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Effect of an Electric Field on Transfer Processes in Axially Symmetric Magnetic Traps[1)]

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Expressions for the particle and energy fluxes perpendicular to a strong magnetic field are obtained for the case of a weak collision plasma and axially symmetric toroidal systems. Besides the longitudinal solenoidal and radial potential electric fields, allowance is made for deviations of the equipotential surfaces from the magnetic surfaces caused both by equilibrium variations of the potential and by low frequency plasma oscillations. An equation is derived which defines the magnitude of the self-consistent radial electric field. It is shown that diffusion involves the appearance of a velocity of the plasma as a whole along the magnetic field force lines; the magnitude of the velocity varies with time.

MANY recent papers have been devoted to different problems in the theory of transport processes across a strong magnetic field in a toroidal system. Most of these papers, however, consider the case when the system parameters are such that the self-consistent electric field that is produced in the quasistationary diffusion regime exerts no influence on the character of the motion of the individual particles, and its equipotentials coincide with the magnetic surfaces[2)]. In addition, the question of determining the self-consistent radial field produced in the plasma still remains open, since the condition of ambipolar diffusion is automatically satisfied for a fully-ionized plasma in the lowest order in the Larmor radius (i.e., in $1/B$), and consequently this condition cannot serve as an equation for the determination of this field.

The purpose of the present paper was to fill the indicated gaps, i.e., to obtain expressions for the particle and energy fluxes with allowance both for the possible deviations of the equipotentials from the magnetic surfaces and for the radial electric field. It is also our aim to derive an equation for the value of this field, confining ourselves in the solution of the kinetic equation to the first approximation in the toroidality and in the Larmor radius. We limit ourselves here to the case of axially-symmetrical magnetic traps and sufficiently low collision frequencies, when the hydrodynamics equations no longer hold.

1. FORMULATION OF PROBLEM AND EQUATION FOR LONGITUDINAL VELOCITY

Thus, we consider axially-symmetrical magnetic traps characterized by longitudinal and azimuthal (poloidal) magnetic field components $B_\zeta = B_0/[1 + (r/R)\cos\varphi]$ and B_φ, respectively, and assume that the ratio $\theta = B_\varphi/B_\zeta$ is much smaller than unity and is independent of the small azimuth φ.[3)] We assume furthermore that there exists in the system a longitudinal solenoidal electric field $E_\zeta = E_0/[1 + (r/R)\cos\varphi]$ that produces a longitudinal current and is responsible for the production of the stabilizing field B_φ and a "proper" electric field produced by the plasma itself and characterized by a potential

$$\Phi(r, \varphi) = \Phi_0(r) + \tilde{\Phi}(r, \varphi), \quad \tilde{\Phi} = \Phi_c(r)\cos\varphi + \Phi_s(r)\sin\varphi, \quad (1)$$

which is assumed for concreteness to consist only of the zeroth and first harmonics $\Phi_{c,s}$, while the amplitude of the first harmonic is assumed to be low enough so that the ratio $|e_j\tilde{\Phi}_{c,s}|/T_j$ is much smaller than unity (T_j is the temperature of the particles of type j).

It should be indicated that within the framework of the quasistationary theory considered here there exist two mechanisms of entirely different nature leading to the appearance of azimuthal variations of the potential. The first mechanism is connected with the deviation of the system from symmetry with respect to the small azimuth, a result of the toroidality. The amplitude of these so to speak equilibrium deviations is proportional to the Larmor radius and to the toroidal ratio $\delta = r/R$, and can be consistently calculated within the framework of the theory considered here from the quasineutrality condition for the corrections $\tilde{N}_j$ to the density:

$$\sum_j e_j\tilde{N}_j = 0$$

(for details see[2]). The second source of azimuthal variations of the potential are plasma oscillations. The high-frequency oscillations, whose frequency Ω greatly exceeds the collision frequency, cannot lead directly to an increase of the diffusion or of the thermal conductivity, although in the case of turbulence they can cause an increase of the effective collision frequencies. However, oscillations of sufficiently low frequency—much lower than either the collision frequencies or the characteristic frequencies of the azimuthal motion of the trapped particles—can obviously be regarded as quasistatic ones and consequently can be accounted for with sufficient rigor within the framework of the theory in question[2,4]. The amplitude of the variation of the potential is regarded here as specified (say, determined from experiment)[5].

We start from the drift kinetic equation[2]

$$\frac{\partial F_j}{\partial t} + \dot{r}\frac{\partial F_j}{\partial r} + \dot{\varphi}\frac{\partial F_j}{\partial \varphi} + \dot{u}\frac{\partial F_j}{\partial u} + \dot{w}\frac{\partial F_j}{\partial w} = \mathrm{St}_j, \quad (2)$$

where u and w are the longitudinal and transverse components of the particle velocity relative to the magnetic

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1) A somewhat abbreviated version of this paper was delivered at the Fourth International Conference on Plasma Physics and Controlled Fusion (paper CN-28/C-5, Madison, Wisconsin, June 1971).

2) Exceptions are Stringer's investigations, where similar problems were considered within the framework of the hydrodynamic approximation (see, for example[1]).

3) For convenience, we use here exactly the same notation as in the earlier papers[2,3].

field, St_j is the collision integral, the dot denotes the total derivative with respect to time, and the subscript j = e, i indicates the type of particle (electron or ion) to which the particular quantity pertains. Multiplying this equation by h^2u, where $h = 1 + (r/R) \cos \varphi$, averaging with respect to φ, and integrating with respect to the velocity, we readily obtain[4])

$$\frac{\partial \overline{h^2\langle uF_j\rangle^\varphi}}{\partial t} = \omega_j\theta\overline{h\langle \dot{r}F_j\rangle^\varphi} + \overline{h^2\langle u\mathrm{St}_j\rangle^\varphi} + \frac{e_j}{m_j}\overline{E_\zeta h^2\langle F_j\rangle^\varphi} - \frac{1}{r}\frac{\partial}{\partial r}\overline{rh^2\langle u\dot{r}F_j\rangle^\varphi},$$

where the angle brackets denote the operation of integration in velocity space, so that, for example, $\langle F_j\rangle$, $\langle uF_j\rangle$, and $\langle \dot{r}F_j\rangle$ are the density and longitudinal and radial fluxes of the particles, etc., while the bar with the superscript φ denotes averaging with respect to the azimuth φ. In deriving (3) we have neglected for simplicity quantities of order $\theta^2 \ll 1$ and the electric particle drift connected with the solenoidal electric field (the drift velocity is $\theta E_\zeta/B_\zeta$) in comparison with the toroidal drift; this corresponds, in particular, to neglecting the rate of plasma contraction as a result of the usual pinch effect in comparison with the velocity of the diffusion motion.

Since the radial drift velocity $\dot{r}$ is a quantity of first order of smallness in the toroidality $\delta = r/R$ and in $1/B$ (i.e., in the Larmor radius), it suffices, to determine the particle and energy fluxes in the first nonvanishing approximation, to determine the distribution function likewise only in first order in δ and $1/B$. Accordingly, we can neglect in the kinetic equation the dependence of all the quantities on the time and the corrections to the azimuthal motion, necessitated by the toroidal drift; in the equation for the electrons we can neglect also the corrections due to the electric drift[5]). If we assume furthermore that the solenoidal field E_ζ is small enough so that there are no runaway electrons, and that the energy acquired in this field by the trapped particles is much smaller than the thermal energy, then we can rewirte Eq. (2) after changing over from the variables u and w to the variables

$$\mu = w^2/2B, \quad \mathscr{E} = {}^1\!/_2(u^2 + w^2) + e_j\Phi/m_j$$

in the form

$$\frac{u}{r\omega_j}\frac{\partial hu}{\partial \varphi}\frac{\partial F_j}{\partial r} + [\theta u - V_E]\frac{1}{r}\frac{\partial F_j}{\partial \varphi} = \frac{e_jE_\zeta u}{T_j}F_j^M + \mathrm{St}_j, \qquad (4)$$

where

$$u = \{2\mathscr{E} - 2e_j\Phi/m_j - 2\mu B\}^{1/2}, \quad B \approx B_\zeta,$$

$$\omega_j = \frac{e_jB_0}{m_j}, \quad V_E = -\frac{1}{B_0}\frac{\partial \Phi_0}{\partial r};$$

$$F_j^M = \frac{N_j}{[2\pi v_j^2]^{3/2}}\exp\left\{-\frac{\mathscr{E} - e_j\Phi/m_j}{v_j^2}\right\},$$

$v_j = [T_j/m_j]^{1/2}$ is the average thermal velocity and N_j is the particle density. In the equation for the electron distribution function (j = e) we shall neglect also the velocity of the electric drift V_E in comparison with θu.

This equation will serve us as the starting point for the determination of the radial particle and energy fluxes S_j and Π_j averaged over the magnetic surface.

It is easy to verify that when $|V_E| \ll |\theta v_j|$ relation (3) takes on the following form in the lowest order in the Larmor radius, i.e., in the same approximation in which Eq. (4) is valid:

$$\theta\omega_jS_j = -\overline{h^2\langle u\,\mathrm{St}_j\rangle^\varphi} - \frac{e_j}{m_j}\overline{E_\zeta h^2N_j^\varphi}. \qquad (5)$$

On the other hand, inasmuch as the collision integral St_j satisfies the momentum conservation law, i.e., in the absence of neutral particles, we have

$$\sum_j m_j\langle u\,\mathrm{St}_j\rangle \equiv 0, \qquad (6)$$

it follows directly from (5) that for a fully ionized plasma, by virtue of the quasineutrality condition $\Sigma e_jN_j = 0$, the condition for diffusion ambipolarity

$$\sum_j e_jS_j = 0 \qquad (7)$$

is identically satisfied. In other words, it is a direct consequence of the initial equation (4), and consequently cannot serve as an equation for the determination of the radial electric field $E_r = -\partial\Phi_0/\partial r$ produced in the plasma.

There are two possible ways out of this difficulty. First, one could expect a change from the first approximation in $(1/B)$ considered by us to a more exact solution of the kinetic equation (2) to give rise to a dependence of the fluxes on the electric field or on its derivatives, so that the quasineutrality condition (7) acquires a deeper meaning and ceases to be a trivial consequence of the initial equation. This way, however, is very complicated and requires, in particular, that account be taken of the dependence of all the quantities on the time, or that additional and generally speaking artificial stationary conditions be imposed[6].

The second and, in our opinion, more consistent way to resolve the indicated difficulty is to use Eq. (3).

Indeed, multiplying (3) by m_j, summing over j, using the ambipolarity condition (7), and neglecting small quantities of order δ^2 and $(m_e/m_i)^{1/2}$, we get

$$\frac{\partial N_iU_0}{\partial t} = -\nu_{i0}N_iU_0 - \frac{1}{r}\frac{\partial}{\partial r}rP, \qquad (8)$$

where

$$U_0 = \sum_j m_j\langle uF_j\rangle \Big/ \sum_j m_j\langle F_j\rangle$$

is the average "hydrodynamic" plasma velocity,

$$P = \overline{h^2\langle u\dot{r}F_i\rangle^\varphi}, \qquad (9)$$

and ν_{i0} is the frequency of collisions between the ions and the neutral particles, which we shall henceforth assume to be small in comparison with the effective frequency of the ion-ion collisions[6]).

It is interesting to note that in the case considered here, that of a weak-collision plasma and not excessively small poloidal fields, when $V_E/\theta v_i < 1$, the quantity P turns out to be very simply connected with the diffusion flux of the ions in the lowest order in the toroidality δ, namely,

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4)This relation was first derived in[2], where it was used to prove the ambipolar character of the diffusion of a fully ionized plasma in the lowest approximation in the Larmor radius.

5)As a rule, the inequality $|\theta v_e| \gg |E_r/B_0|$ is always satisfied with a large margin. For ions, on the other hand, this is far from always the case.

6)On the other hand, this condition is obviously not necessary for the derivation of (8).

$$P = V_E S_i / \theta. \quad (10)$$

Since we shall show later on (see, incidentally[3]) that the velocity U_0 is not uniquely connected with the velocity of the azimuthal electric drift V_E, Eq. (8) can be regarded as the missing equation for the radial electric field E_r. Furthermore, since $\dot{r}$ is of first order of smallness both in $1/B$ and in the toroidality δ, it suffices for calculation of the quantity P which enters in (8) in the lowest order in the toroidality to determine the distribution function likewise in first order in $1/B$ and in the toroidality δ, i.e., precisely in the same approximation as used to determine the fluxes S_j and Π_j of interest to us[7]. It is important, however, that since Eq. (8) is valid in a higher approximation than the kinetic Eq. (4), Eq. (8) contains additional information even in this lowest approximation and consequently makes it possible to obtain the missing equation without resorting to a more accurate solution of the kinetic equation.

The physical meaning of Eq. (8) is quite simple: it represents the plasma-momentum conservation law in the presence of diffusion loss of particles, and indicates, in particular, that diffusion is accompanied in the entire plasma by a velocity component along the magnetic-field force lines. In a weak-collision plasma, the presence of such "acceleration" can readily be understood if it is recognized that the main contribution to the diffusion losses is made in this case by the "trapped" or "almost trapped" particles, which have, in the presence of a radial electric field E_r, a nonzero average longitudinal velocity $u \sim V_E/\theta$, and consequently continuously carry a definite momentum out from the plasma; the rate of such a "departure" of momentum from the plasma is determined precisely by the last term in the right-hand side of (8). It should be indicated that this effect was first pointed out in[7] (see also[8]), where experimental data were cited pointing to the presence in the plasma of of a longitudinal velocity whose nature, in all probability, is connected with the "acceleration" mechanism considered above.

We now turn to a determination of the particle and energy fluxes S_j and Π_j, the radial electric field $E_r = B_0 V_E$, and the longitudinal plasma velocity U_0. We start here from Eq. (4), take the collision integral St_j in the form given and used in[2,3], and confine ourselves to relatively low collision frequencies satisfying the conditions

$$r\nu_j^{eff} \ll |\theta| v_j, \quad (11)$$

where

$$\nu_e^{eff} = \frac{4\sqrt{2\pi}}{3}\frac{e_e^2 e_i^2 N_i \lambda}{m_e^{1/2} T_e^{3/2}}, \quad \nu_i^{eff} = \frac{4\sqrt{\pi}}{3}\frac{e_i^4 N_i \lambda}{m_i^{1/2} T_i^{3/2}} \quad (12)$$

are the effective frequencies of the electron-ion and ion-ion collisions, as used in ordinary hydrodynamics. In addition, we assume that $m_e \nu_e^{eff}/m_i \nu_i^{eff} \ll 1$ and that the poloidal field B_φ is not too small, so that

$$\sqrt{\delta}|\theta| v_i < |V_E| < \sqrt{\delta}|\theta| v_e, \quad (13)$$

and we accordingly neglect the drift velocity V_E in comparison with θu in the kinetic equation (4) for the electrons.

[7]It should be noted that since ambipolarity results, as a rule, in $S_j \ll \Pi_j T_j^{-1}$, it is necessary in many cases to add to the quantity P defined by (10) also terms of higher order in the toroidality (see footnote 10 below).

Recognizing that the procedure for solving (4) is rather standard (see, for example,[2,3]) and being unable to dwell on it here in any detail, we confine ourselves below to a simple list of the final expressions that are valid in various limiting cases, and to brief discussions. (The most essential steps in the solution of Eq. (4) are given in the Appendix.)

2. EXPRESSIONS FOR THE PARTICLE AND ENERGY FLUXES AND EQUATION FOR THE ELECTRIC FIELD

A. We first introduce expressions for the electron fluxes. If the deviations of the equipotentials from the magnetic surfaces are sufficiently small, so that

$$\mathscr{V}_e = [e_e^2(\Phi_c^2 + \Phi_s^2)]^{1/2} / T_e \ll r/R \quad (14)$$

and the collision frequency satisfies the condition

$$r\nu_e^{eff} \ll (r/R)^{3/2} |\theta| v_e, \quad (15)$$

then the expressions for the fluxes take the form[8]

$$S_e = -2{,}1 N_e \left(\frac{r}{R}\right)^{1/2} \frac{E_\zeta}{B_\varphi} - 2{,}1 \nu_e^{eff} N_e \left(\frac{r}{R}\right)^{1/2} \frac{\rho_e^2}{\theta^2} \mathscr{L}_e^{(-0,38)}, \quad (16)$$

$$\Pi_e = 1{,}1 S_e T_e - 2{,}1 N_e T_e \left(\frac{r}{R}\right)^{1/2} \frac{E_\zeta}{B_\varphi} - 2{,}4 \nu_e^{eff} N_e T_e \left(\frac{r}{R}\right)^{1/2} \frac{\rho_e^2}{\theta^2} \frac{\partial \ln T_e}{\partial r}, \quad (17)$$

where $\rho_j = v_j/\omega_j$ is the Larmor radius of the particle, and $\mathscr{L}_e^{(\alpha)}$ denotes the quantity

$$\mathscr{L}_e^{(\alpha)} = \frac{\partial \ln N_e T_e^\alpha}{\partial r} + \frac{\theta \omega_e}{v_e^2}\left[U_0 - \frac{V_E}{\theta}\right]. \quad (17')$$

In the region of intermediate collision frequencies, when

$$(r/R)^{3/2} |\theta| v_e \ll r\nu_e^{eff} \ll |\theta| v_e, \quad (18)$$

but the condition (14) is satisfied as before, we get

$$S_e = -3{,}4 N_e \left(\frac{r}{R}\right)^2 \frac{|\theta| v_e}{r\nu_e^{eff}} \frac{E_\zeta}{B_\varphi} - 1{,}25 \frac{|\theta| v_e}{r} N_e \left(\frac{r}{R}\right)^2 \frac{\rho_e^2}{\theta^2} \mathscr{L}_e^{(1,5)}, \quad (19)$$

$$\Pi_e = 3 S_e T_e - 4{,}1 N_e T_e \left(\frac{r}{R}\right)^2 \frac{|\theta| v_e}{r\nu_e^{eff}} \frac{E_\zeta}{B_\varphi} - 3{,}75 \frac{|\theta| v_e}{r} N_e T_e \left(\frac{r}{R}\right)^2 \frac{\rho_e^2}{\theta^2} \frac{\partial \ln T_e}{\partial r}. \quad (20)$$

On the other hand, if the variations of the electrostatic potential are sufficiently large, i.e.,

$$\mathscr{V}_e \gg r/R, \quad (21)$$

then the decisive role is played not by the toroidal but by the electric drift, and formulas (15)—(20) take the form

$$S_e = -2{,}2 N_e \mathscr{V}_e^{1/2} \frac{E_\zeta}{B_\varphi} - 3{,}8 \nu_e^{eff} N_e \mathscr{V}_e^{1/2} \frac{\rho_e^2}{\theta^2} \mathscr{L}_e^{(-0,9)}, \quad (22)$$

$$\Pi_e = 0{,}62 S_e T_e - N_e T_e \mathscr{V}_e^{1/2} \frac{E_\zeta}{B_\varphi} - 2{,}2 \nu_e^{eff} N_e T_e \mathscr{V}_e^{1/2} \frac{\rho_e^2}{\theta^2} \frac{\partial \ln T_e}{\partial r} \quad (23)$$

at $r\nu_e^{eff} \ll \mathscr{V}_e^{3/2} |\theta| v_e$ and

$$S_e = -0{,}62 \frac{|\theta| v_e}{r\nu_e^{eff}} N_e \mathscr{V}_e^2 \frac{E_\zeta}{B_\varphi} - 0{,}62 \frac{|\theta| v_e}{r} N_e \mathscr{V}_e^2 \frac{\rho_e^2}{\theta^2} \mathscr{L}_e^{(-0,5)}, \quad (24)$$

[8]Some differences between the numerical coefficients given here and the corresponding coefficients of[3] are due to the fact that we have used here the exact expressions for the collision frequencies, whereas in[3] it was assumed that $\nu_{jj} \sim 1/v^3$. On the other hand, in [9] it was assumed that the collision frequency does not depend at all on the particle energy.

$$\Pi_e = 2S_eT_e - 0{,}94N_eT_e\mathscr{V}_e^2\frac{|\theta|v_e}{rv_e^{eff}}\frac{E_\zeta}{B_\varphi} - 1{,}25\frac{|\theta|v_e}{r}N_eT_e\mathscr{V}_e^2\frac{\rho_e^2}{\theta^2}\frac{\partial\ln T_e}{\partial r} \quad (25)$$

at $\mathscr{V}_e^{3/2}|\theta|v_e \ll r\nu_e^{eff} \ll |\theta|v_e$.

We present, finally, an expression for that part of the charge density $\tilde{q}_e$ which varies with the azimuth φ and is connected with the electrons. This expression can be useful for the calculation of equilibrium deviations of the potential (i.e., deviations not connected with low-frequency plasma oscillations). It turns out to be the same for all the cases considered above:

$$\tilde{q}_e = -(e_e^2N_e/T_e)[\Phi_c\cos\varphi + \Phi_s\sin\varphi]. \quad (26)$$

On the other hand, the condition at which the low-frequency oscillations with frequency Ω can be regarded as quasistatic for the electrons is

$$\Omega \ll \nu_e^{eff}. \quad (27)$$

B. We now turn to the ion fluxes. The number of limiting cases turns out to be twice as large here, since the ratio $\mathscr{V}_E/|\theta|v_i$ can be much less as well as much larger than unity.

Let us discuss first the case of greatest practical interest, that of sufficiently large poloidal fields, when

$$|V_E|/|\theta|v_i \ll 1. \quad (28)$$

The expression for the variable part of the charge density has a form perfectly analogous to (26), namely

$$\tilde{q}_i = -(e_i^2N_i/T_i)\tilde{\Phi}$$

and it follows from the quasineutrality condition $\tilde{q}_e + \tilde{q}_i = 0$ that the equilibrium deviations of the potential in the approximation in question are equal to zero. In other words, in the case when the condition (28) is satisfied, $\mathscr{V}_j$ must be taken to mean the potential variation, connected only with the low-frequency oscillations. For ions they must be taken into account only if

$$\Omega \ll \nu_i^{eff}. \quad (29)$$

On the other hand, if $\Omega \gg \nu_e^{eff}$, then, as already noted above, they do not make a direct contribution to the fluxes and it can be assumed that $\mathscr{V}_j = 0$. At $\nu_e^{eff} \gg \Omega \gg \nu_i^{eff}$ the variations of the potential must be taken into account in the expressions for the electron fluxes, and when $\Omega \ll \nu_i^{eff}$ they must be taken into account also in the expressions for the ion fluxes.

Let the variations of the potential be sufficiently small so that

$$\mathscr{V}_i = [e_i^2(\Phi_c^2 + \Phi_s^2)]^{1/2}/T_i \ll r/R. \quad (30)$$

Then in the region of small collision frequencies, when

$$r\nu_i^{eff} \ll (r/R)^{3/2}|\theta|v_i, \quad (31)$$

the expressions for the ion flux density S_i and for the ion energy Π_i take the form

$$S_i = -0{,}17\nu_i^{eff}\Lambda N_i\left(\frac{r}{R}\right)^{1/2}\frac{\rho_i^2}{\theta^2}\mathscr{L}_i^{(-0{,}15)}, \quad (32)$$

$$\Pi_i = -0{,}24\nu_i^{eff}\Lambda N_iT_i\left(\frac{r}{R}\right)^{1/2}\frac{\rho_i^2}{\theta^2}\mathscr{L}_i^{(0{,}75)}, \quad (33)$$

where

$$\mathscr{L}_i^{(\alpha)} = \frac{\partial\ln N_iT_i^\alpha}{\partial r} + \frac{\theta\omega_i}{v_i^2}\left(U_0 - \frac{V_E}{\theta}\right),$$

$$\Lambda = \ln\left\{\frac{32|\theta|v_i}{r\nu_i^{eff}}\max\left[\left(\frac{r}{R}\right)^{3/2}, \mathscr{V}_i^{3/2}\right]\right\}. \quad (34)$$

On the other hand, if low-frequency oscillations exist in the plasma and satisfy the condition (29), and if their amplitude is large enough to have

$$\mathscr{V}_i \gg r/R, \quad (35)$$

then we get at low collision frequencies

$$r\nu_i^{eff} \ll \mathscr{V}_i^{3/2}|\theta|v_i. \quad (36)$$

The formulas for the ion fluxes take the form

$$S_i = -0{,}2\nu_i^{eff}\Lambda N_i\mathscr{V}_i^{1/2}\frac{\rho_i^2}{\theta^2}\mathscr{L}_i^{(-0{,}6)}, \quad (37)$$

$$\Pi_i = -0{,}35\nu_i^{eff}\Lambda N_iT_i\mathscr{V}_i^{1/2}\frac{\rho_i^2}{\theta^2}\mathscr{L}_i^{(0{,}3)}. \quad (38)$$

On the other hand, if the poloidal field B_φ is large enough so that

$$V_E \ll |\theta|v_i\max[(r/R)^{1/2}, \mathscr{V}_i^{1/2}],$$

i.e., the electric drift is negligible also for the ions, then the coefficients 0.17, 0.24, 0.2, and 0.35 in formulas (32), (33), (37), and (38) must be replaced by 1, 1.4, 1.2, and 2, respectively.

At intermediate collision frequencies, when

$$(r/R)^{3/2}|\theta|v_i \ll r\nu_i^{eff} \ll |\theta|v_i \quad \text{for} \quad r/R \gg \mathscr{V}_i,$$

$$\mathscr{V}_i^{3/2}|\theta|v_i \ll r\nu_i^{eff} \ll |\theta|v_i \quad \text{for} \quad r/R \ll \mathscr{V}_i, \quad (39)$$

the expressions for the ion fluxes will be determined by formulas (19) and (20) at $r/R \gg \mathscr{V}_i$ and (24) and (25) at $r/R \ll \mathscr{V}_i$, and the subscript e in these formulas must be replaced by i. In view of the inequality $m_e\nu_e^{eff} \ll m_i\nu_i^{eff}$, the term proportional to E_ζ/B_φ and describing the compression under the influence of the solenoidal electric field should be neglected in comparison with the remaining terms, just as was done in the derivation of formulas (32)–(38).

C. We turn now to the derivation of the equation for the electric field E_r. As already noted, in all the cases considered above the quantity P in (8) is connected with the corresponding ion flux by the relation (10).

As to the longitudinal plasma velocity U_0 and, in particular, its connection with the radial electric field, it can also be obtained in principle by solving the kinetic equation (4). This method, however, leads to complicated and rather cumbersome calculations. There is a simpler procedure. Indeed, if we recall that Eq. (5), together with the ambipolarity of the diffusion, is a direct consequence of (4) if $|V_E|/|\theta| \ll v_i$, then the connection between the longitudinal velocity U_0 and the plasma parameters can be obtained simply from the ambipolarity condition (7) or, inasmuch as the ionic diffusion coefficient of the ions is, as a rule, much larger than the electron diffusion coefficient, from the vanishing of the corresponding ion flux S_i.

We substitute now the expression obtained for U_0 into Eq. (8) and take (10) into account. This yields a final expression for the time variation of the radial electric field E_r (or V_E)[9]. By way of an example, let us consider the case when the equipotentials coincide with the magnetic surfaces (i.e., $\mathscr{V}_j \ll r/R$), and the collision

[9]We must not forget, however, that the equation will be valid only so long as $U_0 \ll v_i$. This is due to the fact that the form we used for the collision integral is also valid only when $U_0 \ll v_i$.

frequencies ν_j^{eff} satisfy the conditions (15) and (31). Neglecting small quantities on the order of $(m_e/m_i)^{1/2}$ in comparison with unity, we obtain from the ambipolarity condition (7)

$$U_0 = \frac{V_E}{\theta} - \frac{v_i^2}{\theta\omega_i}\frac{\partial \ln N_i T_i^{-0,15}}{\partial r}. \quad (40)$$

On the other hand, assuming for simplicity that the quantity V_E/θ does not depend on the radius r, and recognizing that $S_e = S_i$ and $\partial N_i/\partial t = -r^{-1}\partial(rS_i)/\partial r$, we get from (8)[10)]

$$\frac{\partial U_0}{\partial t} + \nu_{i0}U_0 = \frac{v_i^2}{\theta\omega_i}\frac{\partial \ln N_i T_i^{-0,15}}{\partial r}\frac{\partial \ln N_i}{\partial t}. \quad (41)$$

This equation indeed determines the sought dependence of U_0, and consequently also of E_r, on the time and on the plasma parameters. It follows from it, in particular, that if $\partial \ln N_i T_i^{-0.15}/\partial r < 0$, then the plasma velocity U_0 increases monotonically with time, and the radial electric field is initially negative (if $U_0 = 0$ at $t = 0$), and then goes to zero and becomes positive.

Analogous equations can be obtained also for the other cases considered above.

D. Let us consider finally the case of small poloidal fields B_φ, when

$$z = |V_E/\theta v_i| \gg 1. \quad (42)$$

In this case, as shown by calculations, there is practically no parameter region in which the main contribution is made by "trapped" or "almost trapped" particles, and the fluxes of the energy and of the particles are determined entirely by the untrapped particles with longitudinal velocities $u \lesssim v_i \ll |V_E/\theta|$[11)]. As will be shown below, the quantity $r^{-1}\partial(rP)/\partial r$ turns out in general to be of the same order of smallness as the term $\theta\omega_i S_i$, and consequently the ambipolarity condition (7) is no longer a consequence of the initial kinetic equation (4), and therefore enables us to determine the radial electric field E_r. Accordingly, the fluxes turn out to be independent of the longitudinal velocity in the lowest approximation in $\theta v_i/V_E$; as before, the value of this velocity is determined by Eq. (8).

Thus, if the inequality (42) is satisfied and the ratio of r/R to ν_i^* is arbitrary, we obtain the following expressions for the particle and energy fluxes S_i and Π_i[12)]:

$$S_i = -0,2\nu_i^{eff} N_i\left(\frac{r}{R}\right)^2\frac{v_i^2}{V_E^2}\rho_i^2\left\{\frac{\partial \ln N_i T_i^{0,75}}{\partial r} - \frac{\omega_i V_E}{v_i^2}\right\}, \quad (43)$$

$$\Pi_i = \frac{e_i\Phi_c R}{r}S_i - 0,45\nu_i^{eff}N_i T_i\left(\frac{r}{R}\right)^2\frac{v_i^2}{V_E^2}\rho_i^2\left\{\frac{\partial \ln N_i T_i^{1,4}}{\partial r} - \frac{\omega_i V_E}{v_i^2}\right\}. (44)$$

That part of the electric-charge density which is variable in the azimuth φ takes the form

$$\tilde{q}_i = -\frac{e_i^2 N_i\tilde{\Phi}}{T_i}\frac{v_i^2}{\omega_i V_E}\frac{\partial \ln N_i}{\partial r} + \frac{2\delta e_i N_i v_i^2}{\omega_i V_E}\left[\frac{\partial \ln N_i T_i}{\partial r} - \frac{\omega_i V_E}{v_i^2}\right]\cos\varphi, (45)$$

and the quantity P in Eq. (8) is a rapidly decreasing function of V_E given by

$$P = 3\theta v_i^2\Pi_i / T_i V_E. \quad (46)$$

At not too large values of $(V_E/\theta v_i)^2 \ll (m_i/m_e)^{1/2}$ it follows from the ambipolarity condition that

$$V_E = \frac{v_i^2}{\omega_i}\frac{\partial \ln N_i T_i^{0,75}}{\partial r}, \quad (47)$$

and consequently the term $\theta\omega_i S_i$ in (3) is of the same order as $r^{-1}\partial(rP)/\partial r$.

Using, finally, formulas (26) and (45) and the quasineutrality condition $\tilde{q}_e + \tilde{q}_i = 0$, we obtain the following expressions for the amplitudes of the equilibrium deviations of the potential:

$$e_i\Phi_s = 0,$$

$$e_i\Phi_c = -2\delta\frac{e_i}{e_e}T_e\left(1 - \frac{v_i^2}{\omega_i V_E}\frac{\partial \ln N_i T_i}{\partial r}\right)\bigg/\left(1 - \frac{v_e^2}{\omega_e V_E}\frac{\partial \ln N_i}{\partial r}\right). \quad (48)$$

It follows therefore that in the absence of a gradient of the ion temperature the equilibrium deviations of the potential are equal to zero. They can become appreciable only in the vicinity of the point where

$$\frac{T_i}{e_i}\frac{\partial \ln N_i T_i^{0,75}}{\partial r} = \frac{T_e}{e_e}\frac{\partial \ln N_i}{\partial r},$$

i.e., only in the region where $\partial \ln N_i/\partial \ln T_i < 0$. Such a situation, however, does not seem very likely.

APPENDIX

Let us consider the solution of (4). In the case of sufficiently large poloidal fields, when

$$|V_E| \ll \sqrt{\delta}|\theta|v_i, \quad (A.1)$$

the electric field has no appreciable influence on the character of motion of the individual particles, and the factor with V_E in the second term of the left side of (4) can be neglected in comparison with θu. The solution of such an equation is well known and is given, for example, in[3]. We confine ourselves therefore only to the case of sufficiently small θ, when

$$|V_E| \gg \sqrt{\delta}|\theta|v_i, \quad (A.2)$$

and it is important to take the electric field into account.

Recognizing that in the absence of collisions and of a solenoidal electric field E_ζ the quantity

$$\mathcal{I} = \int^r \theta\, dr - hu/\omega_i \quad (A.3)$$

10)It should be noted that for the case of a weak-collision plasma and sufficiently strong poloidal fields B_φ, when the condition (31) is satisfied, $T_e = T_i$ and $\nu_{i0} = 0$, an analogous equation was obtained also in [10]. However, unlike in the present paper, no account was taken in [10] of the electron-ion collisions, because the lowest-order terms in the toroidality (i.e., $\sim\delta^{1/2}$) were not included in [10]. Our calculations, on the other hand, were made with allowance for the electron-ion collisions, but in the lowest order in the toroidality, so that the terms proportional to the higher powers of the toroidal ratio (i.e., $\sim\delta^{3/2}$) were not taken into account by us. Accordingly, our results and the results of [10] have different regions of applicability. Thus, assuming as an estimate that $T_e = T_i$, $eE_r r \simeq T_i$, and $\partial\ln N/\partial r = \partial\ln T/\partial r = 1/r$, we find from a comparison of the corresponding formulas that the results of [10] are applicable in the case of a strongly ionized plasma, when

$$\nu_{i0} \ll \frac{1}{5}\frac{\delta^2}{\theta^2}\frac{\rho_i^2}{r^2}\nu_i^{eff},$$

and sufficiently "steep" toruses, when the inequality

$$\delta = r/R \gg 8\sqrt{m_e/m_i} \approx 1/5$$

is satisfied. In the opposite case of sufficiently small toroidality, $\delta \ll 1/5$, it is necessary to use our results.

We indicate also that, generally speaking, it is necessary to add to (8) and (41) terms corresponding to the usual viscosity (resulting from allowance for the rapid Larmor rotation), which obviously cannot be obtained from the drift kinetic equation (2).

11)Thus, for example, in the region where condition (31) is satisfied, the trapped particles make the decisive contribution only at values of z satisfying the inequality $\delta^{3/2}z \ll \exp[-(z^2/2)]$; the "plateau" region exists only if $\delta z^4 \ll 1$.

12)We note, incidentally, that formulas (43)–(45) are valid in the entire region of collision frequencies $r\nu_i^{eff} \ll V_E$.

is an integral of the motion, it is convenient to change over in (4) from the variables r, φ, and μ to new variables r_0, φ, and μ, where r_0 is the minimal radial deviation of the particle, with

$$r = r_0 + \Delta r(r_0, \varphi, \mu). \tag{A.4}$$

In the case of sufficiently strong stabilizing fields, when $\Delta r/r \ll 1$, it is easy to determine the value of Δr as a function of the azimuth φ, by expanding the integral of motion $\mathcal{I}$ in powers of Δr and retaining the terms quadratic in Δr:

$$\Delta r(r_0, \varphi, \mu) = \frac{1}{\theta\omega_j}\frac{1}{\beta}\left\{-\left(u_0 - \frac{V_E}{\theta}\right) + \Delta u\right\},$$

$$\Delta u = \frac{(u_0 - V_E/\theta)}{|u_0 - V_E/\theta|}\left\{\left(u_0 - \frac{V_E}{\theta}\right)^2 + 2|\beta|\left[\tilde{Q}_j(\varphi) - \tilde{Q}_j(\varphi_0)\right]\right\}^{1/2},$$

$$\beta = 1 - \frac{1}{\theta\omega_j}\frac{\partial}{\partial r_0}\frac{V_E}{\theta},$$

$$\tilde{Q}_j(\varphi) = \left[\delta(\mu B + u_0^2)\cos\varphi - \frac{e_j\tilde{\Phi}(r_0, \varphi)}{m_j}\right], \tag{A.5}$$

where $V_E = -B_0^{-1}\partial\Phi_0/\partial r_0$ and u_0 are the values of the electric-drift velocity and of the longitudinal velocity at the point $r = r_0$ and $\varphi = \varphi_0$, while φ_0 is the value of the azimuth at which the radial coordinate of the particle is equal to r_0. We have assumed here that β is not very close to zero, so that $|\beta| \gtrsim \delta^{1/3}$; in the opposite case it is necessary to take into account the expansion terms that are cubic in Δr.

Thus, taking into account the explicit form of the collision integral (see[3]), changing over in (4) to the variables r_0, φ, and μ, and confining ourselves to the lowest order in the toroidality $\delta = r/R$, we obtain

$$\theta\Delta u\frac{\partial F_j}{\partial\varphi} = r_0\nu_j\hat{L}\left\{F_j - \bar{a}_j u_0\bar{F}_j^M\left[1 + \frac{e_j\tilde{\Phi}}{T_j}\right]\right.$$
$$\left.-\left[1 + \frac{u_0}{\theta\omega_j}\frac{\partial\ln\bar{a}_j\bar{F}_j^M}{\partial r_0}\right]\theta\omega_j\bar{a}_j\bar{F}_j^M\Delta r - u_0\bar{a}_j\bar{F}_j^M\delta\cos\varphi + \tilde{a}_j u_0\bar{F}_j^M\right\}, \tag{A.6}$$

where

$$a_j = \frac{U_j}{v_j^2} + \frac{e_j E_\zeta}{T_j\nu_j},$$

$$\bar{F}_j^M = \frac{N_j(r_0)}{(2\pi v_j^2)^{3/2}}\exp\left\{-\left[\frac{\mathcal{E} - e_j\Phi_0(r_0)/m_j}{v_j^2}\right]\right\}, \tag{A.7}$$

U_j and ν_j are determined in the same manner as in[3],

$$\hat{L} = \frac{u}{B}\frac{\partial}{\partial\mu}u\mu\frac{\partial}{\partial\mu}\Bigg|_{r,\varphi,\mu\to r_0,\varphi,\mu}, \tag{A.8}$$

and the quantities $\bar{a}_j$ and $\tilde{a}_j$ denote the average value of a_j and the part variable in the azimuth φ, as functions of the variables r and φ.

Putting now $F_j = \bar{F}_j^M(r_0)\left[1 + \bar{a}_j u_0 + \Psi_j(r_0, \varphi)\right]$, where $\Psi_j(r_0, \varphi) \ll 1$ is a small correction that takes into account the dependence of the solution on the azimuth, we obtain for Ψ_j in the first order in Δr the equation

$$\frac{\partial\Psi_j}{\partial\varphi} = \frac{r_0\nu_j}{\theta\Delta u}\hat{L}\left\{\Psi_j - C_j\Delta r - u_0\bar{a}_j\left[\delta\cos\varphi + \frac{e_j\tilde{\Phi}}{T_j}\right] + u_0\tilde{a}_j\right\},$$
$$C_j = \left[(1 + \bar{a}_j u_0)\frac{\partial\ln\bar{F}_j^M}{\partial r_0} + \left(1 + \frac{u_0}{\theta\omega_j}\frac{\partial\ln\bar{a}_j}{\partial r_0}\right)\theta\omega_j\bar{a}_j\right]. \tag{A.9}$$

Inasmuch as the quantity in the curly bracket is at least of the same order of smallness as Δr, the operator L can be replaced by

$$\hat{L}_0 = \frac{u_0}{B_0}\frac{\partial}{\partial\mu}u_0\mu\frac{\partial}{\partial\mu}. \tag{A.10}$$

We note that in terms of the ordinary variables r, φ, and μ the distribution function, in the same order in the toroidality, is obviously

$$F_j(r, \varphi) = F_j^M(r, \varphi)\{(1 + \bar{a}_j u)(1 - e_j\tilde{\Phi}/T_j)$$
$$- C_j\Delta r + u\bar{a}_j\delta\cos\varphi + \Psi_j\}. \tag{A.11}$$

We now proceed to solve Eq. (A.9) in different limiting cases.

At low collision frequencies, when the inequality (31) or (36) is satisfied:

$$r\nu_j^{eff} \ll |\theta| v_j \max\left\{\left(\frac{r}{R}\right)^{3/2}, \mathcal{V}_j^{3/2}\right\}, \tag{A.12}$$

it is convenient to change over in (A.9) from the variable μ to the variable k, where

$$k = \frac{(u_0 - V_E/\theta)}{2(|\beta|Q_j)^{1/2}}, Q_j = \left\{\left[\frac{e_j\Phi_c}{T_j} - \delta(\mu B + u_0^2)\right]^2 + \left(\frac{e_j\Phi_s}{T_j}\right)^2\right\}^{1/2}, \tag{A.13}$$

and to introduce a new unknown function in accordance with the relation[13)]

$$\Psi_j = \frac{2C_j(|\beta|Q_j)^{1/2}}{\theta\omega_j\beta}[f_j - k]. \tag{A.14}$$

Then, changing over from the variable φ to $\vartheta = (\varphi - \varphi_0)/2$, recognizing that the main contribution to the transport processes is made by the region of values $k \lesssim 1$, and neglecting accordingly the last two terms on the right-hand side of (A.9), which are of higher order of smallness (in the toroidality), we obtain the following equation for the function f_j:

$$\frac{\partial f_j}{\partial\vartheta} = \frac{\varkappa_j}{x}\frac{\partial^2 f_j}{\partial k^2} + \frac{\varkappa_j}{x^4}\sin^2\vartheta,$$
$$x = \frac{k}{|k|}\sqrt{k^2 - \sin^2\vartheta}, \quad \varkappa_j = \frac{r\nu_j\mu B}{4\theta(|\beta|Q_j)^{3/2}}. \tag{A.15}$$

For the untrapped particles corresponding to the region $k^2 > 1$, the solution of (A.15) can be sought, just as in the case $V_E = 0$ (see, for example,[3]) by the method of successive approximations, expanding the solution in powers of the small parameter $\kappa_j \ll 1$. Putting $f_j = \bar{f}_j + \tilde{f}_j(\vartheta)$, where $\bar{f}_j$ does not depend on the angle ϑ, and $\tilde{f}_j \ll \bar{f}_j$ ($\tilde{f}_j$ is a correction proportional to the collision frequency and taking the dependence of the distribution function on ϑ into account), and bearing in mind that for untrapped particles the solution $f_j(\vartheta)$ should be a periodic function of the angle ϑ with a period π, we obtain the following equations for $\bar{f}_j$ and $\tilde{f}_j(\vartheta)$:

$$\frac{\partial\tilde{f}_j}{\partial\vartheta} = \frac{\varkappa_j}{x}\left[\frac{\partial^2\bar{f}_j}{\partial k^2} + \frac{\sin^2\vartheta}{x^3}\right],$$
$$\frac{\partial^2\bar{f}_j}{\partial k^2} = -\overline{\left(\frac{\sin^2\vartheta}{x^4}\right)}^{\vartheta}\Big/\overline{\left(\frac{1}{x}\right)}^{\vartheta}, \quad k^2 > 1, \tag{A.16}$$

where the superior bar with the ϑ denotes averaging of the corresponding quantity with respect to ϑ:

$$\bar{A}^{\vartheta} = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2}A(\vartheta)\,d\vartheta.$$

13)We note that in the case of sufficiently strong poloidal fields $|V_E| \ll |\theta| v_j$, when the condition (28) is satisfied, we can put $\beta = 1$ in (A.14) and the following equations, and

$$C_j = \bar{a}_j\omega_j\theta + \frac{\partial\ln\bar{F}_j^M}{\partial r_0} = \frac{\partial\ln N_j T_j^{-3/2}}{\partial r_0} - \frac{\omega_j V_E}{v_j^2} + \frac{\mathcal{E} - e_j\Phi/m_j}{v_j^2}\frac{\partial\ln T_j}{\partial r_0} + \bar{a}_j\omega_j\theta.$$

Bearing in mind that $\bar{f}_j = 0$ for trapped particles, we find from (A.16) that the quantity $\partial\bar{f}_j/\partial k$ can be represented in the entire range of variation of k in the form

$$\frac{\partial \bar{f}_j}{\partial k} = \varepsilon(k)\int_k^\infty dk\left(\overline{\frac{\sin^2\vartheta}{x^4}}\right)^{\circ} \Big/ \left(\overline{\frac{1}{x}}\right)^{\circ}, \quad \varepsilon(k) = \begin{cases}1, & k>1\\ 0, & k<1\end{cases}. \quad \text{(A.17)}$$

Thus, recognizing that the radial drift velocity is

$$\dot{r} = \frac{1}{r\omega_j}u\frac{\partial hu}{\partial\varphi} \sim \frac{\partial x^2}{\partial\vartheta},$$

we find that the contribution made by the untrapped particles to the radial flux S_j averaged over the magnetic surface is proportional to the integral

$$\int D(k)\left(\overline{x^2\frac{\partial \bar{f}_j}{\partial\vartheta}}\right)^{\circ} dk = \varkappa_j\int_1^\infty dk D(k)\left(\overline{\frac{\sin^2\vartheta}{x^2}}\right)^{\circ}$$

$$+\varkappa_j\int_1^\infty dk[\bar{x}^{\circ}(1)D(1)-\bar{x}^{\circ}(k)D(k)]\left(\overline{\frac{\sin^2\vartheta}{x^4}}\right)^{\circ}\Big/\left(\overline{\frac{1}{x}}\right)^{\circ},$$

where D(k) is a certain function of k and is finite at k = 1. Since

$$\left(\overline{\frac{\sin^2\vartheta}{x^4}}\right)^{\circ} = -\frac{\partial}{\partial k^2}\frac{1}{\sqrt{1-k^{-2}}},$$

this integral is obviously finite and consequently the contribution from the untrapped particles to the particle and energy fluxes is proportional to the small parameter κ_j. On the other hand, as we shall soon verify, the contribution to the fluxes from the trapped particles (i.e., from the region $k^2 < 1$) is proportional to $\kappa_j \ln(1/\kappa_j)$, i.e., it is decisive when $\kappa_j \ll 1$. Accordingly, we shall take into account below only the fluxes determined by the trapped particles, i.e., by the solution of Eq. (A.15) in the region $k^2 < 1$.

We make in (A.15) one more change of variables, from k and ϑ to k′ and ψ, in accordance with the relations

$$k' = k, \quad \sin\vartheta = |k|\sin\psi.$$

Since the right-hand side of (A.15) has a singularity at x = 0, i.e., the solution has a sharp maximum in the vicinity of the point $\psi = \pm\pi/2$, we obtain for f_j, neglecting accordingly the derivative $\partial/\partial k'$ in comparison with $(\sin\psi/k'\cos\psi)\partial/\partial\psi$, the following equation:

$$\frac{\partial^2 f_j}{\partial\psi^2} + W(\psi)\frac{\partial f_j}{\partial\psi} = -\frac{k'}{\cos\psi},$$

$$W(\psi) = \frac{1}{\sin\psi\cos\psi} - \frac{k'|k'|}{\varkappa_j}\frac{\cos^2\psi(1-k^2\sin^2\psi)^{1/2}}{\sin^2\psi}. \quad \text{(A.18)}$$

It is not particularly difficult to find an exact solution of this equation. However, if account is taken of the fact that the fluxes S_j and Π_j averaged over the magnetic surface are proportional to the quantity

$$\overline{\dot{r}f_j^{\circ}} \sim \overline{\frac{\partial x^2}{\partial\theta}}^{\circ} f_j = -\frac{1}{\pi}\int_{-\pi/2}^{\pi/2} x^2\frac{\partial f_j}{\partial\psi}d\psi,$$

i.e., behave like $\kappa_j \ln \kappa_j^{-1}$ as $\kappa_j \to 0$, we can assume, confining ourselves to logarithmic accuracy (i.e., neglecting quantities of the order of unity in comparison with $\ln \kappa_j^{-1}$), that

$$\frac{\partial f_j}{\partial\psi} \approx -\frac{k}{\cos\psi}\frac{W(\psi)}{W^2(\psi)+\partial W(\psi^*)/\partial\psi^*}, \quad \text{(A.19)}$$

where ψ^* is the root of the equation $W(\psi) = 0$.

Substituting now (A.19) in the expression for the fluxes, neglecting the contribution from the untrapped particles, and performing a simple integration, we obtain without difficulty expressions (32)—(38) and relation (10).

In the region of intermediate collision frequencies, when the conditions (39) are satisfied, the main contribution to the solution is made by the vicinity of the point $u_0 = V_E/\theta$, and we can assume that

$$\Delta u = u_0 - V_E/\theta, \quad \Delta r = \hat{Q}_j/\omega_j\theta\Delta u. \quad \text{(A.20)}$$

Changing over further to the complex amplitudes $\{\Psi_j, \hat{Q}_j\} = \mathrm{Re}\{\hat{\Psi}_j, \hat{Q}_j\}e^{\varphi}$ and putting $\hat{L} = \mu B\partial^2/\partial u_0^2$, we obtain for Ψ_j the equation

$$\frac{\partial^2\hat{\Psi}_j}{\partial y^2} - i\frac{y}{\alpha_j}\hat{\Psi}_j = \frac{2\beta_j}{y^3}, \quad \text{(A.21)}$$

where

$$y = \frac{u_0}{v_j} - \frac{V_E}{\theta v_j}, \quad \alpha_j = \frac{r_0 v_j \mu B}{\theta v_j^3}, \quad \beta_j = \frac{C_j\hat{Q}_j}{\theta\omega_j v_j}.$$

A solution of this equation can easily be obtained (see, for example,[2]) and is of the form

$$\hat{\Psi}_j = \beta_j/y - i\beta_j\chi(\alpha_j, y), \quad \text{(A.22)}$$

$$\chi(\alpha, y) = \frac{\alpha}{|\alpha|}\int_0^\infty ds\exp\left[-\left(|\alpha|\frac{s^3}{3} + isy\frac{\alpha}{|\alpha|}\right)\right].$$

Thus, recognizing that at $\alpha \ll 1$ we have

$$\chi(\alpha, y) \approx \pi\frac{\theta}{|\theta|}\delta(y) - \frac{i}{y},$$

where $\delta(y)$ is a delta function, we obtain

$$\hat{\Psi}_j = -i\pi\frac{\hat{Q}_jC_j}{|\theta|\omega_j}\delta\left(u_0 - \frac{V_E}{\theta}\right), \quad \text{(A.23)}$$

from which follow the expressions (19), (20), (24), (25), and the ratio (10).

Finally, at sufficiently small transformation angles, when $|V_E| \gg |\theta| v_j$, it is necessary to take into account all the terms in the right-hand side of (A.9). We can then use the expressions (A.20) for Δu and Δr and, since the trapped particles do not, as a rule, play a significant role when $z = |V_E/\theta v_i| \gg 1$, we can expand fractions of the type $1/(u_0 - V_E/\theta)$ in powers of $\theta u_0/V_E$. The solution of (A.9) can in this case be determined in elementary fashion, and we shall not stop to discuss it here.

In conclusion, we note that in the calculation of the final expressions for the particle and energy fluxes S_j and Π_j we encounter integrals of the type

$$J_l = \int_0^\infty e^{-t}t^l\eta(t)dt, \quad I_l = \int_0^\infty e^{-t}t^{l-1/2}\eta(t)dt, \quad \text{(A.24)}$$

where l = 0, 1, 2, 3, and

$$\eta(t) = \frac{2}{\sqrt{\pi}}\left[e^{-t}t^{1/2} + \left(1-\frac{1}{2t}\right)\int_0^t e^{-s}s^{1/2}ds\right].$$

Taking the relation

$$\frac{1}{\sqrt{\pi}}\int_0 dt\, e^{-\mu t}\int_0^t e^{-s}s^{-1/2}ds = \frac{1}{\mu\sqrt{\mu-1}}$$

into account and integrating by parts and with respect to the parameter μ, we readily obtain

$$J_0=\sqrt{2}-\frac{1}{2}\ln\frac{\sqrt{2}+1}{\sqrt{2}-1}\approx 0{,}52,\quad J_1=\frac{1}{\sqrt{2}},\quad J_2=\frac{9}{4\sqrt{2}},\quad J_3\approx\frac{13}{2\sqrt{2}};$$
$$I_0=\frac{\pi-2}{\sqrt{\pi}},\quad I_1=\frac{1}{\sqrt{\pi}},\quad I_2=\frac{4+\pi}{4\sqrt{\pi}}.$$

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