Photoneutrino Processes in a Strong Field

D. V. Gal'tsov and N. S. Nikitina
Moscow State University
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Photon emission by a neutrino (antineutrino) and disintegration of a photon into a neutrino-antineutrino pair in crossed electromagnetic fields are considered. The matrix elements are calculated in accordance with the Furry picture with exact allowance for the external field. It is demonstrated that only the divergence of the axial current can make a contribution, and this allows one to take into account hadron vacuum polarization (if neutral currents exist). The magnitudes of the effects in the magnetic fields of pulsars are estimated.

NEUTRINO-photon interaction, which is realized in the Feynman-Gell-Mann scheme in first order in the weak interaction, is of great importance in astrophysics[1]. Among the reactions that play a definite role in the energy balance of stars is the decay of a photon into a neutrino-antineutrino pair in the Coulomb fields of nuclei[2]. In view of the discovery of pulsars[3] having magnetic fields close to $B_0 = m^2/e = 4.41 \times 10^{13}$ G, it is useful to consider processes of this type in a constant and homogeneous field. We note incidentally that a near-unity value of the dimensionless parameter $\chi^2 = e^2 m^{-6}(F_{\mu\nu}k_\nu)^2$ (where e and m are the charge and mass of the electron, $F_{\mu\nu}$ is the external-field tensor, and k_μ is the initial momentum), which determines the effectiveness of the influence of the external field, becomes attainable even under laboratory conditions[4].

We consider here the reactions

$$\nu_l(\mathbf{q}) + F \to \nu_l(\mathbf{q}') + \gamma(\mathbf{k}, \mathbf{e}) + F, \tag{1}$$

$$\bar{\nu}_l(\mathbf{q}) + F \to \bar{\nu}_l(\mathbf{q}') + \gamma(\mathbf{k}, \mathbf{e}) + F, \tag{2}$$

$$\gamma(\mathbf{k}, \mathbf{e}) + F \to \nu_l(\mathbf{q}) + \bar{\nu}_l(\mathbf{q}') + F. \tag{3}$$

Here l = e or μ; the symbol F stands for a constant crossed field ($F^2_{\mu\nu} = F_{\mu\nu}F^*_{\mu\nu} = 0$, $F^*_{\mu\nu}$ is the dual tensor)[1)]. The matrix element of process (1) can be written in the form

$$M = 2^{-1/2}G(2q_0 2q_0')^{-1/2} l_\mu \int dx\, e^{i(q-q')x} \langle \gamma(\mathbf{ke}) | \mathbf{J}_\mu(x) + \mathbf{J}_{\mu 5}(x) | 0 \rangle, \tag{4}$$

where $\mathbf{J}_\mu(x)$ and $\mathbf{J}_{\mu 5}(x)$ are the vector and axial currents in the Heisenberg representation (relative to the electromagnetic interaction), $l_\mu = i\bar{u}(q')\gamma_\mu(1+\gamma_5)u(q)$ is the neutrino bracket. We use a Euclidean metric, $a_\mu b_\mu = \mathbf{a}\cdot\mathbf{b} - a_0 b_0$, a system of units with $\hbar = c = 1$, and Hermitian Dirac matrices γ_μ.

The reactions (1)–(3) are kinematically possible if the momenta of all the participating particles are parallel, which means, in the zero-mass case under consideration, proportionality of their 4-momenta: $k_\mu = k_0 q_\mu / q_0$; $q'_\mu = q'_0 q_\mu / q_0$. As a result, $\bar{u}(q') = (q_0'/q_0)^{1/2}\bar{u}(q)$ and the neutrino bracket can be easily calculated:

$$l_\mu = 4(q_0'/q_0)^{1/2} q_\mu. \tag{5}$$

Substituting (5) in (4) and taking the conservation of the vector current into account, we can show that the matrix element is determined entirely by the divergence of the axial current:

$$M = \frac{2^{1/2} iG}{k_0} \int dx\, e^{i(q-q')x} \langle \gamma(\mathbf{ke}) | \partial_\mu \mathbf{J}_{\mu 5}(x) | 0 \rangle. \tag{6}$$

It is furthermore advantageous to use the identity of the "partial conservation" in spinor electrodynamics with allowance for the anomalous term, as established by Adler[6]:

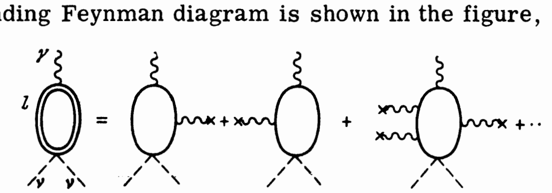
$$\partial_\mu \mathbf{J}_{\mu 5}(x) = 2im\mathbf{J}_5(x) - \frac{e^2}{8\pi^2}\mathcal{F}_{\mu\nu}{}^*\mathcal{F}_{\mu\nu}. \tag{7}$$

Here $J_5(x)$ is the pseudoscalar density, $\mathcal{F}_{\mu\nu} = F_{\mu\nu} + f_{\mu\nu}$, $f_{\mu\nu}$ is the photon field operator, and $F_{\mu\nu}$ the crossed-field tensor. The matrix element of the second term in (7) can be easily calculated and is equal to

$$8i\pi^2 e^2 (2\pi/k_0)^{1/2} (e^* F^* k)\,\delta(q - q' - k), \tag{8}$$

where we use the notation $e^*_\mu F^*_{\mu\nu} k_\nu = (e^* F^* k)$

The matrix element of the pseudoscalar density is further calculated in the lowest order in the interaction with the quantized electromagnetic field (i.e., without radiative corrections), and the interaction with the external field is taken exactly into account. The corresponding Feynman diagram is shown in the figure,



where the double line denotes the casual Green's function of the electron in the crossed field[7]. As expected from general considerations[8,9], the pseudoscalar-density matrix element turns out to be singular in the electron mass (this is indeed the cause of the appearance of the anomalous term in the identity (7)). The final result can be written as follows:

$$M = 16\pi^{5/2} e^2 G \frac{(e^* F^* q)}{q_0 k_0^{1/2}} \delta(q - q' - k) \left(1 - i\pi^{1/2} \int_0^1 z\mathfrak{G}(z)\, du\right), \tag{9}$$
$$z = (\chi u (1-u) k_0/q_0)^{-2/3}, \qquad \chi^2 = e^2 m^{-6} (F_{\mu\nu} q_\nu)^2.$$

Here $\mathfrak{G}(z)$ is the Airy-Hardy function[10]:

$$\mathfrak{G}(z) = \pi^{-1/2} \int_0^\infty \exp\left[-i\left(zt + \frac{t^3}{3}\right)\right] dt.$$

The matrix element of the analogous reaction (2) with participation of the antineutrino is obtained by replacing in (4) the neutrino bracket by the charge-conjugated one, which, by virtue of the specific nature

[1)]As noted by Nikishov and Ritus[5], the results of the calculations in a crossed field are approximately applicable to any homogeneous field if $\chi^2 \gg f, g$, where $f = e^2 m^{-4} F_{\mu\nu} F_{\mu\nu}$ and $g = e^2 m^{-4} F_{\mu\nu} F^*_{\mu\nu}$.

of the kinematics, the matrix. The matrix element of the reaction (3) is obtained by making the substitutions $q \to -q$, $k \to -k$, $e^* \to e$. However, when account is taken of the dispersion effects due to the polarization of the vacuum in the presence of the external field, a difference is observed between (1) and (3). The effect of the external field on the propagation of electromagnetic waves can be described with the aid of the effective refractive indices $n_{1,2}(k)$ (the subscripts 1 and 2 pertain to different polarizations), which are real quantities larger than unity at small χ^2, and which become coupled with a real part less than unity in the case of large χ^2. Such a "phenomenological" approach is justified if the dimensions of the region where the external field is effective are sufficiently large (appropriate estimates are given in[11]), and apparently reduces to a summation of all the self-energy inserts in the external photon lines. At small χ^2 the square of the photon momentum is larger than zero: $k^2 = \mathbf{k}^2 - k_0^2 = k_0^2(n^2 - 1) > 0$. It is easy to show that in this case the reaction (3) is forbidden by the 4-momentum conservation law. Similar reasoning was used by Adler et al.[12] in the analysis of the disintegration of a photon into two photons in an external field.

Let us discuss further the possible contribution of hadronic polarization of vacuum. Such a contribution arises if there exist neutral hadronic currents that interact with the neutrino[13-15]. This possibility is not excluded experimentally at the present time[15]. Since the matrix element is determined in our case by the divergence of the axial current, this contribution can be calculated if we use PCAC with allowance for the anomalous term[6]:

$$\partial_\mu J_{\mu 5}^{(h)} = c\varphi_{\pi^0} - \frac{e^2}{8\pi^2}\bar{Q}\mathscr{F}_{\mu\nu}{}^*\mathscr{F}_{\mu\nu},$$

where φ_π^0 is the operator of the neutral pion field, and Q is the "average number" of the spinor particles, and is estimated, for example, by starting from the probability of the $\pi^0 \to 2\gamma$ decay. Thus, in the presence of neutral hadronic current there appears another contribution that differs from (8) only by a factor.

Let us derive an expression for the probability of the decay (1). As already noted, the 4-momenta of the produced particles are parallel to the initial momentum, i.e.,

$$k_\mu = \lambda q_\mu, \quad q_\mu' = (1-\lambda) q_\mu \quad (0 < \lambda < 1).$$

The differential probability $dw/d\lambda$ thus determines the probability that the photon carries away a fraction λq of the initial neutrino momentum:

$$\frac{dw_{\nu\to\nu\gamma}}{d\lambda} = \frac{e^4G^2}{2\pi^4 q_0}|e^*F^*q|^2(1-\lambda)|g(\chi,\lambda)|^2 \tag{10}$$

If there are no neutral hadronic currents, the function $g(\chi, \lambda)$ coincides with the function

$$1 - i\pi^{1/2}\int_0 z\mathfrak{G}(z)\,du \approx \begin{cases} -{}^1/_{15}\lambda^2\chi^2, & \chi^2 \ll 1 \\ 1, & \chi^2 \gg 1 \end{cases} \tag{11}$$

from (9). If neutral hadronic currents exist, then an increment independent of χ^2 is produced:

$$\Delta g^{(h)} = \bar{Q}. \tag{12}$$

Comparison of (12) and (11) shows that hadronic polarization of vacuum should affect strongly the value of the probability at small χ^2. Substituting (11) in (10), integrating with respect to λ, and summing over the photon polarizations, we obtain the total decay probability in the limiting cases under consideration:

$$w_{\nu\to\nu\gamma} = \frac{e^2G^2m_l^6\chi^{*2}}{4\pi^4 q_0}\begin{cases} \chi^4/(15)^3, & \chi^2 \ll 1 \\ 1, & \chi^2 \gg 1, \end{cases} \tag{13}$$

where

$$\chi^{*2} = e^2 m_l^{-6}(q_\mu F_{\mu\nu}{}^*)^2.$$

We note that when $\chi^2 \gg 1$ the probability is independent of the lepton mass m_l, and is thus the same for the electron and the muon. However, the condition under which the approximation is valid in the case of the muonic neutrino calls for larger neutrino energies or field intensities, in view of the appreciable difference in the masses. On the other hand, if neutral hadronic currents exist, then the probability in the entire interval of χ^2 is equal, in order of magnitude, to the value of (13) in the region $\chi^2 \gg 1$.

Let us turn further to the decay of a photon into a neutrino-antineutrino pair. As indicated, it is meaningful to consider this process only for $\chi^2 \gg 1$. The probability of the decay of an unpolarized photon is equal to

$$w_{\gamma\to\nu\tilde{\nu}} = e^2G^2m_l^6\chi^{*2}(k)/24\pi^4 k_0,$$

where $\chi^{*2}(k) = e^2 m_l^{-6}(k_\mu F^*_{\mu\nu})^2$, and k_μ is the photon momentum. The energy carried away by the neutrino from 1 cm^3 per second as a result of the conversion of the photons into neutrino pairs is given by

$$\mathscr{E}_{\nu\tilde{\nu}} = \int k_0 w(\mathbf{k})\, n(\mathbf{k})\, d^3k \approx 0.1\cdot 10^{-7}\int \chi^{*2} n(\mathbf{k})\, d^3k \ [\text{MeV-sec}^{-1}\text{cm}^{-3}].$$

Here $n(k)$ is the number of photons per cm^3.

If the invariants χ^2 and χ^{*2} greatly exceed the dimensionless field invariants f and g, the obtained formulas are valid for a spontaneous constant and homogeneous field[5]. This enables us to obtain approximate estimates of the effect under pulsar conditions. We assume that the field is purely magnetic; then

$$\chi^{*2} = \left(\frac{k_0 B}{m B_0^{(e)}}\sin\theta\right)^2,$$

where θ is the angle between the direction of the magnetic field and the photon momentum. Putting $B \sim 0.1 B_0^{(e)}$; $k_0 \gg m$; $\sin\theta \sim 1$, we get $\chi^{*2} \gg 1$; f, g $\sim 10^{-2}$. The effective photon mean free path with respect to decay into an electronic neutrino-antineutrino pair is of the order of the pulsar radius (10^6 cm) at a photon energy $\sim 10^{10}$ GeV.

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