Cavitation Stability of Liquid Helium Due to "Electron Bubbles"

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Equilibrium stationary bubbles produced by free electrons in liquid helium are considered. The sizes of the "electron bubbles" are estimated. Cavitation stability of liquid helium is calculated for the case in which the "electron bubbles" are such cavitation centers that rupture of the liquid takes place on them during quasistatic stretching.

It is known[1,2] that it is energetically favorable for a free electron (positron) in liquid helium to be situated inside a spherical bubble, owing to the presence of a potential barrier between the electron (positron) and the liquid helium. The value of this barrier for an electron in liquid helium is about 1.0 eV.[3,4] If the radius of the "electron bubble" is equal to R, then the approximate value of the ground level of the energy of the electron is given by

$$E_1 = \pi^2\hbar^2/2mR^2, \tag{1}$$

where m is the mass of the electron. Then the quantity

$$p_e = -\frac{1}{4\pi R^2}\frac{dE_1}{dR} = \frac{\pi\hbar^2}{4mR^5} \tag{2}$$

determines the pressure created by the electron in a bubble of radius R.[5] The equation of motion of such a spherical bubble, which is usually written down in the form of the Rayleigh equation,[6] which assumes incompressibility of the liquid can, in the case of liquid helium, be represented in the form

$$R\frac{d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right)^2 + \frac{1}{\rho}\left[p - p_v + \frac{2\sigma}{R} - \frac{\pi\hbar^2}{4mR^5}\right] = 0, \tag{3}$$

where p is the pressure in the liquid, p_V the vapor pressure of helium in the bubble, σ the surface tension of helium, and ρ the density of the liquid helium.

We consider quasistatic measurements of the pressure in liquid helium, which lead to such a slowly changing radius of the bubble that the vapor pressure can be regarded as a constant corresponding to the temperature of the liquid T, and the inertial terms in Eq. (3) can be neglected. The condition of the dynamic equilibrium of such a bubble then follows from (3):

$$p = p_v - 2\sigma/R + \pi\hbar^2/4mR^5. \tag{4}$$

Analysis of this equation shows that the bubble will be stable for decrease in the pressure to some critical value p_c; for further decrease in the pressure, the bubble begins to expand without limit, i.e., it becomes unstable (see Fig. 1). Here it is easy to follow the analogy with the stability of a bubble in an ordinary liquid,[7] only in this case, in place of the gas pressure in the bubble, we have the action of the electron.

We determine the critical radius R_c and the critical pressure p_c from Eq. (4) by differentiating with respect to R and equating the derivative to zero. This pressure determines the formation of an unstable bubble for decrease in the pressure, i.e., it determines cavitation in liquid helium:

$$R_c = \left(\frac{5\pi\hbar^2}{8m\sigma}\right)^{1/4}, \quad p_c = p_v + \frac{\pi\hbar^2}{4mR_c^5} - \frac{2\sigma}{R_c} = p_v - \frac{4}{5}\frac{2\sigma}{R_c}. \tag{5}$$

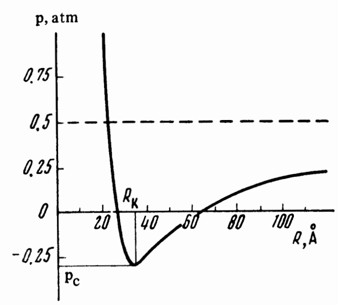

FIG. 1. Connection of the equilibrium radius of the bubble R and the pressure p in liquid helium at a temperature T = 3.6°K. Here σ = 0.165 dyne/cm, p_V = 0.5 atm (shown by the dashed line).

The radius of the equilibrium bubble is also determined from Eq. (4):

$$R_0 = (\pi\hbar^2/8m\sigma)^{1/4} = 5^{-1/4}R_c. \tag{6}$$

The value of σ for liquid helium depends essentially on the temperature, and decreases with increasing temperature.[8] Therefore, the quantities R_0 and R_c are also functions of the temperature. In the case in which the pressure in the liquid exceeds the equilibrium vapor pressure, which corresponds to a compression of the liquid which is realized in bubble chambers and some cryogenic systems, the value of the equilibrium radius R_0 is determined from the equation

$$R_0^4 = \frac{\pi\hbar^2}{8m\sigma}\left(1 + \frac{R_0\Delta p}{2\sigma}\right)^{-1}, \tag{7}$$

where $p = p_0 - p_V$ is known as the overcompression of the liquid, p_0 is the hydrostatic pressure.

Equation (6) allows us to determine the value of the critical (threshold) pressure for which cavitation arises in liquid helium on electron bubbles as the cavitation centers. The experimental determination of the cavitation threshold seems very simple and accurate if a powerful acoustic (ultrasonic) field is excited. In this case, the maximum value of the alternating pressure p_{ac} necessary for the cavitation formation is known as the value of the cavitation stability. For liquid helium with electron bubbles, this value is

$$p_{ac} = p_0 - p_c = \Delta p + {}^8/_5\sigma(8m\sigma/5\pi\hbar^2)^{1/4}. \tag{8}$$

In different experiments with liquid helium, the case without compression where $\Delta p = 0$ is most frequently encountered. In this case

$$p_{ac} = {}^8/_5\sigma(8m\sigma/5\pi\hbar^2)^{1/4}. \tag{9}$$

The cavitation stability of liquid helium, controlled by the electron bubbles and computed in this fasion, is shown in Fig. 2. It can be seen that the value of the cavitation stability increases and, reaching the value of about 1.5 atm at T_λ, remains practically constant

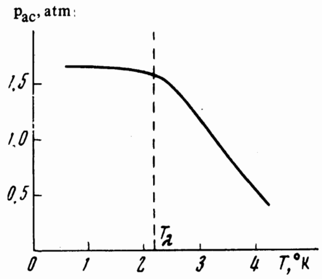

FIG. 2. Temperature dependence of the cavitation stability of liquid helium due to electron bubbles; $\Delta p = 0$.

over the range of temperatures of He II.

It must be noted that other cavitation centers can exist in liquid helium in addition to the electron bubbles. Of special importance in the experimental determination of cavitation stability are vaporous cavitation centers, produced by heterophase temperature fluctuations. If the liquid helium is at $\Delta p = 0$ and in phase equilibrium with the vapor, the dimensions of such cavities can exceed the dimensions of the electron bubbles and cavitation can consequently arise on them first for smaller values of the cavitation stability.[9] However, in overcompressed liquid helium, the vaporous cavitation centers formed by these fluctuations play a much lesser role, and the cavitation stability can be governed essentially by the electron bubbles. In contrast with other cavitation centers in liquid helium, the electron bubbles can change dimensions when the electron is excited by an external electromagnetic field. If the energy of the electron exceeds the potential barrier, then the existence of the electron bubble becomes unstable. In pulsations of such a bubble in the field of an ultrasonic wave, the appearance of instability is also possible at the moment of collapse[10]. This can affect the cavitation stability of the liquid helium. Experimental studies of these phenomena are of definite interest.

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225