_0\int G(q)\frac{d^4q}{(2\pi)^4} + in_0\left(\frac{4\pi}{M}\mathscr{F}_0\right)^2\int \mathscr{G}(p-q) \tag{10}$$
$$\times [G(q) + G(-q) + \hat{G}(q) + \check{G}(q)]\frac{d^4q}{(2\pi)^4},$$

which already accounts for the fact that the amplitude can be regarded as constant at low momenta. The second term in (10) is subtracted to take into account the renormalization of the Born scattering amplitude in second order in the interaction. We note incidentally
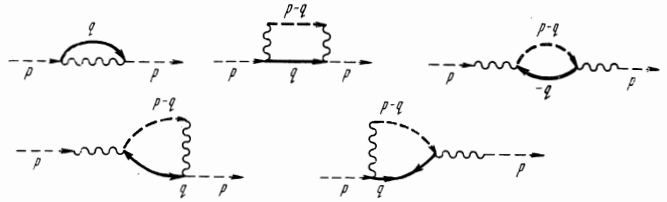

FIG. 5

that if we introduce, in addition to the nondiagonal interaction matrix element characterized by the quantity $V(q)$ in Fig. 1, the interaction $U(q)$ corresponding to simple scattering of an excited atom by an unexcited one, then it is necessary to make in (10) the substitution $\mathscr{F} \to \mathscr{F}_V + \mathscr{F}_U$. The need for this may arise if, as noted in the preceding section, the amplitude corresponding to the excitation exchange is not anomalously large.

Integrating (10) with respect to the frequency, we obtain

$$\Sigma(p) = \frac{4\pi}{M} n_0\mathscr{F}_0 - n_0\left(\frac{4\pi}{M}\mathscr{F}_0\right)^2 \oint \frac{dq}{(2\pi)^3}\left(\frac{p^2}{4M} - \frac{q^2}{M}\right)^{-1}$$
$$+ \frac{4\pi}{M}\mathscr{F}_0\int \frac{dq}{(2\pi)^3}\frac{-\varepsilon(\mathbf{q}) + \varepsilon_0(\mathbf{q}) + 4\pi n_0 f_0/M}{2\varepsilon(\mathbf{q})} \tag{11}$$
$$+ n_0\left(\frac{4\pi}{M}\mathscr{F}_0\right)^2\int \frac{dq}{(2\pi)^3}\frac{\varepsilon_0(\mathbf{q})/\varepsilon(\mathbf{q})}{\tilde{p}_0 - \varepsilon(\mathbf{q}) - \varepsilon_0(\mathbf{p}-\mathbf{q}) + i\delta}$$

We call attention to the structure of the poles in the integrand of (11). If $\tilde{p}_0$ is replaced by $\epsilon_0(p) = p^2/2M$, the denominator may vanish only if the conservation laws[8]

$$\varepsilon_0(\mathbf{p}) = \varepsilon_0(\mathbf{p}-\mathbf{q}) + \varepsilon(\mathbf{q}). \tag{12}$$

are satisfied. It is easy to verify that this equation is first satisfied only for $p = p_c$, when $v_c = (4\pi n_0 f_0)^{1/2}/M$ and plays the role of the speed of sound. Thus, when $\tilde{p}_0 < p_c^2/2M$ the denominator does not vanish and the imaginary part of (11), together with the damping of the excitation, is equal to zero.

In the limiting case $p \ll p_c$ the expression takes the form

$$\Sigma(p) = \Sigma(0) + \tfrac{1}{2}\Delta(1/M)p^2,$$

where

$$\Sigma(0) = \frac{4\pi}{M} n_0\mathscr{F}_0\left\{1 + \frac{4}{3\pi^2}\left(\frac{f_0}{\mathscr{F}_0}\right)^{1/2}[n_0(4\pi\mathscr{F}_0)^3]^{1/2}\right.$$
$$\left. + \frac{1}{3\pi^2}[n_0(4\pi f_0)^3]^{1/2}\right\},$$

and $\Delta(1/M)$ is the increment to the reciprocal of the effective mass:

$$\Delta\left(\frac{1}{M}\right) = -\frac{8}{45\pi^2 M}\left(\frac{\mathscr{F}_0}{f_0}\right)^{1/2}[n_0(4\pi\mathscr{F}_0)^3]^{1/2}.$$

Near p_c we have

$$\Sigma(p) = \Sigma(p_c) + \left(\frac{\mathscr{F}_0}{f_0}\right)^{1/2}[n_0(4\pi\mathscr{F}_0)^3]^{1/2}\varepsilon_0(p_c)\left[\left(\frac{1}{4} - \frac{2}{\pi^2}\right)\frac{p_c-p}{p_c}\right.$$
$$\left.+ \left(\frac{1}{16} - \frac{1}{4\pi^2}\right)\left(\frac{p_c-p}{p_c}\right)^2 - \frac{2}{3\pi^2}\left(\frac{p_c-p}{p_c}\right)^3\ln\frac{p_c-p}{p_c}\right] \quad (p < p_c),$$

$$\Sigma(p) = \Sigma(p_c) + \left(\frac{\mathscr{F}_0}{f_0}\right)^{1/2}[n_0(4\pi\mathscr{F}_0)^3]^{1/2}\varepsilon_0(p_c)\left[-\left(\frac{1}{4} - \frac{2}{\pi^2}\right)\frac{p-p_c}{p_c}\right.$$
$$+ \left(\frac{1}{16} - \frac{1}{4\pi^2}\right)\left(\frac{p-p_c}{p_c}\right)^2 + \frac{2}{3\pi^2}\left(\frac{p-p_c}{p_c}\right)^3\ln\frac{p-p_c}{p_c}$$
$$\left.- i\frac{2}{3\pi}\left(\frac{p-p_c}{p_c}\right)^3\right] \quad (p > p_c).$$

Here

$$\Sigma(p_c) = \frac{4\pi}{M} n_0\mathscr{F}_0\left\{1 - \frac{1}{2}\left(\frac{f_0}{\mathscr{F}_0}\right)^{1/2}[n_0(4\pi\mathscr{F}_0)^3]^{1/2} + \frac{1}{3\pi^2}[n_0(4\pi\mathscr{F}_0)^3]^{1/2}\right\}.$$

The obtained formulas pertain to a model in which the excited state is not degenerate. For excitons corresponding to the He* atom in the P state, allowance for degeneracy would lead to a splitting into two branches corresponding to longitudinal and transverse excitons with a splitting on the order of $4\pi n_0 \mathscr{F}_0 / M$. We note also that in order to neglect radiative decay processes we must add $n_0 \lambda^3 \gg 1$ to the condition (4).

OPTICAL PROPERTIES OF LIQUID He II

The results obtained above are, of course, connected with a model. At the density of real He II it is impossible to obtain quantitative relations between, say, the effective mass and the mass of the atom. Nonetheless, several qualitative considerations can be advanced. First, at T = 0 the ground state of He II has Galilean invariance. It might seem, therefore, that strongly excited states, if stable, should be classified not only with respect to their momentum, which corresponds to the motion of the "center of gravity," but also by their angular momentum, i.e., the optical spectrum of the quantum liquid He II is similar to a certain degree to the atomic spectrum of He. At first glance it seems that with decreasing interaction this spectrum goes over adiabatically into the spectrum of the atom. We have shown above, however, that in an ideal Bose gas (the atoms in the ground state do not interact with one another) the exciton loses its momentum as it knocks out particles from the condensate. Only in the presence of a sufficient interaction in the Bose gas can the "exciton" be stable and have a quadratic dispersion law

$$\varepsilon(\mathbf{p}) = \tilde{\omega}_0 + \mathbf{p}^2 / 2M_{\mathrm{eff}}.$$

Therefore the optical spectrum of He II, generally speaking, should not correspond exactly to the atomic spectrum.

At finite temperatures, a stable excited level will have a width governed by the collisions with the phonons and rotons. To calculate the collision times it is necessary to know the corresponding interaction amplitudes. To obtain estimates, we therefore proceed as follows. The excitation-interaction energy is expressed in the form $U_0 \delta(\mathbf{r}_1 - \mathbf{r}_2)$, and the scattering amplitude

$$F_0 = M_{\mathrm{eff}} U_0 / 2\pi\hbar^2$$

is estimated by using the value $|U_0|^2 = 3.5 \times 10^{-76}$ (erg-cm^3)^2 obtained by Khalatnikov[9] from data on the roton viscosity of He II. Then, taking $M_{\mathrm{eff}} = M_{\mathrm{He}}^4$, we obtain $F_0 \approx 4 \times 10^{-8}$ cm, which corresponds to the interatomic distances in helium. The mean free path with respect to scattering by rotons[9] is

$$l \sim 1 / N_\nu \sigma \sim M_{\mathrm{eff}} / N_\nu F_0^2 \mu,$$

where μ is the roton mass and N_r is the number of rotons per unit volume. At T = 0.6°K, the mean free path for scattering by rotons is of the order of several centimeters, as was indeed observed in the experiments.[1] Above the λ point, an estimate of the mean free path from data on scattering by neighboring atoms would lead to collision times $\tau \sim 10^{-13} - 10^{-12}$ sec.

Optical measurements in He II at T = 1.7°K were made by Hill et al.[10] Optical excitation from the ground state into an excited state connected with the ground state by a dipole transition is apparently difficult to observe, owing to the strong absorption in this frequency region. The helium was therefore bombarded in [10] by electrons, and the emission and absorption spectra in the ultraviolet were then investigated for transitions in which the lower state was metastable. In particular, one of the lines was interpreted in [10] as a transition from 2^3P into 2^3S. The emission and absorption lines were shifted by approximately 70 cm^{-1} relative to each other and their widths were approximately of the same order. As to the line width, if only the level 2^3P is broadened, then, estimating the time of collisions with rotons at T = 1.7°K as before, we get $\tau \sim 10^{-8}$ sec. We would have obtained the correct order of magnitude of the line width $\delta\lambda/\lambda \sim 10^{-2}$ at $\tau \sim 10^{-12}$ sec. The small relative shift of the maxima of the absorption and emission lines should presumably correspond to interatomic collision frequencies $1/\tau \sim 10^{12} - 10^{13}$ sec^{-1}, i.e., to shifts $10^2 - 10^3$ cm^{-1}, which is close to the experimental data. Of course, the electronic transition is accompanied by emission and absorption of a large number of phonons and rotons. Indeed, taking the roton energy as the characteristic energy, we would obtain only $\Delta\nu \sim 6$ cm^{-1}. The relatively small value of the observed shift (see the calculation in [11]) and also the good agreement between the picture of the rotational structure of the excited He$_2^*$ molecules (see below) and the observed spectrum in the region $\lambda \approx 21\,000$ Å, is evidence, in the opinion of the authors of [10], in favor of formation of a "bubble" around the excitation in He II, although the foregoing estimates show that we obtain the correct order of magnitude for the line broadening and shift in a liquid by estimating the frequency of the collisions between the atoms. In other words, the optical characteristics of the spectrum are not sensitive to the transition through the λ point.

As already mentioned, the electronic transition is accompanied by simultaneous emission or absorption of many phonons and rotons. Therefore the spectral lines in He II, unlike the atomic spectrum, will have a structure corresponding to these processes even at low temperatures.

We consider now the dielectric constant $\epsilon(\omega, \mathbf{k})$, which is connected with the "polarization operator" of the medium $\Pi(\omega, \mathbf{k})$ by the relation (see, for example, [6])

$$\varepsilon(\omega, \mathbf{k}) - 1 = \frac{4\pi}{c^2 k^2} \Pi(\omega, \mathbf{k}).$$

The general form of $\Pi(\omega, \mathbf{k})$ is quite complicated and includes, in particular, derivative diagrams such as 6a, which correspond precisely to a transition accompanied by emission and absorption of ordinary excitations in He II. It is interesting, however, that the frequencies corresponding to the transition from the ground state, the expression for $\Pi(\omega, \mathbf{k})$ must also contain the diagram of Fig. 6b, which is proportional to the Green's function of the exciton
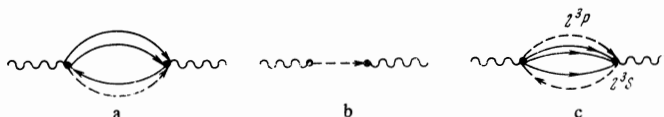

FIG. 6

$$a \Big/ \left(\omega - \tilde{\omega}_0 - \frac{k^2}{2M_{\text{eff}}} + \frac{i}{\tau} \right).$$

At low temperatures, when the collision time is inversely proportional to the normal density in He II, the dielectric constant must thus have a pole term

$$\varepsilon(\omega, k) = \varepsilon_0 - \frac{4\pi n_0}{\hbar} \frac{d^2}{\omega - \tilde{\omega}_0 - k^2/2M_{\text{eff}}}, \qquad (13)$$

where d is the effective dipole matrix element, n_0 is the number of particles in the condensate, and ω_0 is the frequency corresponding to the energy of the first excited level (exciton). Therefore at sufficiently low temperatures ($1/\tau \ll \Delta_r$) the absorption lines in this frequency region must contain, in addition to a certain smeared structure, also a sharp peak corresponding to expression (13), i.e., to a transition without emission or absorption of rotons. Expression (13) indicates that at the frequencies ω_0 corresponding to the transition from the ground state there could appear effects connected with spatial dispersion. Unfortunately, owing to the smallness of the speed of sound in helium, these phenomena are apparently difficult to observe. Figure 6c shows the diagram making the contribution to $\epsilon(\omega, k)$ for the transition from 2^3S to 2^3P. It is obvious that in this case the line width is not determined by the width $1/\tau$ of the 2^3P level. At the same time, the intensity is proportional to the number of atoms in the metastable state 2^3P.[1] In light of the foregoing, it would be of interest to trace the variation of the line width at $T < 1.7°K$. Is this width really independent of the temperature?

In [10] they also observed a detailed spectrum in the region $\lambda = 21\,100$ Å, which was interpreted as the analog of the rotational structure for the $(0-0)^3\Sigma_u \to {}^3\Pi_u$ transition of an excited He_2 molecule. It is interesting that the positions of the lines in He II can be identified with the aid of the molecular spectra of He_2^*, a fact favoring the "bubble" model. The "bubble" model, however, does not agree well with the rather large width of the rotational levels, $\delta\lambda/\lambda \sim 10^{-2}$.

Our preceding reasoning was based on the fact that the probability of nonradiative electronic transition is small. As to the excitation of the rotational levels, the corresponding energy interval is much smaller here. According to [12], the rotational quantum of the He_2^* molecule is $B \approx 10°K$, i.e., it is comparable with the characteristic roton energy $\Delta_r \approx 8.6°K$. Therefore for rotational levels with low numbers, there are two possible types of processes that can lead to damping of the rotation, namely interaction with the normal excitations and decay, i.e., transition to a lower rotational level with simultaneous emission of several rotons and phonons. The width of the intensity peaks in [10] turns out to be of the same order as the distance between levels $\delta\lambda/\lambda \sim 10^{-2}$, and requires a collision time of the "normal" order $\tau \sim 10^{-12}$ sec. In other words, in this case, too, the broadening mechanism is apparently not connected with the distinctive feature of He II (the decrease of the normal density) and reduces to a conversion of the rotational part of the energy into translational energy of the helium atoms. In other words, the "exchange" of

the rotational energy is accompanied by acceleration of the helium atoms to energies on the order of several dozen degrees K, and they then lose their energy and are transformed into ordinary rotons and phonons. The observation of the rotational structure in excitations such as the $He_2^{*3}\Sigma_g$ molecule raises the question of the possible presence in He II of a fraction of the He_2 molecule in the ground state, a question that calls for optical experiments in the far infrared.

Let us return once more to the question of the level width. The estimated interatomic collision frequency $1/\tau \sim 10^{12} - 10^{13}$ shows that at such low temperatures one can state that the He II spectrum corresponds to a quantum liquid with the spectrum of rarefied helium (the line broadening and shift are not too large). On the other hand, the question of the existence of the exciton, which is a high-energy excitation of small width (smaller than $1/\tau$), is solved differently for different types of excitations. In the first part of the paper we considered mainly the first excited level, which is connected with the ground state by an allowed transition. Such a level, owing to resonant reabsorption processes, should possess a small width at sufficiently low temperatures and constitutes a new type of excitation in He II. (We neglect the probability of the nonradiative electronic transition.) The existence of a metastable level (of the 2^3S type) is a hypothesis based on the Galilean invariance of the ground state of He II, which is apparently confirmed by experiment.[1,2,3,10] The statement that this level has zero width at low temperatures includes the assumption that the electronic configuration in He II, corresponding to such an excitation, is formed in such a way as to forbid optical transitions and transitions with emission of ordinary excitations in He II. (For such excitations, the processes of reradiation in He II are insignificant.) Transitions with absorption of thermal excitations disturb the metastability of the state. Finally, excited states of higher energy, for which transitions to intermediate states are possible (for example, 2^3P) cannot have a width smaller than the natural width, on the order of $1/\tau \sim 10^8$. According to (13), the collision time with respect to rotons is of the same order of magnitude already at temperatures close to $1.7°K$. All these excitations take part in the normal motion of the superfluid liquid and could, in principle, be separated in a helium stream because of their different lifetimes at low temperature.[2] In [11] the current, due apparently to metastable states such as the He_2^* molecule, increases with decreasing temperature (below $0.6°K$) approximately in inverse proportion to the normal density. In [3], to the contrary, the fraction of radiation captured by the metastable states remains constant at given instrument dimensions below $0.9°K$. The free path with respect to collisions is already very large under these conditions. In [1], bombardment with high-energy α particles produced, near the source, an ordinary recombination cascade that included also transitions to the states 2^3S of He^* or $^3\Sigma_g$ of He_2^*. Therefore, if the metastability of these states is disturbed primarily by a radiative transition with absorption of rotons (and not two-photon emission), then the results of Hereford et al.[3] become understandable.

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[1] A diagram of the type of Fig. 6b could arise at extremely low temperatures if the configuration of the 2^3S states were sufficient for their "Bose condensation."

[2] We are indebted to A. I. Shal'nikov for this remark.

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