Parametric Magnetization Instability of the Electron Liquid in Metals

G. B. Teĭtel'baum
Kazan' Physico-technical Institute, USSR Academy of Sciences
Submitted October 21, 1971
Zh. Eksp. Teor. Fiz. 62, 1513–1520 (April, 1972)

Radio-frequency magnetization of metals is investigated by taking into account the nonlinearity due to interaction between Fermi-liquid spin waves. It is found that the magnetization is unstable during excitation by an alternating field whose amplitude exceeds a certain threshold value. The magnitude of the threshold field strongly depends on the spin correlation parameters of the electron-liquid quasi-particles.

1. At present there are several known weakly-damped electromagnetic excitations in nonferromagnetic metals situated in a constant magnetic field[1-4], the study of which yields valuable information on the electron structure of conductors. These include magnetization excitations due to spin magnetism of the conduction electrons[4-7]. Owing to the relative smallness of such effects, they are investigated at high-power levels of the exciting radio-frequency (RF) field, and this can lead to a number of important singularities in the investigated phenomena. It has been established, for example, that the dependence of the amplitude of the resonant magnetization on the exciting field, under conditions when the latter causes saturation of the electron-spin system, is significantly altered in comparison with the one that takes place in the absence of saturation[5]. Azbel' et al.[5], however, did not consider Fermi-liquid effects, i.e., they neglected the interaction between the conduction electrons. Yet it is known that the spin correlation of the quasiparticles of an electronic Fermi liquid makes the propagation of unique magnetization spin waves possible[6,8]. In turn, at large excitation levels, the interaction between the spin waves can become significant. Such nonlinear phenomena have not yet been considered in metals, whereas their investigation in ferromagnetic dielectrics, initiated by Suhl[9], have been greatly expanded.

It is clear that if we disregard the very fact of propagation of spin waves, the physical situation, in nonferromagnetic metals differs significantly at least outwardly, from that considered by Suhl. Indeed, the Fermi-liquid spin waves are collective excitations of moving charged quasiparticles, and the exciting RF field in such a medium is highly inhomogeneous. Finally, we note that the electromagnetic field in the interior of the conductor (in the vicinity of the spin resonance) is produced entirely precisely by the spin waves, as was observed experimentally[8]. It is therefore of interest to trace the behavior of the magnetization of a Fermi-liquid metal at a sufficiently high value of the exciting RF field, when nonlinear effects become significant.

We consider in this paper non-equilibrium magnetization $\mathbf{M}_1$ of a flat metallic plate of thickness L, unbounded in the two other dimensions, in which a constant magnetic field produces a magnetic induction $\mathbf{B}_0$. We introduce the coordinate system xyz with z axis parallel to $\mathbf{B}_0$, from the origin of which we draw a semi-axis ζ directed into the interior of the metal normal to its surface. The inclination of the constant magnetic field to the surface of the metal is characterized by the angle ϑ between the axes ζ and z. Assume that on the surface $\zeta = 0$ there is an exciting field of radio frequency ω with components $\tilde{B}_x \sim \cos\omega t$ and $\tilde{B}_y \sim \sin\omega t$.

2. The magnetization $\mathbf{M}_1$ can be obtained by simultaneously solving the equations of motion and the system of Maxwell's equations, in which we omit the displacement currents. In the solution we neglect the surface spin relaxation of the quasiparticles of the electronic Fermi liquid, obtaining a boundary condition $\partial \mathbf{M}_1/\partial\zeta|_{\text{sur}} = 0$, which indicates that there is no flux of magnetization through the surface. From symmetry considerations it is clear that $\mathbf{M}_1$ inside the sample depends only on ζ and, in accordance with the boundary condition, it can be continued in even fashion into the region $-L \le \zeta < 0$. We can therefore write the following Fourier expansion for the magnetization (an analogous expansion is valid for the non-equilibrium magnetic induction)

$$\mathbf{M}_1(\zeta) = \sum_q \mathbf{m}_q e^{iq\zeta}, \tag{1}$$

$$\mathbf{m}_q = \frac{1}{L}\int_0^L d\zeta\, \mathbf{M}_1(\zeta)\cos q\zeta, \tag{1a}$$

the summation in (1) is over the discrete values of the wave number $q = \pi n/L$, $n = 0, \pm 1, \pm 2, \ldots$

The spatial Fourier transform of the non-equilibrium magnetization $m_q^{\pm} = m_q^x \pm i m_q^y$ satisfies the equation derived by Silin in the long-wave ($qv_f/\omega_c < 1$) approximation[10] 1), where we took into account also the nonlinear terms (we neglect the nonlinearities in the expressions proportional to the weak spatial inhomogeneity):

$$\dot{m}_q^{\pm} \pm i\left(\omega_s \mp iq^2 D_q \mp \frac{1+\beta_0}{T_1}\right)(m_q^{\pm} - \chi b_q^{\pm}) - \gamma \sum_{q'} [\mathbf{m}_{q-q'}\mathbf{b}_{q'}] = 0. \tag{2}$$

Here b_q is the Fourier transform of the non-equilibrium magnetic induction, χ is the static magnetic susceptibility of the Fermi liquid, $\gamma = ge\hbar/2mc$ (g is the spectroscopic factor), the diffusion coefficient near the spin-resonance frequency ω_d is equal to

$$D_q = \frac{v_f^2}{3\omega_c}\frac{(1+\beta_0)}{(1-A)}\Big[i(1 - A\cos^2\vartheta_q) + \frac{A^{1/2}}{\tau\omega_c}\frac{(1-A)^2\cos^2\vartheta_q + (1+A)\sin^2\vartheta_q}{(1+\beta_1)(1-A)}\Big] \tag{2a}$$

$$A = \omega_c^2(1+\beta_0)^2(1+\beta_1)^2/\omega_s^2(\beta_0-\beta_1)^2,$$

τ and T_1 are the relaxation times of the momentum and spin of the quasiparticles of the Fermi liquid, β_0

1)The sign of ω_s in our paper is the opposite of the sign in[10].

and β_1 are the quasiparticle interaction parameters, ω_c is the cyclotron frequency, and ϑ_q is the angle between B_0 and the ζ axis. We assume that the spin-wave propagation conditions are satisfied, i.e.,

$$|(\beta_0-\beta_1)\tau\omega_s/(1+\beta_0)|\gg 1. \tag{2b}$$

The magnetic field, which enters in the expression for the induction b_q in (2), is produced both by the exciting RF field and by the magnetization spin waves. Since the exciting field is localized near the surface of the metal in a thin layer with thickness much smaller than the length of the spin waves, we have for the Fourier components of the magnetic induction produced by the field

$$\tilde{\mathbf{b}}_q=\tilde{\mathbf{b}}_0=\frac{\delta}{L}\tilde{\mathbf{B}}(0),\qquad \delta=\frac{\tilde{\mathbf{B}}(0)}{\tilde{B}^2(0)}\int_0^L d\zeta\,\mathbf{B}(\zeta), \tag{3}$$

where $\mathbf{B}(0)$ is the RF field on the surface of the metal and δ is the effective depth of its penetration[3]. As to the magnetic induction produced by the magnetization spin waves, it can be easily obtained by assuming that the exciting RF field does not penetrate at all into the metal, i.e., $L\gg|\delta|$. For example, it follows from Maxwell's equations that its zeroth spatial Fourier component is equal to zero. For the Fourier transforms at $q\neq 0$ we can use Maxwell's equations in the magnetostatic approximation[10], since the spin velocity of the spin waves is low in comparison with the velocity of light, and we can express the induction in terms of the magnetization. Taking all this into account, we find that the total non-equilibrium magnetic induction which enters in (2) is

$$\mathbf{b}_q=\begin{cases}\tilde{\mathbf{b}}_0, & q=0.\\ \tilde{\mathbf{b}}_0-4\pi\mathbf{q}(\mathbf{m}_q\mathbf{q})/q^2+4\pi\mathbf{m}_q, & q\neq 0.\end{cases} \tag{4}$$

Here $\mathbf{q}$ is a vector directed along the ζ axis of length q.

From the form of (2) it is clear that near the spin resonance $\omega\cong\omega_s$ the most appreciable contribution to the spatial distribution of the magnetization is made by the zeroth Fourier components $m_0^{\pm}$, and it can therefore be realized by the method proposed by Suhl[9]. To this end, we solve first Eq. (2) at $q=0$, neglecting the nonlinear terms. With the aid of (4) we find that ($\tilde{b}_0^{\pm}\sim e^{\mp i\omega t}$)

$$m_0^{\pm}\approx\frac{-\chi\omega_s\tilde{b}_0^{\pm}}{\omega-\omega_s\pm i(1+\beta_0)/T_1}. \tag{5}$$

Naturally, this relation is valid only when the RF field does not cause saturation of the spin resonance, i.e., when

$$\tilde{B}(0)<\tilde{B}_{\text{нас}}=\sqrt{2}(1+\beta_0)/\gamma T_1\delta. \tag{6}$$

In the present paper we confine ourselves to just such a situation.

Under this condition we can put

$$M_1^z\approx -M_1^+M_1^-/2M_0,$$

where M_0 is the static magnetization of the Fermi liquid. Then, recognizing that the main contribution to the sum over q is made by the $m_0^{\pm}$ which have already been determined by us, and using expressions (1) and (1a), we arrive at

$$m_q^z=-\frac{1}{2M_0}\sum_{q'}m_{q-q'}^{+}m_{q'}^{-}\approx-\frac{1}{2M_0}(m_q^+m_0^-+m_0^+m_q^-). \tag{7}$$

In analogous fashion, taking (4) into account, we can linearize the entire equation (2). After its linearization at $q\neq 0$, confining ourselves to the terms bilinear in $m_0^{\pm}$ and neglecting the damping throughout, with the exception of $m_0^{\pm}$, we obtain

$$m_q^+ + im_q^+A_q + im_q^-B_q - i\tilde{b}_0^+C_q + E_q + G_q = 0;$$

$$A_q=\Omega_q\left(1-4\pi\chi+2\pi\chi\frac{q^+q^-}{q^2}\right),\qquad B_q=2\pi\chi\left(\frac{q^+}{q}\right)^2,$$

$$C_q=\chi\Omega_q-\gamma|m_0^+|^2/M_0,\qquad \Omega_q=\omega_s+q^2\,\mathrm{Im}\,D_q, \tag{8}$$

$$E_q=-i2\pi\gamma(q^z/q^2)\left[m_0^+(m_q^+q^-+m_q^-q^+)+q^+(m_0^+m_q^-+m_0^-m_q^+)\right],$$

$$G_q=i\tilde{b}_0^+F_q(m_0^+m_q^-+m_q^+m_0^-)+i\frac{2\pi\gamma}{M_0}|m_0^+|^2\left[2m_q^+-\frac{q^+}{q^2}(m_q^+q^-+m_q^-q^+)\right],$$

$$F_q=\frac{\gamma}{2M_0}\left[1-4\pi\left(\frac{q^+q^-}{q^2}\right)\frac{m_0^+}{\tilde{b}_0^+}\right]. \tag{9}$$

The equation obtained for m_q^- from (8) by complex conjugation with replacement of q by $-$q will be called the associated equation. We recall that the vector q is directed along the ζ axis. The components $q^{\pm}=q^x\pm iq^y$ and q^z are taken in the xyz system.

To solve the obtained equations, we change over to new variables a_q and a_{-q}^* which are connected with the old ones by the Holstein-Primakov transformation

$$m_q^+=u_qa_q+v_q^*a_{-q}^*,\qquad m_q^-=u_q^*a_{-q}^*+v_qa_q, \tag{10}$$

where $|u_q|^2-|v_q|^2=1$. We choose the coefficients u_q and v_q such that the transformation of the three first terms of (8) leads to an equation that includes only the variables a_q (in the associated equation only a_{-q}^*). It is easy to verify that this condition is satisfied by

$$u_q=\left(\frac{A_q+\omega_q}{2\omega_q}\right)^{1/2},\qquad v_q=-\frac{B_q}{|B_q|}\left(\frac{A_q-\omega_q}{2\omega_q}\right)^{1/2}, \tag{11}$$

where $\omega_q=(A_q^2-|B_q|^2)^{1/2}$, and the arbitrary phase factor in u_q and v_q is chosen equal to unity.

Equation (8), transformed with allowance for the relations in (10), takes the form

$$\dot{a}_q+i\omega_qa_q=i(u_q^*\tilde{b}_0^+C_q+v_q^*\tilde{b}_0^-C_{-q}^*)-[u_q^*(E_q+G_q)-v_q^*(E_{-q}^*+G_{-q}^*)], \tag{12}$$

i.e., it is a differential equation with a right-hand side and with variable coefficients (we must remember the time dependence of $m_0^{\pm}$ and $b_0^{\pm}$, which enter in E_q and G_q). We put

$$a_q=x_q(t)\exp(-i\omega_qt), \tag{13}$$

where $x_q(t)$ is a function that modulates the oscillations with frequency ω_q. The expression for the velocity of the slowly-varying function $x_q(t)$, obtained by substituting (13) in (12), can be simplified by time-averaging the rapidly oscillating terms in its right-hand part[11]. Taking formulas (5) and (9) into account and taking the time dependence of all the quantities in explicit form, we find as a result of the averaging that $x_q(t)$ depends on the time only in the regions near $\omega\sim\omega_q$ and $\omega\sim 2\omega_q$, namely

$$\dot{x}_q=\begin{cases}ix_qS_q-ix_{-q}^*R_q\exp[2i(\omega_q-\omega)t]+iQ_q\exp[i(\omega_q-\omega)t], & \omega\sim\omega_q. \quad (14a)\\ ix_{-q}^*P_q\exp[i(2\omega_q-\omega)t], & \omega\sim 2\omega_q. \quad (14b)\end{cases}$$

Here

$$P_q=4\pi\gamma\bar{m}_0^+u_q^*(q^z/q)(v_q^*q^-+u_q^*q^+),$$

$$Q_q = u_q^* \bar{b}_0^+ C_q, \qquad R_q = \bar{m}_0^+ \bar{b}_0^+ (u_q^*)^2 F_q,$$
$$S_q = 2\pi\gamma(|\bar{m}_0^+|^2/M_0 q^2)|q^- u_q + q^+ v_q|^2 - (|u_q|^2 f_q + |v_q|^2 f_q^*), \quad (15)$$
$$f_q = \bar{m}_0^-(\bar{b}_0^+ F_q + 4\pi\gamma|m_0^+|/M_0),$$

where $m_0^\pm$ and $b_0^\pm$ denote the corresponding time-independent amplitudes. We note that in (14b) we have confined ourselves to terms linear in $m_0^\pm$.

3. We consider the case when $\omega \sim \omega_q$. Differentiating (14a) with respect to time and using the corresponding associated equation, we obtain for x_q a differential equation with constant coefficients:

$$\ddot{x}_q + 2[\operatorname{Im} S_q - i(\omega_q - \omega)]\dot{x}_q - [|R_q|^2 - |S_q|^2 + 2S_q(\omega_q - \omega)]x_q = \{Q_q[(\omega_q - \omega) - S_{-q}^*] - Q_{-q}^* R_q\}\exp[i(\omega_q - \omega)t]. \quad (16)$$

Its general solution is

$$x_q(t) = K_1(q)\exp(\lambda_1 t) + K_2(q)\exp(\lambda_2 t) + \bar{x}_q, \quad (17)$$

where $K_1(q)$ and $K_2(q)$ are time-independent constants, $\bar{x}_q$ is a particular solution of (16), equal to

$$\frac{\{Q_q[(\omega_q - \omega) - S_{-q}^*] - Q_{-q}^* R_q\}\exp[i(\omega_q - \omega)t]}{[(\omega_q - \omega) - \operatorname{Re} S_q]^2 - |R_q|^2 + (\operatorname{Im} S_q)^2}$$

and the roots of the characteristic equation are

$$\lambda_{1,2} = -\operatorname{Im} S_q + i(\omega_q - \omega) \pm \{|R_q|^2 - [(\omega_q - \omega) - \operatorname{Re} S_q]^2\}^{1/2}.$$

The connection between $x_q(t)$ and the spatial magnetization $\mathbf{M}_1$ is given by formulas (1), (10), and (13), and therefore the determination of the behavior of $\mathbf{M}_1$ in time includes an investigation of the stability of the obtained solution. In the study of the conditions for the stability of $x_q(t)$, we must remember the attenuation of the magnetization (which was not taken into account in the derivation of (17)), which can be taken into account by adding to ω_q in (13) the imaginary part of the spin-wave energy, which is equal to $-i\Gamma_q$, $\Gamma_q \cong (1 + \beta_0)/T_1 + q^2 \operatorname{Re} D_q$. Obviously, the instability of the non-equilibrium magnetization $\mathbf{M}_1$ arises if the growth increment exceeds the damping decrement of the spin waves, i.e., at $\operatorname{Re} \lambda_{1,2} > \Gamma_q$ or

$$\operatorname{Re}\{|R_q|^2 - [(\omega_q - \omega) - \operatorname{Re} S_q]^2\}^{1/2} > \Gamma_q + \operatorname{Im} S_q. \quad (18)$$

It follows from (5), (6), and (15) that at the maximum, reached at $\omega = \omega_s$, we have

$$\operatorname{Im} S_q = \frac{\gamma\chi\omega_s T_1}{2M_0(1+\beta_0)}|\bar{b}_0^+|^2 = \frac{1+\beta_0}{T_1}\left|\frac{\mathcal{B}^+(0)}{\mathcal{B}_{\text{sat}}^+}\right|^2, \quad (19)$$

i.e., it becomes comparable with the minimum value of Γ_q only under saturation conditions. Since in our case the exciting RF field is smaller than the saturating field, it is clear that the presence of Im S_q in the inequality (18) is immaterial.

With the aid of (9) and (15), at $\omega \approx \omega_s$, we rewrite (18) in the form

$$|\bar{b}_0^+| > \frac{[2(1+\beta_0)]^{1/2}\{(\Gamma_q + \operatorname{Im} S_q)^2 + [(\omega_q - \omega) - \operatorname{Re} S_q]^2\}^{1/4}}{\gamma|u_q|T_1^{1/2}[1 + (4\pi\sin^2\vartheta_q\chi\omega_s T_1)^2/(1+\beta_0)^2]^{1/4}}. \quad (20)$$

According to this inequality, we can introduce the threshold value of the exciting RF field on a metallic surface; when this threshold is exceeded, the solution (17) is unstable, i.e., parametric resonance sets in[12]. It follows from (20) that the value of the threshold is minimal when the following condition is satisfied

$$(\omega_q - \omega) - \operatorname{Re} S_q = 0 \qquad (\operatorname{Re} S_q \ll \omega_q, \omega), \quad (21)$$

from which we get a dispersion equation for the parametrically excited spin waves, under the condition that the length of these waves is such that $\Gamma_q \sim (1 + \beta_0)/T_1$. Neglecting Im S_q in comparison with Γ_q and using (3) and (6), we arrive at the following smallest threshold RF field

$$|\mathcal{B}^+(0)|_{\text{thr}} = |\mathcal{B}_{\text{sat}}^+|[1 + (4\pi\sin^2\vartheta_q\chi\omega_s T_1)^2/(1+\beta_0)^2]^{-1/4}. \quad (22)$$

We assume here that $|u_q| = 1$, corresponding to neglecting the quantity χ in comparison with unity in (11).

Expression (22) indicates that the solution (17), and hence also the nonequilibrium magnetization $\mathbf{M}_1$, can become unstable even at an excitation level much lower than saturation, provided that

$$4\pi\sin^2\vartheta_q\chi\omega_s T_1/(1+\beta_0) \gg 1. \quad (23)$$

We shall discuss below the requirements that this inequality imposes on the parameters that enter in it.

We proceed now to consider the frequencies $\omega \sim 2\omega_q$. With the aid of (14b), proceeding just as in the first case, we obtain the following equation for the modulating function:

$$\ddot{x}_q - i(2\omega_q - \omega)\dot{x}_q - |P_q|^2 x_q = 0. \quad (24)$$

There is no need to present here the general solution of this equation, and we write down only the condition for its instability (with allowance for the spin-wave attenuation)

$$|P_q|^2 > \Gamma_q^2 + (\omega_q - \omega/2)^2. \quad (25)$$

The smallest value of the threshold RF field on the surface of the metal, causing instability of the magnetization, is reached at $\omega = 2\omega_q$ ($\vartheta_q \neq 0, \pi/2, \pi$) and its order of magnitude at $\omega = \omega_s$, with allowance for expressions (3), (5), and (15), is

$$|\mathcal{B}_+(0)|_{\text{thr}} = |\mathcal{B}_{\text{sat}}^+|\Gamma_q/\chi\omega_s. \quad (26)$$

From the dispersion equation $\omega = 2\omega_q$ we can obtain the wave vectors of the most excited parametric spin waves. Rough estimates show that $q^2 \sim (\beta_1 - \beta_0)\omega_s^2/v_f^2$ $(\beta_1 - \beta_0 < 1)$, and $\Gamma_q \sim (\beta_1 - \beta_0)/2\tau$. Consequently, the damping of these waves is given by $\Gamma_q \gg (1 + \beta_0)/T_1$, and the instability threshold is much higher than the saturation level. Therefore parametric instability is impossible at $\omega = \omega_s = 2\omega_q$ within the framework of our analysis.

4. The reason for the instability observed by us lies in the fact that, according to (14a), spin waves with equal but opposite wave vectors are nonlinearly related with each other. Such a nonlinearity, which increases with increasing power of the exciting RF field, leads indeed to the exponential growth of the magnetization with time. Naturally, this growth stops when the system goes over into a new stationary state, which is not considered in the present paper.

We note that the interaction between the spin waves, which leads to the instability, is due to their connection with the homogeneous magnetization $m_0^\pm$. Therefore the instability threshold is minimal at $\omega = \omega_s = \omega_q$, when the interaction of the waves with the homogeneous magnetization, and consequently with one another, is particularly strong. It is clear therefore that the most unstable are those magnetization waves which propagate at an angle to $\mathbf{B}_0$ such that Im $D_q = 0$. According to (2a) this angle is determined from the relation $1 - A\cos^2\vartheta_q = 0$, i.e., is determined completely by the constants characterizing the spin correlation of the quasiparticles of the electron liquid. The threshold value of the exciting RF field depends on the parameters responsible for the dissipative processes in the system. Let us make a few remarks concerning these parameters.

The collision time τ should ensure satisfaction of the inequality (2b). The damping of the parametrically excited spin-wave mode should satisfy the condition $\Gamma_q \sim (1 + \beta_0)/T_1$, which determines the minimum thickness of the sample. As follows from (23), particularly stringent requirements are imposed on the spin-relaxation time T_1. We note that in a sufficiently thick sample, relaxations via the spin waves[13] or surface relaxation[14] is immaterial. Therefore the most effective are the impurity or spin-lattice relaxation mechanisms[15], the latter being realized through the spin-orbit coupling. It is known that in lithium relaxation on impurities predominates ($T_1 \sim 10^{-6}$ sec). By using exceedingly pure single crystals of lithium we can suppress this type of spin damping and bring T_1 to 5×10^{-5} sec, since estimates of the spin-lattice relaxation time at low temperatures yield for lithium precisely this value. Under these conditions we find that the value of the RF field capable of parametrically exciting the first spin-wave mode ($q = \pi/L$) in a plate 2×10^{-1} cm thick is smaller by one order of magnitude that the saturation field and is approximately equal to $10^{-3} B_0$ ($\chi \sim 2 \times 10^{-6}$, $\tau \sim 5 \times 10^{-9}$ sec, $\omega_S \sim 10^{11}$ sec^{-1}).

In conclusion we add that parametric instability of the magnetization of an electron liquid, the feasibility of which is the main conclusion of the present paper, gives rise to a number of new singularities of such phenomena in metals as spin resonance and its saturation, spin echo, and polarization of nuclei.

The author is grateful to S. A. Al'tshuler and E. G. Kharakhash'yan for a useful discussion of a number of questions.

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Translated by J. G. Adashko
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