On the Generation of Long Wavelength Phonons by Electromagnetic Waves

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A kinetic theory of the transformation of electromagnetic energy into sound on the surface of a metal in the presence of specular reflection of the electrons from the boundary is constructed. It is shown that the coefficient of the transformation has a sharp maximum associated with the frequency at which the wavelength of the sound coincides either with the mean free path or else with the depth of the skin layer. It is ascertained that, in the presence of specular reflection, all of the electrons with the Fermi energy participate in the transformation process, even under the conditions of the anomalous skin effect.

THE transformation of electromagnetic energy into sound by electrons has been repeatedly observed experimentally[1-7] and has been theoretically explained.[7-10] In the cited articles (both theoretical and experimental) the question is the excitation in the metal of a sound wave of frequency ω by an electromagnetic wave (EMW) which is incident on the surface of the metal--that is, a linear transformation of the waves. The amplitude of the excited sound wave is proportional to the amplitude of the incident EMW. The electromagnetic field in the metal essentially does not depend on the angle of incidence. This makes it possible to consider the transformation for normal incidence of the EMW on the metal ($z > 0$).

If the surface of the metal, $z = 0$, does not coincide with a plane of symmetry of the crystal (it possesses larger crystallographic indices), then three sound waves are excited upon the incidence of an EMW of arbitrary polarization on the surface. In this case the equation describing the excitation and propagation of sound has the form ($i = 1, 2, 3$)

$$\frac{d^2u_i}{dz^2} + \omega^2\rho\lambda_{il}^{-1}u_l = \lambda_{il}^{-1}F_l(z). \tag{1}$$

Here u_i is the vector displacement, ρ is the density of the metal, the tensor $\lambda_{il} \equiv \lambda_{zilz}$ (λ_{iklm} is the modulus of elasticity tensor), and $F_i(z)$ is the density of the force due to the EMW incident on the metal. The principal values of the tensor $\rho\lambda_{il}^{-1}$ coincide with $1/s_j^2$, where s_j is the velocity of sound polarized along the j axis. The boundary condition on Eqs. (1) is the vanishing of the field intensities on a free boundary, where an "entanglement" of the waves is possible on the boundary, viz, the EMW excites a wave of only one polarization, but waves of the other polarizations are excited at the boundary. If the thickness of the sample is much larger than the attenuation length of the sound waves, then only waves traveling from the surface of the metal should exist in the interior of the metal, and the problem of the transformation of electromagnetic energy into sound reduces to a calculation of the amplitudes of the sound waves in the interior of the metal (as $z \to \infty$).

In the present article we shall assume that the z axis coincides with one of the "good" directions of the crystal. The EMW does not give rise to an electric field normal to the surface of the metal, and the electromagnetic energy is transformed into a single sound wave whose velocity is given by $s_t \equiv s$. The problem is reduced thereby to the calculation of $u_\infty^t \equiv u_\infty$.

Major attention will be given to the excitation of hypersound—that is, long wavelength phonons. The utilization of the macroscopic equations of the theory of elasticity naturally limits our investigation to frequencies $\omega \ll \omega_D$ (ω_D is the Debye frequency of the lattice vibrations of the metal, $\omega_D \approx 10^{13}$ sec^{-1}). It should be emphasized that the transformation mechanism studied here is not intrinsically limited to low frequencies. The decrease of the transformation coefficient with increasing frequency (see below) is due to the electronic properties of the metal. The formulas given below enable us to estimate the feasibility of the excitation of a coherent stream of phonons by an EMW.

The excitation of sound is associated due to the forces acting on the ionic lattice of the metal. Therefore it is natural to separate the excitation mechanisms in accordance with the nature of the forces that responsible for generation of the sound. In metals, as is well known,[11-12] besides the direct action of the electric field on the lattice, the forces exerted on the lattice by the electrons, which are driven out of the equilibrium state by the EMW,[1)] also play an essential role. Therefore, in investigating the nature of the forces responsible for the transformation of electromagnetic energy into sound, it is natural to pose the question of the role of the various groups of conduction electrons in the transformation mechanism.

Collisions of the electrons with the surface of the metal, accompanied by transfer of the momentum acquired from the electric field of the EMW, create a δ-shaped surface force; however, the surface force vanishes if the electrons are specularly reflected from the boundary (in this article we shall consider only this case).[2)] The volume force in the right hand side of (1) the equation of dynamical theory of elasticity (Eq. (1)) can be expressed in the form of two terms:

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2. \tag{2}$$

The first force is given by

[1)]The interaction of the equilibrium electrons with the lattice enters into the "renormalized" moduli of elasticity.[12]

[2)]The reflection of electrons from a metal-vacuum interface depends essentially on the structure of the interface (see, for example, the review article by Andreev[15]). If the interface coincides with a crystallographic symmetry plane and has the period of the bulk crystal, then the projection of the momentum on the surface should be conserved during reflection (in the absence of Umklapp processes), and the reflection will be specular. Besides its simplicity, the choice of specular reflection is dictated by the desire to clarify the role of volume forces in the transformation.

$$\mathbf{F}_1 = \frac{m_0}{e}\frac{\partial \mathbf{j}}{\partial t}, \tag{3}$$

where m_0 and e denote the mass and charge of a free electron, and $\mathbf{j}$ is the current density excited in the metal by the EMW. If the metal is located in a constant magnetic field $\mathbf{H}_0$, then it necessary to add to this term the pondermotive force $c^{-1}\mathbf{H}_0 \times \mathbf{j}$ due to the electromagnetic pressure, and

$$F_{2\alpha} = -\frac{2}{(2\pi\hbar)^3}\int \Lambda_{\alpha z}\frac{\partial f_1(z)}{\partial z}d^3p. \tag{4}$$

Here and in what follows f_1 is the nonequilibrium correction to the Fermi distribution function f_0, and $\Lambda_{ik} = \Lambda_{ik}(\mathbf{p})$ is a tensor of the second rank whose components depend on the quasimomentum $\mathbf{p}$ of the electrons. Calculation of the components of the tensor $\Lambda_{ik}(\mathbf{p})$ is possible, of course, only by using a specific model for the metal. This problem is outside the scope of the present article.

We note that the tensor $\Lambda_{ik}(\mathbf{p})$ formally combines two physically-different mechanisms producing forces exerted on the lattice by the electrons: the interaction of the electrons with the deformed lattice[11] and the transport of quasimomentum by the electrons.[12] In the free electron approximation, in the absence of a magnetic field and in a constant electric field, the transport of momentum to the surface of the metal turns out to be the only mechanism producing the forces and the effects associated with them.[13] It is convenient to use the tensor $\Lambda_{ik}(\mathbf{p})$ in those cases when the necessity to distinguish between both mechanisms does not arise. Since f_1 contains the factor $\partial f_0/\partial\epsilon$ (see[14]), it is essential to know the value of $\Lambda_{ik}(\mathbf{p})$ on the Fermi surface. From general considerations it is clear that for Fermi electrons $|\Lambda_{ik}| \sim \epsilon_F$ (ϵ_F denotes the Fermi energy). Unfortunately, this assertion is not enough to estimate the transformation coefficient, since its value depends on the anisotropy of the tensor $\Lambda_{ik}(\mathbf{p})$ (see below). We emphasize that the nature of the forces of interaction between the electrons and the lattice becomes manifest only if the dependence $\Lambda_{ik} = \Lambda_{ik}(\mathbf{p})$ is concretely specified. If the tensor Λ_{ik} is assumed to be known, then in limiting cases one is able to solve the problem in general form, without making any kind of special assumptions (more precisely, one can express the limiting values of the transformation coefficient in terms of integrals of the components of the tensor $\Lambda_{ik}(\mathbf{p})$ over the Fermi surface, see below).

The current density $\mathbf{j}(z)$ and the electron distribution function $f_1(z)$ in expression (2) for the force (see also (3) and (4)) should be found from the kinetic theory of the skin effect ([14], part IV). Assuming that the electrons are specularly reflected from the boundary (later on we shall make a few remarks about the role of diffuse scattering in the transformation), and by using the kinetic equation with the collision integral in the τ-approximation ($1/\tau \equiv \nu$ denotes the average frequency of the collisions), one can easily obtain ($\mathrm{v} = \overline{\nu}\epsilon$; ϵ denotes the energy of an electron with quasimomentum $\mathbf{p}$)

$$f_1(z) = -\frac{2\omega eH(0)}{i\pi c}v_x\frac{df_F}{d\varepsilon}\int_0^\infty \frac{(\nu - i\omega)\cos kz + kv_z\sin kz}{[k^2 - 4\pi i\sigma(\omega,k)\omega/c^2][(kv_z)^2 + (\nu - i\omega)^2]}dk, \tag{5}$$

$$j_x(z) = \frac{2\omega H(0)}{i\pi c}\int_0^\infty \frac{\sigma(\omega,k)\cos kz\,dk}{k^2 - 4\pi i\sigma(\omega,k)\omega/c^2}, \tag{6}$$

where $H(0)$ denotes the value of the magnetic field of the EMW on the boundary of the metal;

$$\sigma(\omega,k) \equiv \sigma_{xx}(\omega,k) = \frac{2e^2(\nu - i\omega)}{(2\pi\hbar)^3}\oint_{\varepsilon=\varepsilon_F}\frac{dS}{v}\frac{v_x^2}{(kv_z)^2 + (\nu - i\omega)^2}. \tag{7}$$

The x axis is chosen along the direction of the electric field in the EMW, where, as we have already mentioned, it is assumed that the electric field is directed along one of the principal directions of the crystal.[3)] The integration in the last formula is carried out over the Fermi surface, where dS denotes the element of area on this surface. Using formulas (2)—(7), let us determine the nonvanishing component of the force density $F_x = F$, $F_y = F_z = 0$:

$$F = \frac{\omega eH(0)}{\pi c}\int_{-\infty}^{+\infty}\frac{e^{ikz}\,dk}{k^2 - 4\pi i\sigma(\omega,k)\omega/c^2} \times \frac{2}{(2\pi\hbar)^3}\oint_{\varepsilon=\varepsilon_F}\frac{dS}{v}\frac{m_0\omega v_x^2(\nu - i\omega) - \Lambda_{xz}kv_x(\nu - i\omega - ikv_z)}{(kv_z)^2 + (\nu - i\omega)^2}. \tag{8}$$

The solution of Eq. (1), subject to the additional boundary condition

$$\partial u/\partial z = 0 \quad \text{for} \quad z = 0, \tag{9}$$

has the following form:

$$u(z) = \lim_{\gamma\to 0}\int_{-\infty}^{\infty}\Phi(k)\frac{e^{ikz} - e^{i\omega z/s}ks/\omega}{k^2 - \omega^2/s^2 - i\gamma}dk \tag{10}$$

$$\Phi(k) = -\frac{\omega eH(0)}{\pi c\rho s^2}\left(k^2 - \frac{4\pi i\sigma(\omega,k)\omega}{c^2}\right)^{-1} \times \frac{2}{(2\pi\hbar)^3}\oint_{\varepsilon=\varepsilon_F}\frac{dS}{v}\frac{m_0\omega v_x^2(\nu - i\omega) - \Lambda_{xz}kv_x(\nu - i\omega - ikv_z)}{(kv_z)^2 + (\nu - i\omega)^2}. \tag{11}$$

We are interested in the asymptotic value of the function $u(z)$ for large values of the coordinate z. One can show that

$$\lim_{z\to\infty} u(z) = u_\infty e^{i\omega z/s}; \tag{12}$$

$$u_\infty = \frac{i\pi s}{\omega}\Phi_s\left(\frac{\omega}{s}\right) - \frac{2s}{\omega}\int_0^\infty \frac{k\Phi_a(k) - \omega s^{-1}\Phi_a(\omega/s)}{k^2 - \omega^2/s^2}dk, \tag{13}$$

$$2\Phi_s = \Phi(k) + \Phi(-k), \qquad 2\Phi_a = \Phi(k) - \Phi(-k). \tag{14}$$

According to Eq. (11)

$$\Phi_s(k) = -\frac{\omega eH(0)}{\pi c\rho s^2}\frac{1}{k^2 - 4\pi i\sigma(\omega,k)\omega/c^2} \times \frac{2}{(2\pi\hbar)^3}\oint_{\varepsilon=\varepsilon_F}\frac{dS}{v}\frac{m_0\omega v_x^2(\nu - i\omega) + i\Lambda_{xz}v_xv_zk^2}{(kv_z)^2 + (\nu - i\omega)^2}, \tag{15}$$

$$\Phi_a(k) = \frac{\omega eH(0)}{\pi c\rho s^2}\frac{k}{k^2 - 4\pi i\sigma(\omega,k)\omega/c^2}\frac{2}{(2\pi\hbar)^3}\oint_{\varepsilon=\varepsilon_F}\frac{dS}{v}\frac{\Lambda_{xz}v_x(\nu - i\omega)}{(kv_z)^2 + (\nu - i\omega)^2}. \tag{16}$$

By virtue of the boundary (formally because of the second integral term in formula (13)), the nonresonant transformation of an EMW into a sound is possible without conservation of the wave vector ($k = \omega/s$). However, comparison of formulas (16) and (15) shows that always $\Phi_S(k) \neq 0$ not only on account of the term containing the mass of the free electron, but also because the tensor Λ_{xz} necessarily contains a part of the parity[4)] of v_xv_z. Therefore the integral of this

[3)]The geometry is such that $j_y = j_z = 0$, and the axes x, y, and z coincide with the principal directions of the tensors λ_{ik}, σ_{ik}, and of the impedance ζ_{ik}.

term over the Fermi surface certainly does not vanish. In the symmetric case under consideration, $\Phi_a(k)$ then evidently vanishes for any arbitrary Fermi surface. The feasibility and consequences of the nonresonant nature of the bulk transformation will be analyzed in a special article (see the remarks at the end of this communication).

Thus, here we shall investigate only the resonant part of the amplitude of the excited sound wave (the first term in formula (13)):

$$u_\infty \equiv u_\infty^{\text{res}} = -\frac{ieH(0)}{\rho cs}\frac{J(\omega,k)}{k^2-4\pi i\sigma(\omega,k)\omega/c^2}, \quad k=\frac{\omega}{s}, \tag{17}$$

$$J(\omega,k) = \frac{2}{(2\pi\hbar)^3}\oint_{\varepsilon=\varepsilon_F}\frac{dS}{v}\frac{m_0\omega(\nu-i\omega)v_x^2+ik^2\Lambda_{xz}v_xv_z}{(kv_z)^2+(\nu-i\omega)^2}. \tag{18}$$

From formulas (17) and (18) it is evident that there are a large number of characteristic frequencies in the problem, and consequently the dependence of u_∞ on the frequency is extremely complicated. In addition to the frequencies characterizing the electron gas proper, namely, ν, $s\nu/v_F$, $s^2\nu/v_F^2$ (v_F denotes the Fermi velocity of the electrons, $s \ll v_F$), and the plasma frequency[5] $\omega_0 = (4\pi ne^2/m)^{1/2}$, there is also the characteristic "electrodynamic" frequency ω_{em} at which the wavelength s/ω of the sound coincides with the skin depth $\delta = 1/|k_{em}|$ (k_{em} is the root of the equation $k^2 = 4\pi i\sigma(\omega, k)\omega/c^2$). If $\omega \ll \omega_{em}$, then it is necessary to keep in the denominator of formula (17) the term containing the conductivity, and in the opposite case one keeps the k^2 term. In calculating the electrical conductivity (according to formula (8)) it is, of course, necessary to take into account the relation between the frequency ω of the EMW and the characteristic frequencies of an electron gas (ν, $s\nu/v_F$, and so forth). Depending on the magnitude of the mean free path l, the electrodynamic frequency ω_{em} falls in one frequency interval or another:

$$\begin{aligned} &\omega_{эм} \ll s^2\nu/v_F^2 \quad \text{for} \quad l \ll \delta_0 = c/\omega_0,\\ &s^2\nu/v_F^2 \ll \omega_{эм} \ll s\nu/v_F \quad \text{for} \quad \delta_0 \ll l \ll \delta_0(v_F/s)^{1/2},\\ &s\nu/v_F \ll \omega_{эм} \ll \nu \quad \text{for} \quad \delta_0(v_F/s)^{1/2} \ll l. \end{aligned} \tag{19}$$

Now let us go on to the evaluation of the integral (18). At low frequencies we can neglect the term $(kv_z)^2$ in the denominator and the frequency ω in comparison with ν,

$$J(\omega,k) \approx \frac{2}{(2\pi\hbar)^3}\oint_{\varepsilon=\varepsilon_F}\tau^2\frac{dS}{v}\left(m_0\omega\nu v_x^2+\frac{\omega^2}{s^2}\Lambda_{xz}v_xv_z\right). \tag{20}$$

At extremely low frequencies $\omega \ll \nu s^2/v_F^2$

$$J\left(\omega,\frac{\omega}{s}\right) \approx \frac{2}{(2\pi\hbar)^3}\oint_{\varepsilon=\varepsilon_F}\tau\frac{dS}{v}v_x^2m_0\omega = \frac{m_0}{e^2}\sigma_0\omega, \tag{21}$$

where σ_0 is the static electrical conductivity. At higher frequencies ($s^2\nu/v_F^2 \ll \omega \ll s\nu/v_F$)

$$J \approx \frac{2i}{(2\pi\hbar)^3}\oint_{\varepsilon=\varepsilon_F}\tau^2\frac{dS}{v}\Lambda_{xz}v_xv_z\frac{\omega^2}{s^2}. \tag{22}$$

[4]We refer to that part of the tensor Λ_{xz} which is connected with the transport of quasimomentum. For this mechanism $\Lambda_{xz} = p_xv_z$.[13]

[5]For all estimates we assume a quadratic dispersion law, an effective mass $m \approx m_0$, and a radius of the Fermi sphere $p_F = mv_F \approx \hbar n^{1/3}$, where n is the density of the electrons.

In order to estimate J one can utilize the expressions

$$J \approx n\omega/\nu \quad \text{for} \quad \omega \ll \nu s^2/v_F^2, \tag{21'}$$

$$J \approx n(\omega v_F/\nu s)^2 \quad \text{for} \quad \nu s^2/v_F^2 \ll \omega \ll \nu s/v_F. \tag{22'}$$

In these estimates we have taken into consideration the fact that $|\Lambda_{xz}| \sim \epsilon_F$ on the Fermi surface.

In the range of frequencies $\omega \ll \nu s/v_F$ under consideration, the electrical conductivity $\sigma(\omega, k)$ coincides with its static value

$$\sigma\left(\omega,\frac{\omega}{s}\right) \approx \sigma_0 \approx \omega_0^2/4\pi\nu, \quad \omega \ll s\nu/v_F. \tag{23}$$

The last expression is convenient for estimates. Electrons with small values of v_z begin to play a specific role at comparatively high frequencies $\nu s/v_F \ll \omega \ll \nu$:

$$\sigma\left(\omega,\frac{\omega}{s}\right) \approx \frac{2\pi se^2}{(2\pi\hbar)^3\omega}\int_0^{2\pi}p_F^2\left(\frac{\pi}{2},\varphi\right)\cos^2\varphi\,d\varphi, \tag{24}$$

where $p_F^2(\theta, \varphi)$ denotes the Gaussian curvature of the Fermi surface. The integration is carried out over the "strip" on which $v_z = 0$ (see[14], Sec. 33).

Expression (24) takes into consideration only the spatial dispersion of the electrical conductivity: in this case $\sigma(\omega/s) \approx (0, \omega/s)$. Frequency dispersion appears at higher frequencies $\omega \gg \nu$, when

$$\sigma\left(\omega,\frac{\omega}{s}\right) \approx \frac{2\pi e^2 s}{(2\pi\hbar)^3\omega}\int_0^{2\pi}p_F^2(\varphi)\cos^2\varphi\,d\varphi, \tag{25}$$

where $p_F^2(\varphi) \equiv p_F^2[\theta(\varphi), \varphi]$. The "strip" over which the integration is carried out ($\theta(\varphi) = \theta$) is determined by the condition

$$s/v_F(\theta,\varphi) = \cos\theta. \tag{26}$$

since $s/v_F \ll 1$, expressions (24) and (25) differ insignificantly from each other. For estimates one can utilize one and the same value

$$\sigma\left(\omega,\frac{\omega}{s}\right) \approx \frac{\omega_0^2 s}{\omega v_F}, \quad \omega \gg \frac{s}{v_F}\nu. \tag{24'}$$

Now let us return to an investigation of the integral $J(\omega, k)$. It is convenient to rewrite formula (18) by separating the specific electrical conductivity:

$$J\left(\omega,\frac{\omega}{s}\right) = \frac{m_0\omega}{e^2}\sigma\left(\omega,\frac{\omega}{s}\right) + J_1\left(\omega,\frac{\omega}{s}\right), \tag{27}$$

$$J_1\left(\omega,\frac{\omega}{s}\right) = i\frac{\omega^2}{s^2}\frac{2}{(2\pi\hbar)^3}\oint_{\varepsilon=\varepsilon_F}\frac{dS}{v}\frac{\Lambda_{xz}v_xv_z}{(kv_z)^2+(\nu-i\omega)^2}. \tag{27'}$$

Since the numerator vanishes at $v_z = 0$, the anomalous role of the grazing electrons (with $v_z = 0$) is significantly underestimated. If it is assumed that

$$\Lambda_{xz} \approx \tilde{m}v_xv_z, \tag{28}$$

then for $\nu s/v_F \ll \omega \ll \nu$

$$J_1 \approx \frac{2i}{(2\pi\hbar)^3}\oint_{\varepsilon=\varepsilon_F}\tilde{m}v_x^2\frac{dS}{v}. \tag{29}$$

Comparison of J_1 with the term containing the electrical conductivity indicates that J_1 is v_F/s times larger. Thus, formula (29) gives an approximate value of the entire integral J. For estimates one can assume that $\tilde{m} = m$. Then

$$J \approx n \quad \text{for} \quad \nu s/v_F \ll \omega \ll \nu. \tag{29'}$$

Finally, let us consider $J(\omega, \omega/s)$ for extremely high frequencies (for $\omega \gg \nu$) assuming, of course, that $\nu \ll \omega \ll \omega_D$. As a rule the condition $\nu \ll \omega_D$ is satisfied at low temperatures in comparatively pure metal samples. The collision frequency $\nu = \omega_D$ corresponds to an electron mean free path l equal to $v_F a/s \approx 10^{-5}$ cm. It is convenient to separate the nonresonant term in J:

$$J\left(\omega,\frac{\omega}{s}\right) \approx \frac{2i}{(2\pi\hbar)^3}\oint\limits_{\varepsilon=\varepsilon_F}\frac{dS}{v}v_x^2\tilde{m} + \frac{2i}{(2\pi\hbar)^3}\oint\limits_{\varepsilon=\varepsilon_F}\frac{dS}{v}v_x^2\frac{\tilde{m}-m_0}{(v_z/s)^2-1-2i\nu/\omega}. \tag{30}$$

Now one can easily show that thanks to the inequality $v_F \gg s$ the contribution of the nonresonant term is considerably larger than that of the resonant term (the second term). Therefore formulas (29) and (29′) are also valid for $\omega \gg \nu$. Thus we see that the quantity $J(\omega, \omega/s)$ is determined by all of the electrons on the Fermi surface even under the conditions of the extremely anomalous skin effect, when the electrodynamic characteristics (conductivity, impedance) are determined only by the electrons in the "strip" ($v_z = 0$).

The found values of $J(\omega, \omega/s)$ and of the electrical conductivity $\sigma(\omega, \omega/s)$ permits us to calculate u_∞ for any arbitrary ratio of the parameters. We restrict our attention to presenting order of magnitude values (complex factors of the order of unity are omitted) of u_∞, and also of the transformation coefficient γ.

The transformation coefficient is defined as the ratio of the current density Q_{s0} of the sound energy to the current density Q_{em} of the electromagnetic energy incident on the metal:[6)]

$$\gamma = \rho s|\dot{u}_\infty|^2/2Q_{em} \tag{31}$$

If $l \ll \delta_0 = c/\omega_0$, then $\omega_{em} = s^2\omega_0^2/c^2\nu$ is the smallest of the characteristic frequencies of the problem, and

$$\frac{u_\infty}{u_\infty^0} \approx \begin{cases} 1 & \omega \ll \omega_{em}; \\ \omega_{em}/\omega, & \omega_{em} \ll \omega \ll s^2\nu/v_F^2 \\ \omega_{em}v_F^2/s^2\nu, & s^2\nu/v_F^2 \ll \omega \ll s\nu/v_F; \\ \omega_{em}\nu/\omega^2, & s\nu/v_F \ll \omega \ll \omega_D. \end{cases} \tag{32}$$

Under these same conditions

$$\frac{\gamma}{\gamma_0} \approx \begin{cases} \omega^2/\omega_D^2, & \omega \ll \omega_{em}; \\ (\omega_{em}/\omega_D)^2, & \omega_{em} \ll \omega \ll s^2\nu/v_F^2; \\ (\omega_{em}\omega v_F^2/s^2\nu\omega_D)^2, & s^2\nu/v_F^2 \ll \omega \ll s\nu/v_F; \\ (\omega_{em}\nu/\omega_D\omega)^2, & s\nu/v_F \ll \omega. \end{cases} \tag{33}$$

Here and in what follows

$$u_\infty^0 = \frac{mc}{\rho se}H(0), \quad \gamma_0 = \frac{m}{M}\frac{mv_F^2}{e^2/a}\frac{sc}{v_F^2},$$

where H(0) denotes the value of the magnetic field of the EMW on the boundary of the sample, $M = \rho a^3$, where a denotes the size of the crystal cell. In the present case ($l \ll \delta_0$) the transformation coefficient reaches its maximum value for $\omega = s\nu/v_F$, that is,

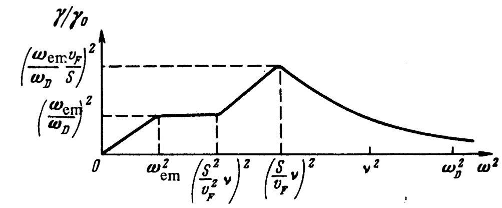

FIG. 1. Frequency dependence of the transformation coefficient for $l \ll \delta_0$.

when the mean free path l and the wavelength $\lambda_{s0} = s/\omega$ of the sound are equal (see Fig. 1). The maximum value of the transformation coefficient is given by

$$\gamma_{max} \approx \frac{m}{M}\frac{mv_F^2}{e^2/a}\frac{sc}{v_F^2}\frac{l^2a^2}{\delta_0^4} \approx \frac{m}{M}\frac{l^2a^2}{\delta_0^4}, \tag{34}$$

if it is assumed that $mv_F^2 \approx e^2/a$ and $sc \approx v_F^2$.

If $\delta_0 \ll l \ll \delta_0(v_F/s)^{1/2}$, then ω_{em} is determined by the previous formula, but under these conditions $s^2\nu/v_F^2 \ll \omega_{em} \ll s\nu/v_F$ and

$$\frac{u_\infty}{u_\infty^0} \approx \begin{cases} 1 & \omega \ll s^2\nu/v_F^2; \\ \omega v_F^2/s^2\nu, & s^2\nu/v_F^2 \ll \omega \ll \omega_{em}; \\ \omega_{em}v_F^2/\nu s^2, & \omega_{em} \ll \omega \ll s\nu/v_F; \\ \omega_{em}\nu/\omega^2, & s\nu/v_F \ll \omega; \end{cases} \tag{35}$$

$$\frac{\gamma}{\gamma_0} \approx \begin{cases} (\omega/\omega_D)^2, & \omega \ll s^2\nu/v_F^2; \\ \omega^4v_F^4/\nu^2s^4\omega_D^2, & s^2\nu/v_F^2 \ll \omega \ll \omega_{em}; \\ \omega_{em}^2\omega^2v_F^4/\nu^2s^4\omega_D^2, & \omega_{\text{эм}} \ll \omega \ll s\nu/v_F; \\ \omega_{em}^2\nu^2/\omega_D^2\omega^2, & s\nu/v_F \ll \omega. \end{cases} \tag{36}$$

The maximum value of the transformation coefficient occurs, just as in the previous case, when $\omega = s\nu/v_F$ (see Fig. 2) and is determined by the same formula (34).

If $\delta_0(v_F/s)^{1/2} \ll l \ll \delta_0(v_F/s)^{3/2}$, then

$$\omega_{em} = \frac{s}{c}\left(\frac{s}{v_F}\right)^{1/2}\omega_0, \quad \frac{s}{v_F}\nu \ll \omega_{em} \ll \nu,$$

and the amplitude and the transformation coefficient are given by the formulas

$$\frac{u_\infty}{u_\infty^0} \approx \begin{cases} 1 & \omega \ll s^2\nu/v_F^2, \\ \omega v_F^2/\nu s^2, & s^2\nu/v_F^2 \ll \omega \ll s\nu/v_F, \\ v_F/s, & s\nu/v_F \ll \omega \ll \omega_{em}, \\ v_F\omega_{em}^2/s\omega^2, & \omega_{em} \ll \omega; \end{cases} \tag{37}$$

$$\frac{\gamma}{\gamma_0} \approx \begin{cases} \omega^2/\omega_D^2, & \omega \ll s^2\nu/v_F^2, \\ \omega^4v_F^4/\omega_D^2\nu^2s^4, & s^2\nu/v_F^2 \ll \omega \ll s\nu/v_F, \\ \omega^2v_F^2/\omega_D^2s^2, & s\nu/v_F \ll \omega \ll \omega_{em}, \\ \omega_{em}^4v_F^2/\omega_D^2\omega^2s^2, & \omega_{em} \ll \omega. \end{cases} \tag{38}$$

The maximum value of γ occurs for $\omega \approx \omega_{em}$. It is given by

$$\gamma_{max} \approx \frac{m}{M}\frac{mv_F^2}{e^2/a}\frac{a^2\omega_0^2}{v_Fc}.$$

But since $mv_F^2 \approx e^2/a$ and $a^2\omega_0^2 \approx v_F^2$ (we note that

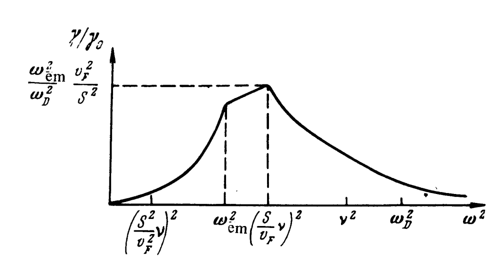

FIG. 2. Frequency dependence of the transformation coefficient for $\delta_0 \ll l \ll \delta_0\,(v_F/s)^{1/2}$.

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6)One can characterize the transformation by the ratio γ' of the acoustic energy flux density Q_{so} to the flux density $Q_{em}\mathrm{Re}\zeta$ of the energy entering into the crystal, where ζ denotes the impedance ($\mathrm{Re}\zeta \ll 1$ for metals):

$$\gamma' = \gamma/\mathrm{Re}\,\zeta.$$

When γ' is close to unity, a more rigorous investigation of the transformation process is required than is given in this article, namely, a self-consistent solution of the equations of electrodynamics and elasticity. The condition for the method utilized here is $\gamma \ll \mathrm{Re}\zeta$. It is essentially satisfied at all frequencies (see, however, below).

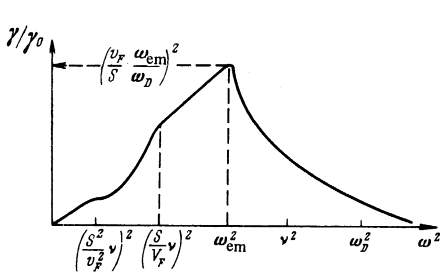

FIG. 3. Frequency dependence of the transformation coefficient for $\delta_0(v_F/s)^{1/2} \ll l \ll \delta_0(v_F/s)^{3/2}$.

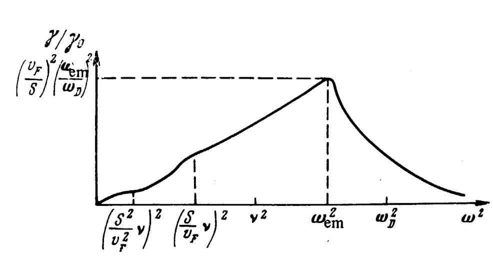

FIG. 4. Frequency dependence of the transformation coefficient for $\delta_0(v_F/s)^{3/2} \ll l$.

$e^2 \approx \hbar v_F$), it follows that $\gamma_{max} \approx mv_F/Mc$ (see Fig. 3).

Finally, if $l \gg (v_F/s)^{3/2}\delta_0$, then the difference from the previous case only consists in the fact that $\omega_{em} \gg \nu$ (see Fig. 4).

Formulas (32)—(38) show that the transformation coefficient significantly depends on the frequency, reaching its maximum either for $\omega \approx s\nu/v_F$ (for $l < \delta_0(v_F/s)^{1/2}$) or for $\omega \approx \omega_{em}$ (for $l > \delta_0(v_F/s)^{1/2}$). γ_{max} increases with increasing mean free path. Just as in all acoustic phenomena in metals (see[14], Appendix II), the limiting frequency is not the collision frequency ν, but the considerably smaller frequency $s\nu/v_F$—a manifestation of spatial dispersion.

The linear nature of the problem permits us to consider the different transformation mechanisms separately, for example, to calculate independently the amplitude of the sound wave excited by the nonresonant transformation mechanism (see formula (13)). This remark does not pertain to allowance for the diffuse scattering of electrons on the metal's boundary, since the form of the scattering essentially determines the nonequilibrium correction to the electron distribution function and by the same token has an influence on all of the transformation mechanisms. Although a detailed analysis of the role of diffusivity will be the subject of a separate communication, we wish to make several remarks:

1. The appearance of a surface force—as a consequence of the diffuse scattering of electrons—significantly increases the transformation coefficient for large frequencies, since here the amplitude of the force acting on the ion lattice of the metal does not tend to zero with increasing values of $k = \omega/s$.

2. Even for partially diffuse scattering of the electrons, the electron distribution function includes a proper solution of the homogeneous kinetic equation (f_1 contains a term that depends on the z coordinate like $\exp[\nu - i\omega)z/v_z]$ for $v_z > 0$). This fact distinguishes the electrons moving away from the boundary, and leads to the possibility of resonant[7)] excitation of sound by the electrons for which $v_z = s$ (for $\omega l/s \gg 1$).

Let us return once more to the nonresonant transformation of sound (the second term in formula (13)). The estimate indicates that at large frequencies allowance for the resonant excitation does not substantially change the amplitude of the sound (even if it is assumed that $\Phi_a \approx \Phi_s$) but allowance for the nonresonant excitation substantially increases the amplitude of the sound: u_∞ tends to infinity[8)] as $\omega \to 0$. This indicates that in this case either a self-consistent (exact) solution of the equations of electrodynamics and elasticity is required, but not the solution in stages (approximate solution) which we have used, or else we need to take the finite thickness of the plate into consideration (since $\delta \to \infty$ as $\omega \to 0$).

One can observe the transformation of electromagnetic energy into sound not only by measuring directly the amplitude of the sound wave, but also from the surface impedance of the metal (more precisely, from the deviation of the latter from the purely electrodynamical value). For a rigorous calculation of the impedance (its real and imaginary parts) one needs, of course, a self-consistent solution; however, if the correction $\Delta\zeta$ to the transformation is small, then the change in the real part of the impedance coincides with the transformation coefficient ($\Delta\zeta \approx \gamma$). The utilization of metal plates of finite thickness d may lead to a number of interesting effects that accompany the transformation, for example: a resonant (for $d = (1/2)\lambda N$, $\lambda = 2\pi s/\omega$, N is an integer) intensification of the role of the transformation; nonexponential transparency of the metallic plate to the EMW, etc. A detailed comparison of the theory with experiment is impossible since there are no experimental data for a single sample over a wide range of frequencies. The temperature dependence of the transformation coefficient is very sensitive to the frequency of the EMW.

In conclusion we express our gratitude to G. Ivanovskiĭ and I. Lifshitz for helpful discussions.

7)We refer here to velocity resonance between the electrons and the sound wave, whereas the term "resonance" was used above (and will be used below) for the interaction of the electromagnetic wave with the sound, denoting equality of the wavelengths.

8)$u_\infty \sim \omega^{-1/2}$ and the transformation coefficient γ depends here on the frequency linearly.

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