Stimulated Mandel'shtam-Brillouin Emission in an Optical Resonator

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Stationary conditions for generation of an arbitrary number of Stokes components of stimulated Mandel'shtam-Brillouin radiation in an optical resonator excited by an external monochromatic beam are considered. General expressions are obtained for the intensities of all radiation components as depending on the intensity of the external incident field. Generation of stimulated Mandel'shtam-Brillouin radiation in the presence of a nonlinear absorber in the cavity is also investigated. It is found that under these conditions and in the steady state the phases of the radiation field components are related to each other in such a way that the omitted radiation consists of a set of successive pulses.

INTRODUCTION

THE process of stimulated Mandel'shtam-Brillouin emission (SMBE) has been so far investigated theoretically to the greatest extent for the case when it occurs in a sample situated outside some optical resonator (see, for example, [1]). The authors of papers devoted to SMBE in optical resonators (see, for example, [2,3]) have confined themselves to generation of one Stokes component, and furthermore only in the nonstationary regime corresponding to the start of its excitation. At the same time, the question of the stationary field oscillations remained open. The present paper is devoted to a theoretical investigation of this question, and the general case of generation of an arbitrary number of radiation components. Just as in [4,5] (where the process of stimulated Raman emission considered under similar conditions), it is assumed below that the mirrors of the open resonator reflect well both at the frequency of the exciting beam incident on one of the slightly-transparent mirrors along the resonator axis, and at all the Stokes frequencies. Under these conditions, we derive equations that describe the dynamics of the generation of the SMBE components, present a general solution of these equations for the stationary generation regime, and investigate on this basis the dependence of the number of the generated components and their intensities on the intensity of the exciting beam.

We discuss also the generation of a number of Stokes components in the presence of a nonlinear absorber in the resonator. It is shown that under these conditions there can occur in the stationary regime a strong parametric (phase-dependent) interaction between the different SMBE components. As a result, the total output radiation takes the form of a sequence of ultrashort pulses. An important role in this synchronization of the SMBE components is played by the nonlinear absorber, in contrast to the stimulated Raman emission in an optical resonator, where such a synchronization of the stimulated-emission components[6] can be obtained also without a nonlinear absorber, as a result of the intrinsic parametric interaction between these components.

1. DERIVATION OF EQUATIONS

Let an optical resonator filled completely with a homogeneous active medium be excited by an external monochromatic beam of given intensity, incident along the resonator axis. We assume that mode selection obtains in the resonator, so that only natural oscillations with the smallest transverse numbers, differing only in the value of the longitudinal (axial) index, can be excited. We assume also that the reflection coefficients of the mirrors can be close to unity for any number of Stokes and anti-Stokes frequencies and at the frequency of the pump wave. We confine ourselves below to the most interesting case, when the Mandel'shtam-Brillouin shift ω_r is close to the difference between frequencies of neighboring resonator modes. In this case, obviously, an external beam of frequency close to one of these frequencies will excite a corresponding axial resonator mode, and also a number of modes whose natural frequencies are close to the frequencies of the scattering components.

Thus, we can write the following expansion for the field in the resonator:

$$\mathbf{E}=\sum_l \mathscr{E}_l(t)\mathbf{E}_l(\mathbf{r}), \tag{1}$$

where l is the order of the SMBE component and $\mathbf{E}_l(\mathbf{r})$ is the coordinate part of the natural mode having a frequency close to the frequency ω_l of this component. For simplicity we shall consider a planar resonator. In this case the expression for the eigenfunction $\mathbf{E}_l(\mathbf{r})$ can be represented with sufficient accuracy in the form[7]

$$\mathbf{E}_l(\mathbf{r})=\mathbf{g}_l(\mathbf{r}_\perp)\sin k_l z;\quad k_l=\pi m_l/L, \tag{2}$$

where L is the distance between mirrors (the z axis coincides with the resonator axis), and m_l is an arbitrary (large) integer. The frequency ω_l corresponding to this mode is equal to $\omega_l = \pi c/Ln(\omega_l)$, where $n(\omega_l)$ is the refractive index of the medium at the frequency ω_l. Under these conditions, obviously, the spectrum of the time-dependent function $\mathscr{E}_l(t)$ is concentrated near the frequency ω_l.

The stimulated Mandel'shtam-Brillouin emission is connected with the change produced in the dielectric tensor of the medium by the hypersonic wave generated in the medium by the external electromagnetic wave, as well as by the excited Stokes and anti-Stokes components themselves. The dielectric tensor ϵ is a function of the deviation ρ of the density in the acoustic wave from its equilibrium value. As usual, we confine ourselves to the zeroth and first-order terms of the expansion of ϵ in powers of ρ:

$$\varepsilon = \varepsilon_0 + (\partial\varepsilon/\partial\rho)_S\rho. \qquad (3)$$

In the hydrodynamic approximation in a medium with a small heat-conduction coefficient, the equation for ρ is (see, for example, [1])

$$\frac{\partial^2\rho}{\partial t^2} - \Delta\left[v^2\rho + \Gamma\frac{\partial\rho}{\partial t}\right] = \overline{Y}_\rho\Delta\mathbf{E}^2, \quad \overline{Y}_\rho = -\frac{1}{8\pi}\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T\rho_0, \qquad (4)$$

where ρ_0 is the equilibrium value of the density, v is the speed of sound in the medium, $\Gamma = (\zeta + {}^4/_3\eta)/\rho_0$, and ζ and η are the viscosity coefficients.[1)]

Substituting the expansions (1) and (2) into the right-hand side of (4) and omitting the terms that are nonresonant with respect to ρ (proportional to $\mathscr{E}_l\mathscr{E}_{l+k}$, $k \neq 0, 1$), we obtain

$$\overline{Y}_\rho\Delta\mathbf{E}^2 = \overline{Y}_\rho\sum 2k_l^2 g_l g_{l+1}\cos[(k_{l+1}+k_l)z]\mathscr{E}_l\mathscr{E}_{l+1}. \qquad (5)$$

We have omitted also the terms proportional to $\mathscr{E}_l\mathscr{E}_l$, since it can easily be verified that these terms, in the case of smooth functions $g_l(\mathbf{r}_\perp)$, do not make a noticeable contribution to the expression for ρ (we define a function as smooth if the scale of its variation along $\mathbf{r}_\perp$ is much larger than the length of the light wave $2\pi/k_l$). Starting from the form (5) of the right-hand side of (4), we can seek a solution for ρ in the form

$$\rho = \sum \rho_l(t)\Lambda_l(\mathbf{r}),$$

$$\Lambda_l(\mathbf{r}) = 2k_l^2 g_l g_{l+1}\cos[(k_{l+1}+k_l)z].$$

Then the equation (4) breaks up into the system

$$\ddot{\rho}_l + 2\overline{h}\dot{\rho}_l + \tilde{\omega}_l^2\rho_l = \overline{Y}_\rho\mathscr{E}_l(t)\mathscr{E}_{l+1}(t), \qquad (6)$$

where $2\overline{h} = 4k_l^2\Gamma$ and $\tilde{\omega}_l^2 = 4k_l^2 v^2$. Writing down Maxwell's equations for the field in the resonator in the form of a system of equations for the time variation of the coefficients $\mathscr{E}_q(t)$ of the expansion in the oscillation modes, we obtain

$$\ddot{\mathscr{E}}_q + \frac{\omega_q}{Q_q}\dot{\mathscr{E}}_q + \omega_q^2\mathscr{E}_l$$
$$= \frac{\omega_q^2}{4\pi N_q}\left(\frac{\partial\varepsilon}{\partial\rho}\right)_S\sum_l a_{ql}\rho_l\mathscr{E}_{2l+1-q} + \mathrm{Re}[\overline{F}_q e^{-ipt}]; \qquad (7)$$

$$a_{ql} = \frac{Lk_l^2}{2}\int(g_l g_{l+1})(g_q g_{2l+1-q})\,d\mathbf{r}_\perp,$$

$$\overline{F}_q = \frac{\omega_q^2}{N_q}\int(\mathbf{P}_0\mathbf{E}_q - \mathbf{M}_0\mathbf{H}_q)\,dv;$$

$$N_q = \frac{1}{4\pi}\int\varepsilon_0\mathbf{E}_q^2\,dv > 0$$

is the norm of the mode $\mathbf{E}_q$; $\mathbf{P}_0$ and $\mathbf{M}_0$ are the complex amplitudes of the extraneous polarization and magnetization, which are determined by the external exciting electromagnetic wave; Q_q is the figure of merit of the mode $\mathbf{E}_q$, and p is the frequency of the exciting beam. The system (6) and (7) describes the dynamics of the SMBE process under the considered conditions.

2. STATIONARY GENERATION OF SMBE COMPONENTS

We make the change of variable $t = t_1/p$ in Eqs. (6) and (7), and for simplicity we again denote the dimensionless time t_1 by t. Then the initial system takes the form

$$\ddot{\rho}_l + \tilde{\Omega}_l^2\rho_l = -2h\dot{\rho}_l + Y_\rho\mathscr{E}_l(t)\mathscr{E}_{l+1}(t),$$
$$\ddot{\mathscr{E}}_q + \Omega_q^2\mathscr{E}_q = -\frac{\Omega_q\dot{\mathscr{E}}_q}{Q_q}$$
$$+\frac{\Omega_q^2}{4\pi N_q}\left(\frac{\partial\varepsilon}{\partial\rho}\right)_S\sum a_{ql}\rho_l(t)\mathscr{E}_{2l+1-q} + \mathrm{Re}[F_q e^{-it}], \qquad (8)$$

where

$$h = \frac{\overline{h}}{p}, \quad \tilde{\Omega}_l = \frac{\tilde{\omega}_l}{p}, \quad Y_\rho = \frac{\overline{Y}_\rho}{p^2}, \quad \Omega_q = \frac{\omega_q}{p}, \quad F_q = \frac{\overline{F}_q}{p^2}.$$

To investigate the stationary solutions of the obtained system of equations, we make the substitutions

$$\mathscr{E}_q = {}^1/_2 Y_q\exp[-i(\Omega_q - \Delta_q)t] + \text{c.c.},$$
$$\rho_l = {}^1/_2 X_l\exp[-i(\Omega_{l+1} - \Omega_l)t] + \text{c.c.}, \qquad (9)$$

where Δ_q are constants and will be determined below. Substituting these expressions in (8) and averaging over the fast oscillations (this is justified because the system (8) is nearly conservative), we obtain

$$\dot{Y}_q = -(\mu_q + i\Delta_q)Y_q +$$
$$+ i\gamma_q[a_q^{(1)}X_{q-1}Y_{q-1} + a_q^{(2)}X_q^*Y_{q+1}] + iF_0\delta_{q0}, \qquad (10)$$
$$\dot{X}_l = -(h + i\delta_l)X_l + iY_\rho Y_{l+1}Y_l^*/2\tilde{\Omega}_l,$$

where

$$\mu_q = \frac{\Omega_q}{2Q_q}, \quad \gamma_q = \frac{\Omega_q}{16\pi N_q}\left(\frac{\partial\varepsilon}{\partial\rho}\right)_S, \quad a_q^{(1)} = a_{q,q-1},$$
$$a_q^{(2)} = a_{q,q}, \quad \delta_l = \Omega_l - \Omega_{l+1} + \tilde{\Omega}_l.$$

We now consider the stationary solutions of (10), putting $\dot{Y}_q = 0$ and $\dot{X}_q = 0$. Eliminating X_q, we obtain the following system of algebraic equations for the Y_q:

$$Y_q[-(\mu_q + i\Delta_q) + (\rho_q^{(1)}|Y_{q+1}|^2 - \rho_q^{(2)}|Y_{q-1}|^2)] + iF_0\delta_{q0} = 0;$$
$$\rho_q^{(1)} = \frac{\gamma_q Y_\rho a_q^{(2)}}{2\tilde{\Omega}_q(h + i\delta_q)}, \quad \rho_q^{(2)} = \frac{\gamma_q Y_\rho a_q^{(1)}}{2\tilde{\Omega}_{q-1}(h + i\delta_{q-1})}. \qquad (11)$$

Each term in the left-hand side of (11) has a definite physical meaning. The first term in the square brackets is due to the intrinsic attenuation of the field in the resonator in the absence of SMBE. The second term corresponds to two-photon Stokes transitions for all pairs $\mathbf{E}_l$ and $\mathbf{E}_{l-1}$ of neighboring radiation components, and the term $iF_0\delta_{q0}$ is due to the external exciting beam. Thus we see that in the stationary generation regime the SMBE components interact only through two-photon transitions, and it is therefore clear (see also below) that at $q \leq -1$ only the modulus of the corresponding amplitude Y_q will be determined for each component, and the phase is arbitrary. The component Y_0, according to (11), should be fully determined, since its phase is determined by the phase of the exciting beam.

The system (11) has the same form as the system of equations for stationary field oscillations in the case of stimulated Raman emission in a dispersive medium. The latter was investigated by one of the authors,[5] and only the results will be given here. For convenience we shall denote the solutions of (11) for stationary amplitudes by a tilde. The values of $|\tilde{Y}_q|^2$ depend on $|F_0|^2$ or, which is the same, on $|Z_0|^2$, where $Z_0 = iF_0/(\mu_0 + i\Delta_0)$ is the complex amplitude of the field oscillations that would be excited in the resonator by an external beam if there were no oscillations at the Stokes frequencies. The aforementioned dependence is, in particular, such that Stokes components of ever-increasing orders are excited in succession with increasing $|Z_0|^2$ (the amplitudes of the anti-Stokes oscillations are equal to zero at any pump-wave intensity).

[1)]When solving Eq. (4) for the case of SMBE in a resonator we can, as usual, disregard the presence of the boundaries of the medium, since the mean free path of the phonons taking part in the SMBE process is much smaller than the dimensions of the medium.

The threshold value of $|Z_0|_m^2$ at which the m-th Stokes component appears is given by

$$|Z_0|_m^2 = \begin{cases} \beta^{(m-1)}/D(\beta^{(m)}), & m=-1,-3,\ldots \\ \beta^{(m)}/D(\beta^{(m-1)}), & m=-2,-4,\ldots \end{cases} \tag{12}$$

where

$$\alpha^{(0)} = \alpha^{(-1)} = 1, \quad \beta^{(0)} = \beta^{(-1)} = 0, \quad \beta^{(-2)} = \beta_{-1}, \quad \beta^{(-3)} = \beta_{-2},$$
$$\alpha^{(q)} = \begin{cases} \alpha_{|q|/2,\,q} & \text{for } q=-2,-4,\ldots \\ \alpha_{|q|-1/2,\,q} & \text{for } q=-3,-5,\ldots \end{cases}$$
$$\beta^{(q)} = \begin{cases} \beta_{|q|/2,\,q} & \text{for } q=-4,-6,\ldots \\ \beta_{|q|-1/2,\,q} & \text{for } q=-5,-7,\ldots \end{cases} \tag{13}$$
$$\alpha_{kq} = \prod_{l=1}^{k} \alpha_{q+2l-1}, \quad \beta_{kq} = \sum_{l=1}^{k-1} \Big(\prod_{l'=l+1}^{k} \alpha_{q+2l'+1} \Big) \beta_{q+2l-1} + \beta_{q+2k-1},$$
$$\alpha_q = \frac{\rho_q^{(2)\prime}}{\rho_q^{(1)\prime}}, \quad \beta_q = \frac{\mu_q}{\rho_q^{(1)\prime}}, \quad D(y) = \frac{\mu_0^2+\Delta_0^2}{(\mu_0+\rho_0^{(2)\prime} y)^2 + (\Delta_0 + \rho_0^{(2)\prime\prime} y)^2}$$

($\rho_q^{(1)\prime}$, $\rho_q^{(2)\prime}$ and $\rho_q^{(1)\prime\prime}$, $\rho_q^{(2)\prime\prime}$ are the real and imaginary parts of the numbers $\rho_q^{(1)}$ and $\rho_q^{(2)}$).

It follows from (12) that the inequalities $|Z_0|_m^2 < |Z_0|_{m-1}^2$ are always satisfied (m is an arbitrary negative integer). In the interval $|Z_0|_m^2 < |Z_0|^2 < |Z_0|_{m-1}^2$, only the amplitudes $|\tilde{Y}_q|$ of the components with $q \geq m$ differ from zero, and the expressions for these quantities differ in accordance with whether the total number m of the generated frequencies is even or odd. At odd m we have

$$|\tilde{Y}_q|^2 = \begin{cases} \dfrac{1}{\alpha^{(q)}} \Big[\Psi\Big(\dfrac{|Z_0|^2}{\beta^{(m-1)}} \Big) - \beta^{(q)} \Big], & q=-1,-3,\ldots \geqslant m \\ \dfrac{1}{\alpha^{(q)}} (\beta^{(m-1)} - \beta^{(q)}), & q=0,-2,-4,\ldots > m \end{cases}; \tag{14}$$

where

$$\Psi(y) = |\rho_0^{(2)}|^{-2} \{ -(\mu_0 \rho_0^{(2)\prime} + \Delta_0 \rho_0^{(2)\prime\prime})$$
$$+ [\,|\rho_0^{(2)}|^2 (\mu_0^2 + \Delta_0^2)(y-1) + (\mu_0 \rho_0^{(2)\prime} + \Delta_0 \rho_0^{(2)\prime\prime})^2]^{1/2} \}.$$

At even m

$$|\tilde{Y}_q|^2 = \begin{cases} \dfrac{1}{\alpha^{(q)}} (\beta^{(m-1)} - \,.\,^{(q)}), & q=-1,-3,\ldots > m \\ \dfrac{1}{\alpha^{(q)}} [\,|Z_0|^2 D(\beta^{(m-1)}) - \beta^{(q)}], & q=0,-2,\ldots \geqslant m \end{cases} \tag{15}$$

The quantities Δ_q (which determine the oscillation frequencies of the corresponding components) are given for all m by the formulas

$$\Delta_q = \rho_q^{(1)\prime\prime} |\tilde{Y}_{q+1}|^2 - \rho_q^{(2)\prime\prime} |\tilde{Y}_{q-1}|^2, \quad q=-1,-2,\ldots,(m+1),$$
$$\Delta_m = \rho_m^{(1)\prime\prime} |\tilde{Y}_{m+1}|^2. \tag{16}$$

It follows directly from the derived expressions that if the total number of Stokes components excited in the resonator is odd, then the intensities of the beams emerging from the resonator with frequencies of the even components do not depend on the pump-wave intensity, whereas the intensities of the odd components increase monotonically with increasing pump intensity. On the other hand, if the total number of excited Stokes components is even, then it is the output intensities of the beams with frequencies of all the odd components which are independent of the amplitude of the external beam, and the intensities of the even components increase with increasing incident intensity, but the dependence, unlike the preceding case, is linear.

To gain a more complete idea of the dependence of the oscillation intensities of the different Stokes components on the intensity of the exciting beam, the figure shows plots of the corresponding quantities. We present also an explicit expression, which is useful in calculations, for $|Z_0|^2$ as a function of the complex amplitude of the field intensity $\mathbf{E}^+$ of the incident beam on the surface of the first mirror:

$$|Z_0|^2 = \frac{|T_1|^2}{(\mu_0^2+\Delta_0^2)(k_0 L)^2} \left| \frac{\int \mathbf{E}^+ g_0\, dS}{\int g_0^2\, dS} \right|^2, \tag{17}$$

where T_1 is the complex transparency coefficient with respect to the electric field for the first mirror at the frequency of the incident beam. With the aid of (12) and (17) we find immediately that for typical media (carbon disulfide, diethyl ether, etc.) at a mirror reflection coefficient on the order of 0.8 and at a resonator length ~2 cm the threshold intensity I_1 of the incident beam, needed to generate the Stokes radiation, is of the order of 1 mW/cm^2. The intensity of the field oscillations inside the resonator are of the order of $|Z_0|_{-1}^2$ (i.e., on the order of 10 mW/cm^2). It also follows from these expressions that with increasing intensity of the exciting beam, at mirror reflection coefficients independent of the frequency, the intensities of the different components in the resonator increase like $|m-q-1|\,|Z_0|_{-1}^2$.

3. GENERATION IN THE PRESENCE OF A NONLINEAR ABSORBER

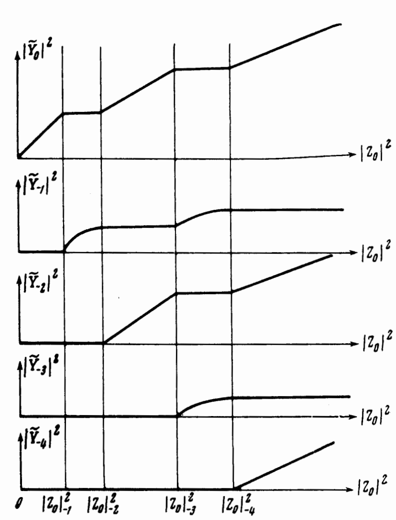
Let us consider now a case when a homogeneously distributed nonlinear absorber is introduced into the resonator (for example, a dye solution), and leads to the appearance of an imaginary increment ϵ'' to the dielectric constant of the active medium in the resonator:

$$\varepsilon'' = 4\pi i \alpha / (1 + \beta \overline{E^2}), \tag{18}$$

where the bar denotes averaging over the fast oscillations, and accordingly the quantity $\overline{E^2}$, expressed in terms of the complex amplitudes Y_q, can be written in the form

$$\overline{E^2} = {}^1\!/_2 \sum_{l,l'} Y_l Y_{l'}^* \exp\{-i(\Omega_l - \Omega_{l'})t\} \mathbf{E}_l \mathbf{E}_{l'},$$

and α and β are real numbers. The relation (18) is usually connected with saturation in a two-level system in the stationary regime, and is therefore valid only if the



time scale of the variation of $\overline{E^2}$ as a function of the time is much larger, at all t, then the time of transverse relaxation, an assumption which we shall make from now on.[2)] In addition, at a sufficiently small nonlinearity coefficient β and a limited value of $\overline{E^2}$ (which is certainly satisfied at a finite number of excited natural modes of the resonator) we can put approximately

$$\varepsilon'' = 4\pi i\alpha(1 - \beta\overline{E^2}). \qquad (19)$$

In the right-hand side of the system (10), for the complex amplitudes of the oscillations of the SMBE components, we should now add the term corresponding to the polarization current due to the nonlinear absorber. Equations (10) then transform into a system of equations of the following type:

$$\dot{Y}_q = -(\mu_q + i\Delta_q) + i\gamma_q[a_q^{(1)} X_{q-1} Y_{q-1} + a_q^{(2)} X_q^* Y_{q+1}]$$
$$-\frac{\Omega_q}{4N_q}\Big\{\alpha \sum_{l''} Y_{l''} \exp[-i(\Omega_{l''} - \Omega_q)t] \int (\mathbf{E}_{l''}\mathbf{E}_q)\,dv$$
$$-\frac{\alpha\beta}{2}\sum_{l,l',l''} Y_l Y_{l'}^* Y_{l''} \exp[-i(\Omega_l + \Omega_{l''} - \Omega_{l'} - \Omega_q)t]\cdot$$
$$\cdot\int (\mathbf{E}_l\mathbf{E}_{l'})(\mathbf{E}_{l''}\mathbf{E}_q)\,dv\Big\} + iF_0\delta_{q0}. \qquad (20)$$

It is easy to verify that the first term in the curly brackets differs from zero only when $l'' = q$, and the second when $l'' = q + l' - l$. Under real conditions, owing to the inversion of the refractive index of the medium, the contribution of the terms with $l' = q + s + k$ and $l = q + s$ decreases with increasing $|s|$ and $|k|$ (owing to the increase of the oscillation frequencies of the corresponding exponential factors in (20)). Taking this into account, we shall henceforth consider for simplicity the following model: we assume that all the terms with $|k| > 1$, $|s| > 1$ are equal to zero (the number s is arbitrary at k = 0, and vice versa), whereas when $|k| \le 1$, $|s| \le 1$ the frequencies of the oscillating terms will be assumed equal to zero. Such an approximation is justified by the fact that we are interested only in the relation between the phases of the radiation components (of the quantities $Y_q = |Y_q|\, e_q^{i\Phi}$), and the exact values of the moduli $|Y_q|$ of the corresponding amplitudes are of no principal significance.

If the frequency of the external exciting beam coincides exactly with one of the natural frequencies of the resonator, if the Mandel'shtam-Brillouin shift is exactly equal to the difference between the frequencies of two arbitrary neighboring resonator modes, and if the reflection coefficients of the mirrors are small at the anti-Stokes frequencies (i.e., $Y_q = 0$ at $q \le 1$), then the equation for the complex amplitudes of the stationary oscillations of the field components take the form ($q \le 0$)

$$\eta_q Y_q + iF_0\delta_{q0} + 2\varkappa_q Y_{q-1} Y_q^* Y_{q+1} + \varkappa_q (Y_{q+1})^2 Y_{q+2}^* + \varkappa_q (Y_{q-1})^2 Y_{q-2}^* = 0;$$

$$\eta_q = -\mu_q + \rho_q^{(1)} |Y_{q+1}|^2 - \rho_q^{(2)} |Y_{q-1}|^2 - \frac{\alpha\Omega_q}{4N_q}\int \mathbf{E}_q^2\,dv + 2\varkappa_q \sum_{s\le 0} |Y_s|^2,$$

$$\varkappa_q = \frac{\alpha\beta\Omega_q}{8N}\int (\mathbf{E}_q\mathbf{E}_{q+1})(\mathbf{E}_{q+1}\mathbf{E}_{q+2})\,dv, \qquad (21)$$

$iF_0 = |F_0|e^{i\Psi_0}$, and Ψ_0 is the phase determined by the external exciting beam. Comparing (21) with (10), we see that in the presence of a nonlinear absorber there is produced between the SMBE components an interaction described by the last three terms of (21). Multiplying both sides of the equation by Y_q^* and summing over q, we can easily verify that the phase of the amplitude Y_0 is equal to Ψ_0, i.e., it is determined by the external exciting beam, just as in the absence of the nonlinear absorber. Taking this into account and solving the system (21) in succession for all $q \le 0$ (starting with q = 0) we verify that the phases Φ_q of the radiation field components Y_q are connected by the relation

$$\Phi_q = \Phi_0 + |q|(\Phi_{-1} - \Phi_0) + \pi m_q,$$

where m_q is a definite integer and the phase Φ_{-1} of the first Stokes component is arbitrary.

Relations (22) mean that the entire aggregate of the SMBE components is broken up into two groups, in one of which

$$\Phi_q = \Phi_0 + |q|(\Phi_{-1} - \Phi_0), \qquad (23)$$

and in the other

$$\Phi_q = \Phi_0 + |q|(\Phi_{-1} - \Phi_0) + \pi. \qquad (24)$$

Since (23) and (24) are analogous, let us consider for concreteness the first of them. If relations (23) are satisfied and the number of generated components is large enough, then the total output radiation will be a sequence of ultrashort pulses separated by the time interval $T = 2\pi/\omega_r$, and the duration of each pulse τ will be of the order of T/N, where N is the number of generated Stokes components. Thus, at a typical Mandel'shtam-Brillouin shift $\omega_r \approx 0.1\ \text{cm}^{-1}$ and $N \sim 100$ we obtain $\tau \approx 3\times 10^{-12}$ sec.

In conclusion, we note that inasmuch as the number of generated Stokes components depends on the intensity of the exciting beam, the duration of the pulses can be varied over a wide range by varying the incident intensity.

[2)]The quantity α in (18) is positive at normal population of the energy levels in question, and negative in the case of inversion. In the latter case, self-excitation of the field oscillations in the resonator is possible without an external exciting beam, i.e., the role of the nonlinear absorber can be played by the active laser medium itself.

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