Stimulated Compton Interaction Between Maxwellian Electrons and Spectrally Narrow Radiation

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Submitted October 19, 1971
Zh. Eksp. Teor. Fiz. **62**, 1392–1408 (April, 1972)

Effects associated with stimulated Compton scattering of high-intensity radiation by free electrons (electron heating, distortion of the radiation spectrum, stimulated radiation pressure) diminish with decrease of the spectral width and angular aperture of the radiation. The integral kinetic equation (its kernel has been found) permits one to determine the electron heating rate and to find an analytic solution for the evolution of intense spectral lines during stimulated Compton interaction for arbitrary spectral widths and angular apertures of the radiation beam.

1. INTRODUCTION

THE radiation of laboratory lasers and also of cosmic masers is concentrated in a narrow range of frequency $\delta/\nu \ll 1$ and solid angle $\Omega \ll 1$ and is distinguished by a high intensity I_ν. The brightness temperatures $kT_b = I_\nu c^2/2\nu^2$ of this radiation are much greater than $m_e c^2$. However, it is just for $kT_b \gg m_e c^2$ (i.e., for very large population numbers $n = kT_b/h\nu$, since $h\nu \ll kT_e \ll m_e c^2$) that the stimulated (induced, enhanced) Compton interaction of the radiation with rarefied matter is particularly important. As a result of this process a heating of the electrons occurs,[1-5] accompanied by a decrease in energy of the low-frequency quanta—by a flux of photons along the energy axis toward low frequencies.[6-9] For the condition $kT_b \gg m_e c^2$ the stimulated radiation pressure on the electrons also becomes important.[10,11]

All of the results enumerated above were obtained on the assumption of a radiation spectrum broad compared to the Doppler profile, $\delta \gg \Delta\nu_D = \nu\sqrt{2kT_e/m_e c^2}$ and do not depend on the electron temperature T_e. The narrowness of the spectrum $\delta \ll \Delta\nu_D$ affects the rate of the processes, but the main conclusion—the decrease in photon energy and heating of the electrons as the result of the stimulated Compton interaction—remains in effect.[8] The photons move downward along the energy axis, but if the line is steeply cut off on the low-frequency side, stimulated scattering cannot shift the photons beyond the limit of the primary profile. The line shifts to the low-frequency edge and narrows (Fig. 1). Since scattering can occur only within the limits of the line, not all electrons take part in the process, and its rate decreases in comparison with that for a broad radiation spectrum with the same brightness temperature. Energy transfer from the radiation to the electrons is reduced, and new regularities appear. In particular, heating of electrons in the field of one laser beam can be no less efficient than heating in the field of crossed or colliding beams. With narrowing of the spectrum, the stimulated radiation pressure also is weakened, but to a lower degree than the electron heating.

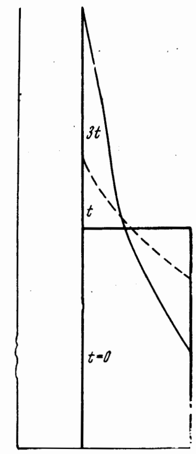

FIG. 1. Evolution of the profile of a narrow intense spectral line in stimulated Compton interaction with free electrons: the initial profile is the rectangle.

2. THE INTEGRAL EQUATION

For a broad radiation spectrum $\delta \gg \Delta\nu_D$ the Compton interaction of Maxwellian electrons with an isotropic radiation field was described by the kinetic equation, which was simplified to the partial differential equation[12]

$$\frac{\partial n}{\partial t}=\frac{\sigma_T N_e h}{m_e c}\frac{1}{\nu^2}\frac{\partial}{\partial \nu}\nu^4\left[n+n^2+\frac{\partial n}{\partial \nu}\frac{kT_e}{h}\right]. \quad (1)$$

Here only the uniform problem was considered; polarization effects were neglected. In the general case with $\delta \lesssim \Delta\nu_D$ and anisotropy of the radiation, this equation is inapplicable and the kinetic equation describing stimulated processes must have an integral form:[8]

$$\frac{\partial n(\nu,\theta,\varphi,t)}{\partial t}=\frac{3}{16\pi}\sigma_T N_e c n(\nu,\theta,\varphi,t) \quad (2)$$

$$\times\int K(\nu,\nu',\alpha)n(\nu',\theta',\varphi',t)$$

$$\times\, d\nu'(1+\cos^2\alpha)\,d\cos\theta'\,d\varphi'.$$

Here N_e is the electron density,

$$^3/_{16}\pi^{-1}\sigma_T(1+\cos^2\alpha)=d\sigma/d\Omega$$

is the differential cross section for scattering,

$$n(\nu,\theta,\varphi)=c^2 I_\nu(\nu,\theta,\varphi)/2h\nu^3$$

is the population number in photon phase space,

$$\varepsilon_\nu=c^{-1}\int I_\nu(\nu,\theta,\varphi)\,d\Omega$$

is the spectral energy density of the radiation, $I_\nu(\nu,\theta,\varphi)$ is the intensity in a given solid angle $d\Omega = d\varphi\, d\cos\theta$. Below

$$\Omega_0=\int_0^{2\pi}d\varphi\int_0^{\theta_0}d\cos\theta=2\pi(1-\cos\theta_0)$$

is the solid angle in which the radiation travels, and $2\theta_0$ is the angular aperture of the radiation beam. The kernel of the equation $K(\nu', \nu, \alpha)$ gives the probability of a transition from one phase-space cell $dk_x dk_y dk_z$, $|\mathbf{k}| = \nu'/c$ to another cell $dq_x dq_y dq_z$, $|\mathbf{q}| = \nu/c$. In the most general case this function depends on both the vectors **k** and **q**. In the case of an isotropic distribution of electrons, the directions of these vectors are not important, K depends on the quantities $|\mathbf{k}| = \nu'/c$, $|\mathbf{q}| = \nu/c$, and the scalar product $\mathbf{k}\cdot\mathbf{q}$, which also leads to a dependence $K(\nu', \nu, \alpha)$, where α is the photon scattering angle. Here

$$\cos\alpha = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\varphi - \varphi'). \tag{3}$$

Note that in the case of isotropic radiation, integration over $d\cos\alpha\, d\varphi'$ is equivalent to integration over $d\cos\theta' d\varphi'$, since the direction of motion of the photon before scattering can be taken as one of the coordinate axes, and the axis can be chosen in the scattering plane. In this case $\theta = 0$ and $\cos\alpha = \cos\theta'$.

We will look for $K(\nu', \nu, \alpha)$, using the known kernel for the case os spontaneous scattering.

a) Spontaneous scattering. The kernel of the integral equation

$$\frac{\partial n(\nu,\theta,\varphi,t)}{\partial t} = -\sigma_T N_e c n(\nu,\theta,\varphi,t) + \frac{3}{16\pi}\sigma_T N_e c \cdot \tag{4}$$

$$\times \int A(\nu',\nu,\alpha)\, n(\nu',\theta',\varphi',t)\, d\nu'(1+\cos^2\alpha)\, d\cos\theta'\, d\varphi',$$

which describes spontaneous Compton scattering and which takes into account the Doppler shift of the photon frequency in scattering by Maxwellian electrons, obtained with neglect of quantum effects, is given in Chandrasekhar's book:[13]

$$A(\nu',\nu,\alpha) = \left[\frac{m_e c^2}{4\pi k T_e \nu\nu'(1-\cos\alpha)}\right]^{1/2} \exp\left\{-\frac{m_e c^2(\nu-\nu')^2}{4kT_e\nu\nu'(1-\cos\alpha)}\right\}$$

$$= \frac{1}{(2\pi)^{1/2}\Delta\nu_D(1-\cos\alpha)^{1/2}} \exp\left\{-\frac{(\nu-\nu')^2}{2\Delta\nu_D^2(1-\cos\alpha)}\right\}, \tag{5}$$

where $\Delta\nu_D = \sqrt{2kT_e/m_e c^2}\,\nu$ is the Doppler width of the spectrum, which characterizes the broadening of a monochromatic line in scattering by Maxwellian electrons. Note that the kernel $A(\nu', \nu, \alpha)$ is symmetric with respect to replacement of ν by ν'.

b) Stimulated scattering. When stimulated processes are taken into account, Eq. (4) takes the form

$$\frac{\partial n(\nu,\theta,\varphi,t)}{\partial t} = -\frac{3}{16\pi}\sigma_T N_e c n(\nu,\theta,\varphi,t) \int A(\nu,\nu',\alpha)\,[1+n(\nu',\theta',\varphi',t)]$$

$$\cdot d\nu'(1+\cos^2\alpha)\, d\cos\theta'\, d\varphi' + \frac{3}{16\pi}\sigma_T N_e c\,[1+n(\nu,\theta,\varphi,t)]$$

$$\cdot \int A(\nu',\nu,\alpha)\, n(\nu',\theta',\varphi',t)\, d\nu'(1+\cos^2\alpha)\, d\cos\theta'\, d\varphi'.$$

Neglecting spontaneous scattering in comparison with stimulated scattering, we obtain an equation of the type of Eq. (2):

$$\frac{\partial n(\nu,\theta,\varphi,t)}{\partial t} = \frac{3}{16\pi}\sigma_T N_e c n(\nu,\theta,\varphi,t) \int [A(\nu',\nu,\alpha) - A(\nu,\nu',\alpha)]$$

$$\times n(\nu',\theta',\varphi',t)\, d\nu'(1+\cos^2\alpha)\, d\cos\theta'\, d\varphi'.$$

If the kernel $A(\nu', \nu, \alpha)$ were absolutely symmetric with respect to replacement of ν by ν', then the kernel $K(\nu', \nu, \alpha) = A(\nu', \nu, \alpha) - A(\nu, \nu', \alpha)$ describing the frequency shift of the photons in stimulated scattering by free electrons would turn out to be identically zero. However, the kernel (5) was found with neglect of quantum corrections of order $h\nu/m_e c^2$, whose inclusion leads to a small asymmetry of the kernel $A(\nu', \nu, \alpha)$ with respect to replacement of ν by ν' and to a difference from zero of the kernel $K(\nu', \nu, \alpha)$. It is possible to repeat the calculations carried out by Chandrasekhar [13] with inclusion of the quantum corrections and to obtain the form of the kernel $K(\nu', \nu, \alpha)$. Here we will limit ourselves to the approximation

$$K(\nu',\nu,\alpha) = A(\nu',\nu,\alpha) - A(\nu,\nu',\alpha) = \frac{\partial A(\nu',\nu,\alpha)}{\partial\nu'}\cdot 2\Delta\nu_1$$

$$= \frac{2h\nu'^2(\nu-\nu')}{(2\pi)^{1/2} m_e c^2 (\Delta\nu_D)^3 (1-\cos\alpha)^{1/2}} \exp\left\{-\frac{(\nu-\nu')^2}{2\Delta\nu_D^2}\right\}, \tag{6}$$

where $\Delta\nu_1 = h\nu'^2(1 - \cos\alpha)/m_e c^2$ is the quantum correction to the Doppler shift of the frequency for Compton scattering by Maxwellian electrons. We will give below a derivation of the formula for electron heating in the field of isotropic radiation with a narrow spectrum, using the kernel K in the form (6). In the Appendix we give an independent derivation of this same formula, which is an indirect confirmation of the validity of the choice of form of the kernel.

c) Properties of the kernel $K(\nu', \nu, \alpha)$. In the case of isotropic radiation with a broad spectrum $\delta \gg \Delta\nu_D$ the integral equation (2) with the kernel (6), as should be expected, is equivalent to the differential equation (1) if we discard in the latter the terms proportional to n, i.e., responsible for spontaneous processes,

$$\frac{\partial n}{\partial t} = \frac{\sigma_T N_e h}{m_e c}\frac{1}{\nu^2}\frac{\partial}{\partial\nu}\nu^4 n^2 = 2\frac{\sigma_T N_e h}{m_e c} n \frac{\partial}{\partial\nu}\nu^2 n. \tag{1'}$$

Actually, integrating (2) by parts and taking into account that

$$(\sqrt{2\pi}\,\Delta\nu_D\sqrt{1-\cos\alpha})^{-1}\exp\left\{-\frac{(\nu-\nu')^2}{2\Delta\nu_D^2(1-\cos\alpha)}\right\}$$

approaches $\delta(\nu - \nu')$—a delta function for $\Delta\nu_D \to 0$, it is easy to obtain Eq. (1′):

$$\frac{\partial n(\nu)}{\partial t} = \frac{3}{16\pi}\sigma_T N_e c n(\nu)\int K(\nu',\nu,\alpha)\, n(\nu')\, d\nu'(1+\cos^2\alpha)\, d\cos\theta' d\varphi'$$

$$= \frac{3}{8\pi}\frac{\sigma_T N_e h n(\nu)}{m_e c}\int\frac{\nu'^2(\nu-\nu')n(\nu')}{(2\pi)^{1/2}\Delta\nu_D^3(1-\cos\alpha)^{1/2}}$$

$$\times \exp\left\{-\frac{(\nu-\nu')^2}{2\Delta\nu_D^2(1-\cos\alpha)}\right\} d\nu'(1+\cos^2\alpha)\, d\cos\alpha d\varphi'$$

$$= \frac{3}{4}\frac{\sigma_T N_e h n(\nu)}{m_e c}\int\frac{\partial}{\partial\nu'}[\nu'^2 n(\nu')]\frac{1}{(2\pi)^{1/2}\Delta\nu_D(1-\cos\alpha)^{1/2}}$$

$$\times \exp\left\{-\frac{(\nu-\nu')^2}{2\Delta\nu_D^2(1-\cos\alpha)}\right\} d\nu'(1-\cos\alpha)(1+\cos^2\alpha)\, d\cos\alpha$$

$$= 2\frac{\sigma_T N_e h n(\nu)}{m_e c}\int\frac{\partial}{\partial\nu'}[\nu'^2 n(\nu')]\,\delta(\nu-\nu')\, d\nu' = 2\frac{\sigma_T N_e h n}{m_e c}\frac{\partial}{\partial\nu} n\nu^2.$$

For a narrow spectrum, replacement of $K(\nu', \nu, \alpha) \times (\nu - \nu')/\Delta\nu_D^2(1 - \cos\alpha)$ by the derivative of the δ function is not permissible; here the behavior for small $(\nu - \nu')$ is important.

The kernel of the integral equation for spontaneous scattering (5) is symmetric with respect to replacement of ν by ν'. The radiation spectrum averaged over angle after spontaneous scattering of a monochromatic line by Maxwellian electrons has an angular point, i.e., a discontinuity of the derivative (see Fig. 2).

The kernel (6) is antisymmetric with respect to replacement of ν by ν'. For the discussion that follows

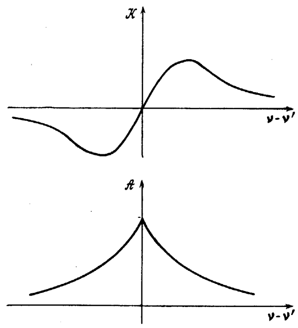

FIG. 2. Properties of the kernels $K(\nu', \nu, \alpha)$ and $A(\nu', \nu, \alpha)$.

Here $\mathscr{K} = \int_0^\pi K(\nu', \nu, \alpha)\, d\cos\alpha$,

$\mathscr{A} = \int_0^\pi A(\nu', \nu, \alpha)\, d\cos\alpha$

it is important that the function

$$F\left(\frac{\nu-\nu'}{\Delta\nu_D}\right) = \frac{2(2\pi)^{1/2} m_e c^2 \Delta\nu_D^2}{h\nu'^2} \int_0^\pi K(\nu', \nu, \alpha)\, d\cos\alpha$$

$$= \frac{\nu-\nu'}{2\Delta\nu_D} \int_0^\pi \exp\left\{-\frac{(\nu-\nu')^2}{2\Delta\nu_D^2(1-\cos\alpha)}\right\} \frac{d\cos\alpha}{\sqrt{1-\cos\alpha}}$$

does not have a discontinuity or a break as $\nu - \nu' \to 0$. Actually,

$$F(x) = \sqrt{2}\, x \exp\left\{-\frac{x^2}{4}\right\} - \sqrt{2\pi}\, x^2 \left[1 - \Phi\left(\frac{x}{\sqrt{2}}\right)\right]_{x\to 0} \to 0$$

in view of the antisymmetry with respect to replacement of x by $-x$: $F(x) = -F(-x)$ which for small x can be represented in the form

$$F(x) = \sqrt{2}\, x - \sqrt{2\pi}\, x|x| = \sqrt{2}\, x(1 - \sqrt{\pi}\, |x|),$$

where as $x \to 0$

$$\frac{\partial F(x)}{\partial x} \to \sqrt{2}, \quad \frac{\partial^2 F(x)}{\partial x^2} \to \mp\sqrt{2\pi},$$

$$\frac{\partial^3 F(x)}{\partial x^3} \to 2\sqrt{2\pi}\, \delta(x),$$

where $\delta(x)$ is the Dirac delta function and $\Phi(t)$ is the probability integral.

In obtaining the Green's function for stimulated scattering, the derivative of the spontaneous-scattering function is taken and is multiplied by the quantum frequency shift; it is essential that this be done for each α with inclusion of the dependence of the quantum shift on α. The derivative of the averaged-over-angle function, which is shown in Fig. 2, multiplied by the average quantum shift, would give an answer with an incorrect singularity near $\nu = \nu'$. Absence of the singularity in the kernel $K(\nu', \nu, \alpha)$ for $\nu \to \nu'$ permits, in the problems of interest to us, replacement in the kernel of the exponential $\exp\{-(\nu-\nu')^2/2\Delta\nu_D^2(1-\cos\alpha)\}$ by unity if two conditions are satisfied: 1) the spectral width of the radiation beam should be less than the Doppler width $\delta \ll \Delta\nu_D$; 2) the maximum possible scattering angle α determined by the beam aperture should be sufficiently large: $1 - \cos\theta_0 > \delta/\Delta\nu_D$.

When these conditions are satisfied the kernel has the form

$$K(\nu', \nu, \alpha) = \frac{2h\nu'^2(\nu-\nu')}{(2\pi)^{1/2} m_e c^2 \Delta\nu_D^3 (1-\cos\alpha)^{1/2}}. \tag{7}$$

We note that the kernel in the form (7) has lost its relation to the Maxwellian distribution function of the electrons, which is explicitly present in (6); obviously it is valid for a broad class of electron velocity-distribution functions, of course for the condition that $\delta \ll \nu\bar{v}/c$, where $\bar{v}$ is the average random velocity of the electrons. A rigorous investigation is desirable for an arbitrary electron spectrum, and in particular, for the spectrum obtained in the action of spectrally narrow radiation on a collisionless plasma.[14]

In addition, absence in the kernel $K(\nu', \nu, \alpha)$ of a singularity at $\nu \to \nu'$ permits replacement of the scattering angle α by its average value $\bar{\alpha}$ in the calculation of integrals of the type (2). This property of the kernel substantially simplifies the numerical evaluations.

3. ELECTRON HEATING

As has already been noted,[8] stimulated Compton scattering leads to heating of the electrons also in the case of a narrow spectrum. The kernel obtained (6) permits determination of the heating of electrons in a high-intensity spectrally narrow radiation field,

$$L^+ = -\frac{2h}{m_e c^3} \int \nu^3 \frac{\partial n(\nu; \theta, \varphi)}{\partial t}\, d\nu\, d\cos\theta\, d\varphi. \tag{8}$$

For the condition $\delta \ll \Delta\nu_D$, using Eqs. (2) and (7), we have

$$L^+ = \frac{3}{4\pi\sqrt{2\pi}} \frac{\sigma_T h^2}{m_e c^4 \Delta\nu_D^3} \int \nu^3 n(\nu, \theta, \varphi)\, \nu'^2 (\nu - \nu') n(\nu', \theta', \varphi') \times d\nu\, d\nu' \frac{d\cos\theta\, d\cos\theta'\, d\varphi\, d\varphi' (1 + \cos^2\alpha)}{\sqrt{1-\cos\alpha}}.$$

For

$$n = \begin{cases} 0 & \text{for } \nu < \nu_0, \\ n_0 & \text{for } \nu_0 < \nu < \nu_0 + \delta, \\ 0 & \text{for } \nu > \nu_0 + \delta \end{cases} \tag{9}$$

and an isotropic radiation field, we obtain[1)]

$$L^+ = \sqrt{\frac{3}{2}}\, \pi \frac{\sigma_T h^2 \nu_0^4 n_0^2}{m_e c^4} \left(\frac{\delta}{\Delta\nu_D}\right)^3 \delta. \tag{8'}$$

The cooling of nonrelativistic electrons in a radiation field is determined by the spontaneous Compton process and therefore does not depend on the form of the radiation spectrum. The rate of cooling is proportional to the total energy density of the radiation;[15] for an isotropic radiation field we have

$$L^- = \frac{4\sigma_T k T_e}{m_e c} \varepsilon_r = \frac{32\pi\sigma_T k T_e}{m_e c^4} h \int n\nu^3\, d\nu.$$

For a spectrum of the form (9) we have

$$L^- = \frac{32\pi\sigma_T k T_e}{m_e c^4} h n_0 \nu_0^3 \delta,$$

and the equilibrium temperature of electrons in the field of isotropic narrow-band radiation of high intensity is

$$\frac{kT_e}{m_e c^2} = \frac{1}{4} \left(\frac{3}{16\pi}\right)^{1/5} \left(\frac{kT_b}{m_e c^2}\right)^{2/5} \left(\frac{\delta}{\nu}\right)^{6/5},$$

where $kT_b = n_0 h\nu_0$ is the brightness temperature of the radiation. We recall that in the case of a broad radiation spectrum $\nu \gg \delta \gg \Delta\nu_D$ of the form (9) we have[2]

$$\frac{kT_e}{m_e c^2} = \frac{h}{4m_e c^2} \frac{\int \nu^4 n^2 d\nu}{\int \nu^3 n d\nu} = \frac{1}{4} \frac{kT_b}{m_e c^2}.$$

1)This result differs from the simple assumption[4] that the heating decreases in proportion to $\delta/\Delta\nu_D$, i.e., $\propto T_e^{-1/2}$.

Thus, while the electrons are cold the heating occurs rapidly, but with increased temperature the condition $\delta > \Delta\nu_D$ is violated and the electron heating rate decreases rapidly as the result of stimulated Compton scattering. Here the electron cooling rate does not depend on the ratio $\delta/\Delta\nu_D$, i.e., the relative role of cooling increases.

Heating by a radiation beam. In the case of electron heating by a laser beam the dependence on the angular aperture of the beam becomes important; for the condition

$$1-\cos\theta_0 \ll \delta^2/2\Delta\nu_D^2, \quad \theta_0 \ll \delta/\Delta\nu_D \tag{10}$$

this dependence becomes dominant. Let us assume a beam angular distribution:

$$n = \begin{cases} 0 & \text{for } \theta > \theta, \\ n_0 & \text{for } 0 < \theta < \theta_0. \end{cases} \tag{11}$$

Integrating (8), using (2) with a kernel K in the form (6) and taking into account that

$$\frac{1}{1-\cos\alpha}\int K(\nu',\nu,\alpha)\,d\nu'$$

approaches the Dirac δ function for $\theta_0\Delta\nu_D/\delta \to 0$, we obtain

$$L^+ = \frac{3}{8\pi}\frac{\sigma_T h^2}{m_e c^4}\int n^2\nu^4 d\nu \int (1+\cos^2\alpha)(1-\cos\alpha)\,d\cos\theta\, d\cos\theta'\, d\varphi\, d\varphi', \tag{12}$$

which for $\theta_0 \ll 1$ and for an angular and spectral distribution of the beam in the form (9) and (11) leads to the expression

$$L^+ = \frac{3\pi}{16}\frac{\sigma_T h^2}{m_e c^4} n_0^2\nu_0^4\theta_0^6\delta. \tag{13}$$

The case (10) presents interest for astrophysics:[5] at a distance r from a luminous sphere of radius $R \ll r$

$$L^+ = \frac{3\pi}{16}\frac{\sigma_T h^2}{m_e c^4}\int n^2\nu^4 d\nu \left(\frac{R}{r}\right)^6$$

and with allowance for the decrease of intensity with distance from the sphere $I_\nu(r) = I_\nu(R)(R/r)^2$, we obtain

$$L^+ = \frac{3\pi}{64}\frac{\sigma_T}{m_e}\int \frac{I_\nu^2(R)}{\nu^2}d\nu\left(\frac{R}{r}\right)^6 = \frac{3\pi}{64}\frac{\sigma_T}{m_e}\int\frac{I_\nu^2(r)}{\nu^2}d\nu\left(\frac{R}{r}\right)^2. \tag{14}$$

For the condition

$$\theta_0 \gtrsim \delta/\Delta\nu_D, \tag{15}$$

which is the inverse of (10), the main role is played by the spectral width of the beam. From (2), (7), and (8) for a beam in the form (9) and (11) we obtain

$$L^+ = \frac{3}{8\pi(2\pi)^{1/2}}\frac{\sigma_T h^2\nu_0^4}{m_e c^4}\left(\frac{\delta}{\Delta\nu_D}\right)^3\delta\int\frac{1+\cos^2\alpha}{\sqrt{1-\cos\alpha}}\,d\cos\theta\, d\cos\theta'\, d\varphi\, d\varphi'.$$

We will make an estimate for the central line of the beam, assuming $\theta = 0$, replacing the integration over the initial angles by multiplication by Ω and integration only over $d\cos\theta'\,d\varphi'$. Then $\cos\alpha = \cos\theta'$ and

$$L^+ = \frac{3}{2}\pi^{1/2}\frac{\sigma_T n_0^2 h^2\nu_0^4\delta}{m_e c^4}\left(\frac{\delta}{\Delta\nu_D}\right)^3\theta_0^3. \tag{16}$$

Equation (16) differs from (13) by the factor $(\delta/\theta_0\Delta\nu_D)^3$. We note that in interaction of electrons with a spectrally narrow radiation beam with aperture $1 \gg \theta_0 \gtrsim \delta/\Delta\nu_D$, heating by one beam is no less efficient than heating in the radiation field of crossed beams. Here there is a strong difference from the case of a low-aperture beam $\theta \ll 1$ for a spectrum $\delta > \Delta\nu_D$, when the presence of the factor $(1-\cos\alpha)$ in Eq. (12) leads to a dependence

$$L^+ = \frac{3\pi}{8}\theta_0^4(1+\cos^2\alpha)(1-\cos\alpha)\int n_1 n_2\nu^4\, d\nu\,\frac{\sigma_T h^2}{m_e c^4}$$

where α is the angle between the beams. In this situation the heating in crossed beams is stronger by a factor $(1+\cos^2\alpha)(1-\cos\alpha)/\theta_0^2$ than in one beam. The rate of cooling in spontaneous Compton scattering of anisotropic radiation is practically the same as in the isotropic case,

$$L^- = \frac{4\pi}{3}\frac{\sigma_T kT_e}{m_e c^4}hn_0\nu_0^3\delta\theta_0^2.$$

Equating L^+ and L^-, we find the equilibrium electron temperature: for the condition (10)

$$kT_e = {}^9/_{64}kT_b\theta_0^4,$$

and for the condition (15)

$$\frac{kT_e}{m_e c^2} = \left(\frac{81}{512\pi}\right)^{1/5}\left(\frac{kT_b}{m_e c^2}\right)^{2/5}\left(\frac{\delta}{\nu}\right)^{6/5}\theta_0^{6/5}.$$

We will give the formulas for heating and for the equilibrium temperature of the electrons in the field of a pulsed laser beam of power W/t (ergs/sec) and pulse duration t. The linear dimension of the beam at the focus of the lens is r. It is assumed that the spectrum and angular distribution have the form (9) and (11), and that $\delta \ll \Delta\nu_D$. Then the population number is

$$n_0 = Wc^2/2\pi^2\theta_0^2r^2t\delta h\nu_0^3,$$

and the brightness temperature

$$kT_b = n_0h\nu_0 = Wc^2/2\pi^2t\theta_0^2r^2\delta\nu_0^2.$$

For the condition (15)

$$L^+ = \frac{3}{8\pi^{7/2}}\frac{\sigma_T W^2}{\theta_0 r^4t^2m_e\nu_0^2\delta}\left(\frac{\delta}{\Delta\nu_D}\right)^3,$$
$$\frac{kT_e}{m_e c^2} = \frac{3^{4/5}}{4\pi}\left(\frac{W}{\theta_0 r^2 t m_e\nu_0^3}\right)^{2/5}\left(\frac{\delta}{\nu_0}\right)^{4/5}, \tag{16a}$$

and for condition (10) in the field of crossed beams

$$L^+ = \frac{3}{32\pi^3}\frac{\sigma_T W^2}{r^4t^2m_e\delta\nu_0^2}(1+\cos^2\alpha)(1-\cos\alpha),$$
$$kT_e = \frac{9}{128\pi^2}\frac{Wc^2}{r^2t\delta\nu_0^2}(1+\cos^2\alpha)(1-\cos\alpha).$$

The time for establishment of the equilibrium temperature (we recall that all the formulas discussed are valid only for $kT_e \ll m_ec^2$)

$$\tau = \frac{3}{2}\frac{kT_e}{L^-} = \frac{3}{2}\frac{kT_e}{L^+}$$

can be substantially less than t.

In the stimulated Compton interaction of Maxwellian electrons with spectrally narrow radiation or for a small beam aperture, only those electrons take part whose velocity directions form sufficiently small angles with the direction of the radiation beam axis.[4] In this connection Bunkin and Kazakov[4] note that for the condition $\delta \ll \Delta\nu_D$ stimulated Compton heating is not efficient, since it leads not to heating of the electrons but only to a rapid distortion of the electron distribution function along the beam axis and to a still sharper decrease in the exchange of energy between the radiation and the plasma. It would appear that the sole condition for validity of the formulas given above is the requirement that the characteristic relaxation time of the electron distribution function be less than the characteristic time of heating of the plasma. In reality, under astrophysical conditions there is always a magnetic field (and in the laboratory it can be created) which, rotating the electrons, leads to constant presence of electrons with momenta directed at small angles to the beam axis. A magnetic field strength is necessary for which the condition exists $1/t_1 < \nu_c < \nu$, where $\nu_c = eH/m_e c$ is the cyclotron frequency, ν is the radiation frequency in the beam, and t_1 is the characteristic time of distortion of the electron distribution function.

4. EVOLUTION OF SPECTRALLY NARROW RADIATION LINES IN INTERACTION WITH FREE ELECTRONS

We discuss spectrally narrow radiation interacting with free electrons. The interaction mechanism is the stimulated Compton effect. The electrons are assumed Maxwellian with a temperature T_e, and the spectral width of the radiation $\delta \ll \Delta\nu_D$ for a sufficiently broad angular distribution of the radiation $\theta_0 > \delta/\Delta\nu_D$. We will use the integral equation (2) with the kernel (7):

$$\frac{\partial n(\nu,t)}{\partial t} = A\frac{\sigma_T N_e h}{m_e c} n(\nu,t)\int \nu'^2 \frac{(\nu-\nu')n(\nu',t)}{\Delta\nu_D^3}d\nu', \qquad (17)$$

where

$$A = \frac{3}{8\pi(2\pi)^{1/2}}\int\frac{1+\cos^2\alpha}{\sqrt{1-\cos\alpha}}d\cos\theta'\,d\varphi'.$$

In the case of an isotropic radiation field $A = 11/5\pi^{1/2} \sim 1$, and for an angular distribution of the form (11) it is easy to make an estimate for the central line of the beam, setting $\theta = 0$ and $\alpha = \theta'$; then $A = 3\theta_0/\sqrt{2\pi} \sim \theta_0$.

We recall that Compton scattering preserves the number of photons in the system, i.e., the photon density

$$N_\nu = \frac{2}{c^3}\int \nu'^2 n(\nu')d\nu'\,d\cos\theta\,d\varphi \qquad (18)$$

does not depend on time. At the same time the energy density of the radiation

$$\varepsilon(t) = \int \varepsilon_\nu d\nu = \frac{2h}{c^3}\int \nu'^3 n(\nu',t)d\nu'\,d\cos\theta\,d\varphi \qquad (19)$$

depends on time. Combining Eq. (17)–(19), we obtain the equation

$$\frac{\partial n(\nu,t)}{\partial t} = Bn(\nu,t)\,[\varepsilon(t) - N_\nu h\nu],$$

whose solution is

$$n(\nu,t) = n_1(\nu)\exp\left\{B\left[\int_0^t \varepsilon(t)\,dt - N_\nu h\nu t\right]\right\}, \qquad (20)$$

where $n_1(\nu)$ is determined by the initial profile of the line, and $B = A\sigma_T N_e c^2/2m_e\Delta\nu_D^3$. The solution, written in the form (20), makes clear the physics of the process: in the spectral line the number of photons with frequency greater than the average at a given moment of time

$$\nu > \frac{1}{hN_\nu t}\int\varepsilon(t)\,dt,$$

decreases, and the number with frequencies less than the average increases. Another form of the solution

$$n(\nu,t) = n_1(\nu)\psi(t)e^{-\alpha\nu t}, \qquad (20')$$

where

$$\alpha = BhN_\nu, \quad \psi(t) = \exp\left\{B\int_0^t\varepsilon(t)\,dt\right\},$$

better demonstrates the law of motion of photons along the frequency axis. For illustration we will present a series of solutions for definite $n_1(\nu)$.

a) Gaussian line profile. The line initially had a Gaussian profile

$$n_1(\nu) = \frac{1}{\sqrt{2\pi}\,\delta}\exp\left\{-\frac{(\nu-\nu_1)^2}{2\delta^2}\right\}, \quad \delta \ll \Delta\nu_D,$$

and then

$$n(\nu,t) = \frac{1}{\sqrt{2\pi}\,\delta}\exp\left\{-\frac{[\nu-(\nu_1-{}^1/_2\,\delta^2BN_\nu th)]^2}{2\delta^2} + \varphi(t)\right\}, \qquad (21)$$

where

$$\varphi(t)/B = -\nu_1 N_\nu t + {}^1/_4\,\delta^2 N_\nu^2 Bt^2 + \int\varepsilon(t)\,dt.$$

It follows from (21) that a Gaussian profile with $\delta \ll \Delta\nu_D$ is not distorted as the result of the stimulated interaction with electrons, but is shifted toward low frequencies with a rate

$$\frac{d\nu}{dt} = -\frac{A}{2}\frac{\sigma_T hc^2}{m_e}N_e N_\nu\frac{\delta^2}{\Delta\nu_D^3} = -\frac{A}{2}\frac{\sigma_T c^2}{m_e}N_e\frac{\delta^2}{\Delta\nu_D^3}\int\frac{\varepsilon_\nu}{\nu}d\nu. \qquad (22)$$

This answer, in principle, can be obtained also without recourse to the language of quantum mechanics; it does not depend on Planck's constant h. For a broad spectrum $\delta \gg \Delta\nu_D$, it follows from Eq. (1) that

$$\frac{d\nu}{dt} = \frac{\sigma_T}{4\pi}\frac{N_e c^2}{m_e}\frac{\varepsilon_\nu}{\nu}, \qquad (23)$$

i.e., because of the narrowness of the line, for the same brightness temperature of the radiation the rate of motion of the photons downward along the energy axis decreases by a factor $(\Delta\nu_D/\delta)^3$.

b) Lorentz profile. For an initial spectrum

$$n_1 = \frac{1}{(\nu-\nu_1)^2+\Gamma^2}$$

the solution (20) has the form

$$n(t) = \psi(t)\frac{e^{-\alpha\nu t}}{(\nu-\nu_1)^2+\Gamma^2}$$

from which it is evident that the line spreads, becomes asymmetric, and gradually disappears as the result of transition of photons to the low-frequency wing.

c) Rectangular line profile. The case in which the line has infinite derivatives at the edges of the line is also interesting for applications. We will present a series of results from solution (20). In those regions where

$$h\nu < \frac{1}{N_\nu t}\int\varepsilon(t)\,dt,$$

the intensity increases, and where the reverse relation holds, it decreases. The line narrows and shifts toward the low-frequency edge. The characteristic time of narrowing of the line is

$$\tau_C \sim \frac{2}{A}\frac{m_e}{\sigma_T c^2 h}\frac{1}{N_e N_\nu}\frac{\Delta\nu_D^3}{\delta} = \frac{1}{A}\frac{1}{\sigma_T N_e c}\left(\frac{m_e c^2}{kT_b}\right)\left(\frac{\Delta\nu_D}{\nu}\right)^3\left(\frac{\nu}{\delta}\right)^2. \quad (24)$$

The line narrows substantially ($\Delta\nu \sim \delta$) when

$$A\sigma_T N_e l\left(\frac{kT_b}{m_e c^2}\right)\left(\frac{\nu}{\Delta\nu_D}\right)^3\left(\frac{\delta}{\nu}\right)^2 \sim 1, \quad (25)$$

where $\tilde{l} \sim r/\theta_0$ is the characteristic length of the interaction.

Is it possible to observe this effect in the laboratory? For a pulsed laser (see the designations near the end of Sec. 3) spectral narrowing of the beam will occur for the condition

$$\frac{W}{t} \gtrsim \frac{8\pi^{5/2}}{3}\frac{\theta_0^2 r m_e \nu_0^3}{\sigma_T N_e}\left(\frac{kT_e}{m_e c^2}\right)^{3/2}\frac{\nu_0}{\delta}. \quad (26)$$

Substituting the optimal parameters $\theta_0 = \delta/\Delta\nu_D$ and $N_e = \pi m_e \delta^2/4e^2$ (when $\delta = 2\nu_{pl}$; ν_{pl} is the Langmuir plasma frequency) into (26) and (16a), we see that a substantial change of the radiation spectrum sets in only for the condition $30 r r_0/\lambda^2 \gg 1$, where r_0 is the classical electron radius and λ is the wavelength of the radiation. In the opposite case the electrons are heated to relativistic temperatures before the spectrum is distorted. Therefore only cosmic masers can apparently satisfy the condition $r \gg \lambda^2/30 r_0$. Another formulation of the problem is also possible: a laser cannot during a pulse heat electrons to relativistic temperatures

$$kT_e = \frac{W\theta_0\delta/\lambda}{\pi r^3 N_e} < m_e c^2. \quad (27)$$

Combining (26) and (27) for the optimal θ_0, we obtain the condition

$$W > \pi^2\left(\frac{3}{2}\right)^{3/2} m_e c^2\left(\frac{\nu}{\delta}\right)^3 (t\delta)^{3/2}\frac{\lambda}{r_0} \quad \text{for} \quad r^2 > 3\left(\frac{\nu}{\delta}\right)^2 (t\delta)\lambda^2\frac{m_e c^2}{kT_e},$$

which can be satisfied only for experimentally large W.

5. STIMULATED PRESSURE ON THE ELECTRONS OF A SPECTRALLY NARROW RADIATION BEAM

Before turning to calculation of the stimulated radiation pressure on a cluster of low-density ionized plasma, we will consider the force of spectrally narrow radiation on a single free electron moving with velocity $v_0 = p_0/m_e$ in the direction of the radiation flux, chosen as the x axis. We will assume also that the angular aperture of the beam is small, $\theta_0 \ll 1$. The expression for this force in the rest system of the electron is easily obtained from the general formula for stimulated radiation pressure:[10]

$$\mathbf{f}_{ind} \approx \frac{3}{16\pi}\sigma_T c\int n(\nu,\mathbf{l})n(\nu,\mathbf{l}')\mathbf{l}'(1-\mathbf{l}\mathbf{l}')[1+(\mathbf{l}\mathbf{l}')^2]\frac{h^2\nu^4}{m_e c^3}\frac{d\nu}{c^3}d\Omega\, d\chi, \quad (28)$$

where $\mathbf{l}$, $\mathbf{l}'$ are unit vectors in the direction of propagation of the photon before and after scattering, $d\chi = d\cos(\mathbf{l}\cdot\mathbf{l}')d\varphi' = d\cos\alpha\, d\varphi'$. The stimulated radiation pressure in the radiation beam (28) is directed in the same direction as the energy flux of the radiation, although in the general case (for example, colliding beams) its direction may not be related to the direction of the radiant energy flux.[10]

In the laboratory system of coordinates, in scattering by a moving electron the frequency of a photon changes as a result of the Doppler effect. Here the change in photon frequency $\Delta\nu_p \approx \nu p_0(\mathbf{l}-\mathbf{l}')/m_e c$ may exceed the spectral width δ of the radiation. In this case, obviously, large-angle scatterings corresponding to a large Doppler shift do not contribute to the stimulated processes; therefore interest is presented only by small-angle scattering:

$$\Delta\nu_p = \frac{1}{2}(\theta'^2 - \theta^2)\frac{p_0\nu}{m_e c} \lesssim \frac{\nu p_0}{m_e c}\theta_0^2, \quad (29)$$

where the inequality $\Delta\nu_p \lesssim \delta$ is satisfied. The radiation pressure is associated with transfer of longitudinal momentum of the photons in the scattering, while the heating—a collection of random energy—occurs in transfer of the lateral component of the photon momentum (perpendicular to the direction of the radiation flux). In the case of the transverse Doppler effect in the problems of interest to us, a more rigid condition (15) is imposed on the angle between the initial and final directions of propagation of the photon than for the longitudinal Doppler effect (29).

Two situations are possible:

$$p_0\theta_0^2 / m_e c < \delta/\nu, \quad (29a)$$

$$p_0\theta_0^2 / m_e c > \delta/\nu. \quad (29b)$$

In the first case, when the angular aperture of the beam is small, its spectrum can be considered broad and, using the fact that the maximum possible photon-scattering angle $\alpha_{max} \approx 2\theta_0$ is equal in order of magnitude to the angular aperture of the beam, it is easy to find from (28) for n in the form (9) and (11)

$$f_{ind} = \frac{3\pi}{32}\frac{\sigma_T h^2\nu_0^4 n_0^2\delta}{m_e c^5}\theta_0^6. \quad (30a)$$

For a laser with power W/t with a focusing zone radius r we obtain from this

$$f_{ind} = \frac{3\sigma_T W^2\theta_0^2}{128\pi^3 r^4 t^2 m_e c\nu_0^2\delta}.$$

At a distance r from a radiating sphere of luminosity L and radius R we have

$$f_{ind} = \frac{3\sigma_T L^2 (R/r)^6}{128\pi(4\pi R^2)^2 m_e c\nu_0^2\delta}.$$

In the second case the scattering angle α is limited by the inequality $\alpha^2_{max} = m_e c\delta/\nu p_0 \ll 1$ and

$$f_{ind} = \frac{3\pi}{32}\frac{\sigma_T h^2\nu_0^4\delta n_0^2}{m_e c^5}\alpha^4_{max}\theta_0^2 \approx \frac{3\pi\sigma_T h^2\nu_0^4 n_0^2\delta}{32 m_e c^5}\left(\frac{\delta}{\nu_0}\frac{m_e c}{p_0}\right)^2\theta_0^2. \quad (30b)$$

Hence for a laser

$$f_{ind} = \frac{3\sigma_T W^2}{128\pi^3 r^4 t^2 m_e c\nu_0^2\delta}\left(\frac{\delta}{\nu_0}\frac{m_e c}{p_0}\right)^2\frac{1}{\theta_0^2}$$

and for a luminous sphere

$$f_{ind} = \frac{3\sigma_T L^2}{128\pi(4\pi R^2)^2 m_e c\nu_0^2\delta}\left(\frac{\delta}{\nu_0}\frac{m_e c}{p_0}\right)^2\left(\frac{r}{R}\right)^2.$$

From (30a) and (30b) it can be seen that for acceleration of free cold electrons to a velocity $v_0 = p_0/m_e$ the optimum angular apertures of the beams are $\theta_0 \approx \sqrt{p_0\delta/\nu_0 m_e c}$, for which f_{ind} is maximal.

The rate of energy transfer from the radiation to cold electrons (having negligible thermal velocities) due to radiation pressure,

$$L_{pr}{}^{+} = f_{ind}N_e p_x / m_e \quad (31)$$

in the second case (29b) is related to the rate of heating

(16) as $(\nu_0/\delta)\times(v_T/v_0)^3\theta_0^{-1}$; in the first case (29a) it is always less than (13) by a factor m_0c/p_0. What change occurs in these derivations in transition to discussion of stimulated radiation pressure on a plasma cluster in which the electrons have appreciable thermal velocities?

Let the electron momentum distribution

$$\Phi(\mathbf{p})=\frac{1}{(2\pi m_e kT_e)^{3/2}}\exp\left\{-\frac{p_z^2+p_y^2+(p_x+p_0)^2}{2m_e kT_e}\right\}$$

describe both the random motion of the electrons with thermal velocities v_T and the systematic motion of the plasma as a whole with velocity $v_0 = p_0/m_e$. We will assume also that in interaction of electrons with the radiation the shape of their momentum distribution does not change, and only T_e and T_0 are variable. Here the pressure on each moving electron taken individually will be described by the formula

$$f_{ind}=\frac{3}{16\pi}\sigma_T c\int n(\nu,\mathbf{I})n[\nu+(\Delta\nu_p+\Delta\nu_D)(\cos\theta-\cos\theta'),\mathbf{I}']$$
$$\times\mathbf{I}'[1-(\mathbf{II}')][1+(\mathbf{II}')^2]\frac{h^2\nu^4}{m_ec^3}\frac{d\nu}{c^3}d\Omega\,d\chi \tag{28'}$$

(where as before $\Delta\nu_D=\nu_0\sqrt{2kT_e/m_ec^2}$, and $\Delta\nu_p = p_0\nu_0/m_ec$) and by the analogs (30a) and (30b); however, the conditions of applicability of these formulas and the specific values of the force will be different for each electron, depending on the magnitude of its momentum and its direction of motion relative to the radiation beam axis. The radiation pressure on the plasma cluster in the sum of the forces acting on the individual electrons.

We will look for the average stimulated radiation force acting on a thermal electron. In the case of interest, the force acting on each electron taken individually is directed along the axis of the radiation beam; the average force must have the same direction

$$\bar f_{ind}=\int f_{ind}\Phi(\mathbf{p})\,d^3p.$$

After some straightforward calculations we obtain by analogy with [13]

$$\bar f_{ind}=\frac{3}{16\pi}\sigma_T\frac{h^2}{m_ec^5}\int n(\nu,\theta,\varphi)n[\nu+(\Delta\nu_p+\Delta\nu_D)(\cos\theta-\cos\theta');\theta',\varphi']$$
$$\times\frac{1}{\sqrt{2\pi}\,\Delta\nu_D(1-\cos\alpha)^{1/2}}\exp\left\{-\frac{(\nu'-\nu[1+p_0(1-\cos\alpha)/m_ec])^2}{2\Delta\nu_D^2(1-\cos\alpha)}\right\}\nu^4\,d\nu\,d\nu'$$
$$\times(1-\cos\alpha)(1+\cos^2\alpha)\,d\Omega\,d\chi. \tag{32}$$

In the limit $\theta_0>\delta/\Delta\nu_D$ the exponential in (32) can be replaced by unity. Then for $\theta_0^2<\delta/(\Delta\nu_p+\Delta\nu_D)<\theta_0$ we obtain

$$\bar f_{ind}=\pi^{1/2}\frac{\sigma_T h^2\nu_0^4 n_0^2\delta}{m_ec^5}\frac{\delta}{\Delta\nu_D}\theta_0^5. \tag{30c}$$

For a laser we have

$$\bar f_{ind}=\frac{\sigma_T W^2}{4\sqrt{2}\pi^{7/2}t^2r^4\nu_0^2m_ec}\sqrt{\frac{m_ec^2}{kT_e}}\theta_0$$

and for a luminous sphere

$$\bar f_{ind}=\frac{\pi^{1/2}\sigma_T L^2}{4\sqrt{2}(4\pi R^2)^2\nu_0^2m_ec}\sqrt{\frac{m_ec^2}{kT_e}}\left(\frac{R}{r}\right)^5.$$

At the same time for $\alpha_{max}^2\approx\delta/(\Delta\nu_p+\Delta\nu_D)<\theta_0^2$ we have

$$\bar f_{ind}\approx\frac{\pi^{1/2}}{8}\frac{\sigma_T h^2\nu_0^4n_0^2\delta}{m_ec^5}\frac{\delta}{\Delta\nu_D}\left(\frac{\delta}{\Delta\nu_p+\Delta\nu_D}\right)^{3/2}\theta_0^2. \tag{30d}$$

For a laser

$$\bar f_{ind}=\frac{\sigma_T W^2}{32\sqrt{2}\pi^{7/2}t^2r^4\nu_0^2m_ec}\sqrt{\frac{m_ec^2}{kT_e}}\left(\frac{\delta}{\nu_0(p_0/m_ec+\sqrt{kT_e/m_ec^2})}\right)^{3/2}\frac{1}{\theta_0^2}$$

and for a luminous sphere

$$\bar f_{ind}=\frac{\pi^{1/2}\sigma_T L^2}{32\sqrt{2}(4\pi R^2)^2\nu_0^2m_ec}\sqrt{\frac{m_ec^2}{kT_e}}\left(\frac{\delta}{\nu_0(p_0/m_ec+\sqrt{kT_e/m_ec^2})}\right)^{3/2}\left(\frac{R}{r}\right)^2.$$

In the limit $\theta_0<\delta/\Delta\nu_D$ the exponential in (32) can be replaced by a δ function and for the additional condition $\theta_0^2<\delta/(\Delta\nu_D+\Delta\nu_p)$ we obtain (30a). In the case $\delta/\Delta\nu_D>\theta_0\gg\theta_0^2>\delta/(\Delta\nu_D+\Delta\nu_p)$, which occurs for $\Delta\nu_p\gg\Delta\nu_D$, Eq. (32) reduces to (30b). Thus, for $\theta_0<\delta/\Delta\nu_D$ the average force on the electrons is equal to the force acting on an individual electron.

The energy transferred per unit time to a plasma cluster with $p_0>\sqrt{kT_em_e}$ as the result of the action of stimulated radiation pressure,

$$L_{pr}{}^+=\int f_{ind}\frac{p_x}{m_e}\Phi(p)\,d^3p,$$

in the case

$$\frac{\delta^2}{\nu_0^2}\frac{m_ec^2}{kT_e}\frac{m_ec}{p_0}<\theta_0^2<\frac{\delta}{\nu}\frac{m_ec}{p_0}$$

exceeds the energy expended in electron heating (16), which opens up the possibility of a number of applications.

We note that for $p_0\ll\sqrt{kT_em_e}$ the collection of energy in acceleration of electrons moving in the direction of the radiation beam is compensated by the giving up of energy to the plasma in slowing down of electrons moving in the opposite direction.

6. LIMITS OF APPLICABILITY OF THE RESULTS OBTAINED

Collective effects can be neglected when the change in photon frequency in scattering exceeds the Langmuir frequency $\nu_{pl}=\sqrt{e^2N_e/\pi m_e}$. For a broad spectrum $\delta\gg\Delta\nu_D$ of isotropic radiation we have $\Delta\nu_D>\nu_{pl}$ or $\nu>\nu_{pl}c/v_T$, and for a narrow spectrum we have $\delta>\nu_{pl}$. For a small angular aperture of the beam we obtain in the case of heating $\nu>\nu_{pl}c/v_T\theta_0$, when $\theta_0<\delta/\Delta\nu_D$, and in the case of pressure $\nu>\nu_{pl}c/v_T\theta_0$, when $\theta_0^2<\delta/(\Delta\nu_D+\Delta\nu_p)$. A limitation on the frequency leads to a limitation on the electron density; thus, for example, for a narrow spectrum when $\delta>\nu_{pl}$, we have $N_e<\pi m_e\delta^2/e^2$.

The formulas presented were obtained in the nonrelativistic approximation $kT_e\ll m_ec^2$, $p_0\ll m_ec$. In derivation of these formulas we did not take into account effects arising in the relativistic oscillatory motion of the electrons in the radiation field, when $eE/2\pi\nu m_ec>1$. Here E is the electric field intensity. Finally, the phases of the waves are assumed to be random.

APPENDIX

ELECTRON HEATING BY HIGH-INTENSITY ISOTROPIC RADIATION

The Fokker-Planck equation for the distribution function of nonrelativistic Maxwellian electrons in an isotropic radiation field and interacting with it by the Compton mechanism has the form[16]

$$\frac{\partial \Phi}{\partial t}=\frac{\partial}{\partial p_i}D_{ik}\frac{\partial \Phi}{\partial p_k}+\frac{\partial}{\partial p_i}A_i\Phi, \qquad (A.1)$$

where p_i and p_k are the components of the electron momentum. The second term on the right depends only on spontaneous processes and describes the slowing down of electrons in the radiation field—electron cooling. The first term on the right depends both on spontaneous and on stimulated processes and describes the heating of electrons as the result of their Compton interaction with the radiation. Assuming a large population number in photon phase space, we will neglect spontaneous processes in the first term. Then the diffusion coefficient is

$$D_{ik}=\int \sigma c n(\mathbf{k}) n(\mathbf{k}')\Delta p_i \Delta p_k\, dk'\, d\Omega, \qquad (A.2)$$

where $\sigma d\Omega = \sigma(\alpha) d\cos\alpha\, d\varphi$ is the differential cross section for scattering, $\int \sigma d\Omega = \sigma_T$, and $\Delta p_i = p_i - p_i' = hk_i - hk_i'$; approximately

$$\Delta \mathbf{p} \approx h\nu(\mathbf{l}-\mathbf{l}')/c; \qquad (A.3)$$

$\mathbf{l}$ is a unit vector in the photon propagation direction. Here we neglect $\Delta\nu$, i.e., corrections for the Doppler effect (of order $ph\nu/m_ec^2$) and quantum effects (of order $(h\nu)^2/m_ec^3$).

By definition,

$$\frac{3}{2}k\frac{dT_e}{dt}=\frac{d\bar{E}_e}{dt}=\int \frac{p^2}{2m_e}\frac{\partial \Phi}{\partial t}d\mathbf{p}=L^+-L^-, \qquad (A.4)$$

the heating L^+ for large $n(\mathbf{k})$ is determined by the stimulated Compton effect, and the cooling L^-—in the nonrelativistic limit $kT_e \ll m_ec^2$—only by the spontaneous process. Writing out L^+ according to (A.4) and (A.1), integrating by parts and substituting $\partial\Phi/\partial p_k = -p_k\Phi/2m_ekT_e$ for a Maxwellian function Φ, we obtain

$$L^+=\int \frac{p^2}{2m_e}\frac{\partial}{\partial p_i}D_{ik}\frac{\partial \Phi}{\partial p_k}d\mathbf{p}=-\int \frac{p_i}{m_e}D_{ik}\frac{\partial \Phi}{\partial p_k}d\mathbf{p}=\int \frac{p_i}{m_e}D_{ik}\frac{p_k}{m_ekT_e}\Phi\, d\mathbf{p}.$$

Using (A.2), we have

$$L^+=\frac{c}{m_ekT_e}\int p_ip_k\sigma n(\mathbf{k})n(\mathbf{k}')\Delta p_i\Delta p_k f\, dk\, d\mathbf{p}\, d\Omega,$$

but according to (A.3) the quantity

$$p_i\Delta p_i=\mathbf{p}\,\Delta\mathbf{p}\approx \mathbf{p}\frac{h\nu}{c}(\mathbf{l}-\mathbf{l}')=\frac{\mathbf{p}}{m_ec}(\mathbf{l}-\mathbf{l}')\nu h m_e\approx h m_e(\nu-\nu')$$

describes the Doppler shift in photon frequency in scattering; therefore

$$L^+=\frac{ch^2}{kT_e}\int \sigma(\nu-\nu')^2 n(\mathbf{k})n(\mathbf{k}')\Phi\, dk'\, d\mathbf{p}\, d\Omega.$$

Since the radiation is assumed isotropic and unpolarized, $dk = 8\pi c^{-3}\nu^2 d\nu$ and

$$L^+=\frac{8\pi h^2}{c^2kT_e}\int \sigma(\nu-\nu')^2\nu'^2 n(\nu)n(\nu')\Phi\, d\nu'\, d\mathbf{p}\, d\Omega. \qquad (A.5)$$

The method of calculating these integrals is well known and is discussed in Chandrasekhar's book;[13] we will follow this method below.

Obviously, not all the differentials $d\nu$, $d\nu'$, dp_x, dp_y, dp_z can be given in arbitrary form; the conservation laws in Compton scattering impose a definite relation on them. If we choose the x, y plane as the scattering plane, in the classical approximation and for $|\mathbf{p}| \ll m_ec$ this relation has the form

$$(1-\cos\alpha)p_x-p_y\sin\alpha=m_ec(\nu/\nu'-1). \qquad (A.6)$$

Here, as previously, α is the photon scattering angle. Excluding integration over $dp_x = (\partial p_x/\partial\nu)d\nu'$, where the derivative $\partial p_x/\partial\nu = m_ec/\nu'(1-\cos\alpha)$ and p_x itself are calculated according to (A.6), and substituting Φ in the form of a Maxwellian function, it is easy to integrate (A.5) over dp_y, dp_z and obtain

$$L^+=\frac{3\pi h^2\sigma_T}{c^2kT_e}\int(\nu-\nu')^2\nu'^2 n(\nu)n(\nu')\frac{1}{(2\pi)^{1/2}\Delta\nu_D(1-\cos\alpha)^{1/2}} \times\exp\left\{-\frac{(\nu-\nu')^2}{2\Delta\nu_D^2(1-\cos\alpha)}\right\}d\nu\, d\nu'(1+\cos^2\alpha)\, d\cos\alpha. \qquad (A.7)$$

In the case of a broad spectrum $\delta \gg \Delta\nu_D$ of isotropic radiation, it is easy to carry out the integration in (A.7) over $d\nu'$, $d\cos\alpha$, and obtain the well known formula for electron heating in the field of low-frequency radiation of high intensity[1,2]

$$L^+=\frac{16\pi h^2\sigma_T}{m_ec^4}\int n^2(\nu)\nu^4 d\nu. \qquad (A.8)$$

In the case of a narrow spectrum $\delta \lesssim \Delta\nu_D$ of isotropic radiation, Eq. (A.8) is inapplicable and Eq. (A.7) must be used in calculations. If $\delta \ll \Delta\nu_D$, then (A.7) is simplified: the exponential can be replaced by unity (see the proof of validity of this substitution above) and the integral can be taken over the scattering angles.

In this case (A.7) reduces to Eq. (8′) which is an indirect demonstration of the validity of the choice of the kernel in the form (7).

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Translated by C. S. Robinson
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