Plasma Wave Synchrotron Instability

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The conditions for the existence of longitudinal waves with phase velocity close to that of light in a magnetoactive plasma are obtained. The increments of such waves are obtained and investigated for the synchrotron instability connected with the presence of isotropically distributed relativistic particles. It is shown that the increments of the longitudinal waves greatly exceed the increments of the transverse-wave synchrotron instability that occurs for the same parameters of the medium.

PLASMA-wave synchrotron instability has been investigated in recent years in a number of studies[1-5]. The instability arises for a definite energy spectrum of isotropically-distributed relativistic particles in a medium with an external magnetic field and with a refractive index that differs from unity. For transverse waves, such a medium can be a "cold" plasma[1] or the system of relativistic particles itself, if the latter have a sufficiently high density[5].

Synchrotron radiation in a medium is characterized by the following: a) The radiation occurs at high harmonics of the rotation frequency of the relativistic particles in the magnetic field (relativistic gyrofrequency $\tilde{\omega}_H$); therefore, if the particle distribution function has a finite width, the spectral characteristics of the radiation are averaged out (smoothed out) over a frequency interval much larger than $\tilde{\omega}_H$. b) Emission of waves with phase velocity v_{ph} close to the velocity of light c is concentrated in a small angle interval $\Delta\theta \sim \max\{m_0c^2/E, (1-n^2)^{1/2}\}$, which includes the instantaneous velocity of the particle (here m_0c^2/E is the ratio of the rest energy to the energy of the relativistic particle and n is the refractive index of the wave). c) The emission of particles of high energy $E/m_0c^2 > (1-n^2)^{-1/2}$ is strongly suppressed because the refractive index n differs greatly from unity.

These features determine the distinct characteristics of the synchrotron instability. Since they are possessed also by radiation of a relativistic particle into plasma (longitudinal) waves if the phase velocity of the latter is close to c, it is natural to expect synchrotron instability to be feasible not only for transverse but also for longitudinal waves. However, while transverse wave with $n \approx 1$ undoubtedly exist, at least at low plasma concentrations, the conditions under which plasma waves with $n \approx 1$ exist in a magnetoactive plasma require a more detailed study.

Plane waves $\exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t)$ in an anisotropic medium are determined in the linear approximation by the equations

$$(n^2\delta_{\alpha\beta} - n_\alpha n_\beta - \varepsilon_{\alpha\beta})E_\beta = 0, \tag{1}$$

where $n \equiv ck/\omega$, $\epsilon_{\alpha\beta}(\omega, k)$ is the dielectric tensor and $\delta_{\alpha\beta}$ is the Kronecker symbol.

We assume that Eq. (1) is expressed in a coordinate system with z axis directed along the wave vector **k** and y axis in the (**k**, $\mathbf{H}_0$) plane ($\mathbf{H}_0$ is the external magnetic field). In an anisotropic medium, a plasma wave is defined as one in which the longitudinal component (relative to the wave vector **k**) of the electric field greatly exceeds the transverse one. If the refractive index $n \equiv ck/\omega$ is large enough (formally, as $n^2 \to \infty$), the approximate dispersion equation $\epsilon_{zz} = 0$ describes precisely such longitudinal waves[6]. In an isotropic plasma the relation $\epsilon_{zz} = 0$ is the exact dispersion equation for longitudinal waves, and waves with $n \lesssim 1$ are possible in this case. It is clear that such waves can exist also in a medium with a weak anisotropy.

The sufficient conditions for the existence of longitudinal waves described by the approximate dispersion equation $\epsilon_{zz} = 0$ can be found from Eq. (1)

$$\begin{gathered}|\varepsilon_{xz}(n^2-\varepsilon_{yy})|, \quad |\varepsilon_{yz}(n^2-\varepsilon_{xx})|, \quad |\varepsilon_{xy}\varepsilon_{yz}|, \quad |\varepsilon_{xy}\varepsilon_{xz}| \ll \Delta, \\ |\varepsilon_{xz}|, \ |\varepsilon_{yz}| \lesssim 1; \quad \Delta = |(n^2-\varepsilon_{xx})(n^2-\varepsilon_{yy}) + \varepsilon_{xy}^2|.\end{gathered} \tag{2}$$

All the components ϵ_{ik} in (2) should be taken at values of ω and k satisfying the equation $\epsilon_{zz} = 0$. For a "cold" magnetoactive plasma, the conditions (2) are satisfied for $n \approx 1$ at frequencies $\omega \approx \omega_p$ if $\omega_p \gg \omega_H$ (ω_p is the plasma frequency and ω_H the gyrofrequency). Thus, longitudinal waves with $v_{ph} \approx c$ are possible in a weakly-anisotropic "cold" plasma ($\omega_p \gg \omega_H$).

1. We consider a medium consisting of a "cold" plasma with concentration n_0 and a relativistic electron component with concentration N_0 with an isotropic distribution function f(p), situated in a constant magnetic field $\mathbf{H}_0$. The tensor component ϵ_{zz} for such a medium is equal to[7]

$$\varepsilon_{zz} = 1 - \frac{\omega_p^2}{\omega^2} - \frac{2\pi^2e^2N_0}{m_0\omega\omega_H}\int_0^\infty p^3\,dp\,\frac{df}{dp} \tag{3}$$

$$\times\int_0^\pi d\varphi\sin\varphi\sum_{s=-\infty}^{\infty}\frac{1}{s-q}[2\cos\theta\cos\varphi J_s + \sin\theta\sin\varphi(J_{s+1}+J_{s-1})]^2;$$

$$q = \omega/\tilde{\omega}_H - kp\cos\theta\cos\varphi/m_0\omega_H.$$

Here $\omega_p^2 = 4\pi e^2n_0/m_0$, $\omega_H = eH/m_0c$, $\tilde{\omega}_H = \omega_H m_0/m$ is the relativistic gyrofrequency, θ is the angle between the wave vector k and the external magnetic field, φ is the angle between the relativistic-particle velocity and the magnetic field, e, m_0 and m are the charge, rest mass, and mass of the electron, and J_s is a Bessel function of order s of the argument $\chi = kp\sin\theta\sin\varphi/m_0\omega_H$. The integration in (3) is carried out for Im > 0 along the real axis; when Im < 0, the integration contour is deformed in such a way that the pole does not cross the contour when the sign of Im ω is reversed.

When calculating that part of ϵ_{zz} which is connected with the relativistic particles, we assume that

$\omega \gg \omega_H$ and $|1 - n\beta| \ll 1$ ($\beta = v/c$ is the ratio of the relativistic-particle velocity to the velocity of light). In this case the summation over s can be replaced approximately by integration with respect to s and with respect to the angular coordinate in momentum space, using a procedure employed by Sazonov[8] to calculate the components ϵ_{xx}, ϵ_{xy}, and ϵ_{yy}. As a result, the component ϵ_{zz} can be transformed into

$$\varepsilon_{zz} = 1 - \frac{\omega_p{}^2}{\omega^2} - \frac{2\pi e^2 N_0 c^2}{\omega^2}\int_0^\infty dE \frac{d}{dE}\left[\frac{N(E)}{E^2}\right]\left\{\ln\frac{4}{|1-n^2\beta^2|}\right. \tag{4}$$
$$\left. -2 + \int_0^\infty \frac{d\nu}{\nu}(e^{i\nu^3/3} - 1)e^{i\nu z} - i\frac{\pi}{2}\left(1 - \frac{1-n^2\beta^2}{|1-n^2\beta^2|}\right)\right\};$$
$$z = (\omega/\tilde{\omega}_H \sin\theta)^{2/3}(1 - n^2\beta^2), \quad \beta = v/c = p/mc.$$

The relativistic-particle distribution function $f(p)$ has been replaced in (4) by the energy spectrum $N(E)$:

$$4\pi p^2 f(p)\,dp = N(E)\,dE, \quad E \approx pc.$$

The Hermitian part of ϵ_{zz} is determined by the "cold" and relativistic components, and the anti-Hermitian part only by the relativistic component. If the anti-Hermitian part is small:

$$\mathrm{Im}\,\varepsilon_{zz} \ll 1, \tag{5}$$

then the plasma waves are weakly damped (growing). Their dispersion characteristics are determined in this case by the equation

$$\mathrm{Re}\,\varepsilon_{zz} = 0, \tag{6}$$

and the increment (decrement) is obtained by perturbation theory:

$$\gamma = -\,\mathrm{Im}\,\varepsilon_{zz} \Big/ \frac{\partial\,\mathrm{Re}\,\varepsilon_{zz}}{\partial\omega}. \tag{7}$$

The values of ω and k in the right-hand side of (7) satisfy Eq. (6).

2. The anti-Hermitian part of ϵ_{zz}, which determines the magnitude and the sign of the dissipation, can be expressed in terms of an integral of the Airy function $v(z)$:

$$\mathrm{Im}\,\varepsilon_{zz} = -\frac{2\pi^{3/2}e^2N_0c^2}{\omega^2}\int_0^\infty dE\frac{d}{dE}\left[\frac{N(E)}{E^2}\right]E^2\int_z^\infty v(z')\,dz' \tag{8}$$

or (after integrating by parts)

$$\mathrm{Im}\,\varepsilon_{zz} = \frac{2\pi^{3/2}e^2N_0c^2}{\omega^2}\int_0^\infty dE\frac{N(E)}{E}\left\{2\int_z^\infty v(z')\,dz' - E\frac{dz}{dE}v(z)\right\}. \tag{8a}$$

The Airy function

$$v(z) = \frac{1}{\pi^{1/2}}\int_0^\infty d\nu \cos\left(\frac{\nu^3}{3} + \nu z\right)$$

is expressed in the case of positive z in terms of a Macdonald function of order $1/3$, and for $z < 0$ in terms of a sum of Bessel functions of order $\pm 1/3$ (see[9]).

In the investigation of the dissipation it is convenient, following Zheleznyakov[1], to break up the relativistic-particle energy interval into two regions $E \ll E^*$ and $E \gg E^*$, where $E^* = m_0c^2|1 - n^2|^{-1/2}$. The electrons with energy $E \ll E^*$ make a positive contribution to the wave damping, equal to

$$\mathrm{Im}\,\varepsilon_{zz} = \frac{4\pi e^2N_0c^2}{3^{1/2}\omega^2}\int_{E\ll E^*}\frac{dE}{E}N(E)\left[\int_y^\infty dy'\,K_{1/3}(y') + yK_{1/3}(y)\right], \tag{9}$$

where $y = 2/3\, z^{3/2}$. In the derivation of (9) we used the well known representation of the Airy integral in terms of the Macdonald function $K_{1/3}$ (for $z > 0$). For waves with phase velocity $v_{ph} = c$, all the energies satisfy the condition $E \ll E^*$; the dissipation of such waves is positive at any energy spectrum of the relativistic particles. The contribution made to $\mathrm{Im}\,\epsilon_{zz}$ by the electrons with energy $E \gg E^*$ is equal to

$$\mathrm{Im}\,\varepsilon_{zz} = \frac{4\pi^{3/2}e^2N_0c^2}{\omega^2}\int_{E\gg E^*}\frac{dE}{E}N(E)\Phi(z), \tag{10}$$

$$\Phi(z) = \int_z^\infty dz'\,v(z') - \frac{1}{3}zv(z), \qquad z = \left(\frac{\omega}{\tilde{\omega}_H\sin\theta}\right)^{2/3}(1 - n^2).$$

Using the asymptotic expressions for $v(z)$[9], we obtain for $z > 0$ and $z \gg 1$

$$\Phi(z) \approx \frac{1}{2}\left(\frac{1}{z^{3/4}} - \frac{1}{3}z^{3/4}\right)\exp\left(-\frac{2}{3}z^{3/2}\right), \tag{11}$$

and for $z < 0$ and $|z| \gg 1$

$$\Phi(z) \approx \pi^{1/2} + \frac{1}{3}(-z)^{3/4}\sin\left[\frac{2}{3}(-z)^{3/2} + \frac{\pi}{4}\right] \tag{12}$$

It follows from these expressions for $\Phi(z)$ that at large $|z|$ and $E \gg E^*$ certain energy intervals in (10) make a negative contribution to the dissipation. At $1 - n^2 > 0$, this is the interval

$$E > E_1 = m_0c^2\frac{\omega_H\sin\theta}{\omega}\left(\frac{z_1}{1-n^2}\right)^{3/2}$$

(here $z_1 \approx 2.0$ is the value of z at which (11) reverses sign). On the other hand, if $1 - n^2 < 0$, then the energy intervals making positive and negative contributions to the dissipation alternate with each other[1)]. The characteristic energy scale of the oscillations of the function $\Phi(z)$ is

$$\Delta E \approx \frac{3\pi}{2}\,\frac{\omega_H\sin\theta}{\omega}\,\frac{m_0c^2}{|1-n^2|^{3/2}}, \qquad \frac{\Delta E}{E} \approx \frac{3\pi}{2z^{3/2}}. \tag{13}$$

We note that an analogous oscillatory relation holds also for the energy radiated by a relativistic particle if $n > 1$; it is due to the nontrivial "superposition" of the synchrotron and Cerenkov effects[11].

As follows from (11), the maximum contribution to the dissipation is reached at $z \approx 2.5$ ($\Phi \approx -0.8 \times 10^{-2}$); electrons with energies $E \gg E_1$ ($z \gg 1$) make a negative exponentially small contribution to the dissipation. The negative energy dissipation of the plasma waves with $n > 1$ can reach an appreciable value if the energy spectrum is narrow enough and the energy spread does not exceed the value given in (13). In the case of a smooth energy spectrum, the oscillations of $\Phi(z)$ are averaged out and result in the usual Landau damping for plasma waves with $n > 1$ in an isotropic relativistic plasma. The figure shows a plot of $\Phi(z)$, from which we determine Im_{zz} for a monoenergetic particle spectrum. In the intervals $z < -5$ and $z > 2$, where negative values of $\Phi(z)$ are possible, we can use with good accuracy the asymptotic expressions (11) and (12).

3. We examine now in greater detail the dispersion equation (6) and the character of the derivative $\partial\mathrm{Re}\,\epsilon_{zz}/\partial\omega$, which also determines the value of the increment (7). The contribution of the relativistic particles to $\mathrm{Re}\,\epsilon_{zz}$ is of the order of

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1)The dissipation of transverse waves with $n > 1$ is of the same type (see[10]).

$$\frac{\Omega_0^2}{\omega^2}\ln\frac{4}{|1-n^2\tilde{\beta}^2|},$$

$$\Omega_0^2=4\pi e^2 N_0\int\limits_0^\infty\frac{c^2dE}{E}N(E),$$

and $\tilde{\beta}$ is a certain value of the quantity $\beta = v/c$ from the energy spectrum under consideration. If

$$n_0\gg N_0\frac{m_0}{\tilde{m}}\ln\frac{4}{|1-n^2\tilde{\beta}^2|},\qquad(14)$$

$$\frac{1}{\tilde{m}}=\int\limits_0^\infty\frac{c^2dE}{E}N(E)$$

the contribution of the relativistic particles to Re ϵ_{zz} is negligibly small and the dispersion properties of the plasma waves are determined by the "cold" plasma;

$$\omega\approx\omega_p,\qquad\frac{\partial\,\mathrm{Re}\,\varepsilon_{zz}}{\partial\omega}\approx\frac{2}{\omega_p}.\qquad(15)$$

The refractive index is determined by the wave number k and can be either larger or smaller than unity. The sufficient condition for the existence of longitudinal waves with $n \approx 1$, as already noted, takes the form $\omega_p \gg \omega_H$.

If the condition inverse to (14) is satisfied, the dispersion properties of the plasma waves are determined by the relativistic plasma:

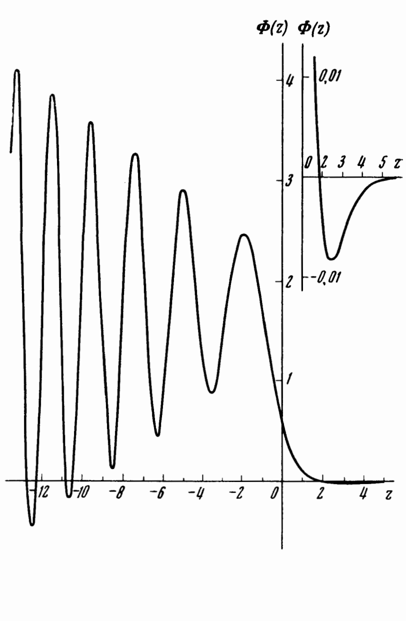
$$1-\frac{\Omega_0^2}{\omega^2}\left(\ln\frac{4}{|1-n^2|}-2\right)=0.\qquad(16)$$

In the derivation of (16) we assumed that the spectrum N(E) lies in the energy interval $E \gg E^*$, so that $1 - n^2\beta^2 \approx 1 - n^2$. In addition, no account was taken in (16) of that part of ϵ_{zz} which depends on $\tilde{\omega}_H$. Numerical estimates show that this part can be neglected if $z > 0.5$ in (4). A dispersion equation such as (16) is given in the book by Silin and Rukhadze[12] for plasma waves with $v_{ph} \approx c$ in an isotropic relativistic plasma.

An analysis shows that when perturbation theory is used to find the imaginary correction to the frequency of plasma waves with $n \approx 1$ in an isotropic relativistic plasma, it is necessary to satisfy a condition more stringent than (5), namely

$$\mathrm{Im}\,\varepsilon_{zz}\ll|1-n^2|.\qquad(5a)$$

For $z \lesssim 0$, this condition cannot be satisfied in practice for any energy spectrum. For sufficiently large positive z, however, this condition is certainly satisfied. We shall therefore consider for a purely relativistic plasma (remaining within the framework of perturbation theory) only longitudinal waves with $v_{ph} > c$. Their refractive index, at $1 - n^2 \ll 1$, is given by[12]



$$n^2\approx1-4\exp(-2-\omega^2/\Omega_0^2),\qquad(17)$$

and in this case[2)]

$$\frac{\partial\,\mathrm{Re}\,\varepsilon_{zz}}{\partial\omega}\approx\frac{2\Omega_0^2}{\omega^3}\frac{1}{1-n^2}.\qquad(18)$$

We now obtain with the aid of (2) the conditions for the existence of plasma waves with $v_{ph} \approx c$ in a relativistic magnetoactive plasma. In the case $\omega \gg \tilde{\omega}_H$ and $|1 - n^2\beta^2| \ll 1$ of interest to us, the components of the tensor ϵ_{ij} are approximately equal to [3)]

$$\varepsilon_{xx}\sim\varepsilon_{yy}\sim1-\frac{\Omega_0^2}{\omega^2},\qquad\varepsilon_{xy}=-\varepsilon_{yx}\sim\frac{\Omega_0^2}{\omega^2}\frac{\tilde{\omega}_H}{\omega}\ln\frac{4}{|1-n^2\beta^2|},$$

$$\varepsilon_{xz}=-\varepsilon_{zx}\sim\frac{\Omega_0^2}{\omega^2}\left(\frac{\tilde{\omega}_H}{\omega}\right)^{1/3},\qquad\varepsilon_{yz}=\varepsilon_{zy}\sim\frac{\Omega_0^2}{\omega^2}\left(\frac{\tilde{\omega}_H}{\omega}\right)^{4/3}z^2.$$

Substituting these estimates in (2), we obtain

$$(\omega/\tilde{\omega}_H)^{1/3}\gg1.$$

Since the plasma-wave frequency is $\omega \sim \Omega_0$, this condition can be rewritten in the form

$$(\Omega_0/\tilde{\omega}_H)^{1/3}\gg1\ \text{or}\ (8\pi NE_0/H_0^2)^{1/6}\gg1.\qquad(19)$$

Thus, in order for longitudinal waves with $n \approx 1$ to exist in a relativistic magnetoactive plasma, it suffices to have the energy of the relativistic particles exceed the energy density of the magnetic field.

4. Using the results of Secs. 2 and 3, we can find the increments of the synchrotron instability in plasma waves for a monoenergetic relativistic-particle spectrum

$$N(E)=\delta(E-E_0)\qquad(20)$$

(at different ratios of the "cold" and "hot" component concentrations) and to compare them with the increments in the transverse waves. If the "cold" plasma concentration is high enough (the condition (14) is satisfied), the plasma-wave frequency equals the Langmuir frequency of the "cold" plasma[4)], and the imaginary correction to the frequency is

$$\mathrm{Im}\,\omega_l\equiv\gamma_l=-\frac{1}{2}\pi^{1/2}\omega\frac{N_0}{n_0}\frac{m_0c^2}{E_0}\Phi(z_0),\qquad(21)$$

$$z_0=\left(\frac{\omega E_0}{\omega_H m_0c^2\sin\theta}\right)^{2/3}(1-n^2).$$

Here N_0 and n_0 are the concentrations of the "hot" and "cold" components, and $\Phi(z)$ is defined in (10). Waves with a refractive index $n < 1$ have a maximum increment at $z_0 \approx 2.5$:

$$\gamma_l^{max}\sim10^{-2}\omega_p\frac{N_0}{n_0}\frac{m_0c^2}{E_0}.\qquad(22)$$

Waves with $n > 1$ are stable up to $z \approx -10$. Therefore, using the asymptotic expression (12) for $\Phi(z)$, we obtain

$$\gamma_l=-\frac{\omega_p}{2}\frac{N_0}{n_0}\frac{m_0c^2}{E_0}\left[\pi+\left(\frac{\pi y_0}{6}\right)^{1/2}\sin\left(y_0+\frac{\pi}{4}\right)\right],$$

$$y_0=\frac{2}{3}\frac{\omega_p}{\omega_H\sin\theta}\frac{E_0}{m_0c^2}|1-n^2|^{3/2}.\qquad(23)$$

[2)]In the calculation of $\partial\mathrm{Re}\epsilon_{zz}/\partial\omega$; it must be recognized that $n^2=c^2k^2/\omega^2\approx1$ in (16).

[3)]Some of the components ϵ_{ij} were estimated by Sazonov[8].

[4)]We assume that the condition for the existence of longitudinal waves with $v_{ph}\approx c$, namely $\omega_p \gg \omega_H$, is also satisfied.

Waves for which $y_0 \gtrsim 20$ can become unstable, and the increments of these waves can exceed the increments of the waves with $n < 1$ by an order of magnitude or more. The dependence of the increment on the refractive index (actually on the wave number, since the plasma-wave frequency is fixed, $\omega \approx \omega_p$) has an oscillatory character; the intervals of the stable and unstable wave numbers alternate with one another. The instability increments of the modes with $n > 1$ depend strongly on the energy scatter of the relativistic particles. Expression (23) is valid for $\Delta E/E_0 \ll 1/y_0$, i.e., for an energy-spectrum width smaller than 5% of E_0. When ΔE increases, the increments decrease rapidly, and all the waves with $n > 1$ become unstable[5)] at a certain value of Δ. The increments of waves with $n < 1$ are less sensitive to the energy scatter of the relativistic particles; thus, for example, the maximum increment remains practically unchanged when the spectrum width $\Delta E/E_0$ increases to 30%.

The over-all picture of the dependence of the increment of "cold" plasma longitudinal waves on the wave number k, for a monoenergetic relativistic-particle spectrum, is the following. At small k ($n^2 < 1$) the plasma waves are unstable with an exponentially small increment. With increasing k, the increment grows, reaches the maximum value (22) at $z_0 \approx 2.5$, and then decreases and reverses sign at $z_0 \approx 2.0$. If the condition $E \gg E^*$ is violated before z_0 reaches the value 2.5, then the instability vanishes earlier and the increment reaches a smaller maximal value. With further increase of k, the refractive index becomes larger than unity and a new unstable wave appears at $z_0 < -10$; the intervals of k with stable and unstable modes alternate with each other. The maximum value of the increment in the instability interval increases with increasing number of the interval (with increasing k). The increments are bounded at large k because of the finite width of the energy spectrum (the instability vanishes). At $|n^2 - 1| \sim 1$ expression (8) for the anti-Hermitian part of ϵ_{zz} ceases to be valid.

As is well known, under the same conditions ($\omega_p \gg \omega_H$, monoenergetic spectrum of relativistic particles) there exist in a cold plasma unstable transverse waves[1] with frequency

$$\omega_{tr} \approx \omega_p(\omega_p / \tilde{\omega}_H)^{1/2}$$

and increment

$$\gamma_{tr} \approx -\frac{\Omega_0^2 \omega_p^2}{\omega_{tr}^3}\Phi_1(z),$$

where $\Phi_1(z)$ is a function with a maximum negative value of the same order as $\Phi(z)$. The ratio of the maximum increments of plasma waves with $n < 1$ and of electromagnetic waves in a system consisting of a "cold" plasma and relativistic particles with a monoenergetic spectrum is

$$\frac{\gamma_l}{\gamma_{tr}} \sim \left(\frac{\omega_{tr}}{\omega_l}\right)^3 \approx \left(\frac{\omega_p}{\tilde{\omega}_H}\right)^{3/2} = \left(\frac{\omega_p}{\omega_H}\frac{E}{mc^2}\right)^{3/2}. \tag{24}$$

The analogous ratio for plasma waves with $n > 1$ can be appreciably larger. The fact that the plasma-wave increments greatly exceed the electromagnetic-wave increments is connected with the following two circumstances: 1) the plasma-wave instability sets in at lower frequencies; 2) a relativistic particle radiates a plasma wave more effectively than an electromagnetic wave, for in the former case the particle moves parallel to the vector **E** in the wave, and in the latter it is perpendicular. Both factors, as follows from the scheme of the Einstein coefficients, make the plasma-wave increment larger than that of the electromagnetic wave.

In the absence of a "cold" plasma (or when a condition inverse to (14) is satisfied), the increment of plasma waves with $v_{ph} > c$ is equal to

$$\gamma_l = -{}^1/_2\omega_l(1-n^2)\Phi(z_0). \tag{25}$$

It was assumed in the derivation of (25) that the spectrum of the relativistic particles is monoenergetic, with energy $E_0 \gg m_0c^2(1 - n^2)^{-1/2}$, and the condition (19) for the existence of longitudinal waves with $v_{ph} > c$ is satisfied. The maximum increment

$$\gamma_l^{max} \sim 10^{-2}\omega_l(\tilde{\omega}_H/\omega_l)^{2/3}$$

is possessed by plasma waves of frequency

$$\omega_l \approx \Omega_0 \ln^{1/2}(\Omega_0/\tilde{\omega}_H).$$

Transverse waves in a sufficiently dense relativistic plasma can also be unstable[5]. The instability sets in if the energy density of the relativistic particles in the source greatly exceeds the energy density of the magnetic field[6)]:

$$(8\pi N E_0 / H_0^2)^{1/2} \gg 1. \tag{26}$$

The frequency of the unstable transverse waves is $\omega_{tr} \approx \Omega_0(\Omega_0/\tilde{\omega}_H)^{1/2}$, and the increment is $\gamma_{tr} \approx -\Omega_0^4\Phi_1(z)/\omega_{tr}^3$. The criterion (26) corresponds to the sufficient condition (19) for the existence of unstable plasma waves with $v_{ph} \approx c$. Thus, in a dense relativistic plasma with a monoenergetic spectrum there exist simultaneously unstable transverse and longitudinal waves. The ratio of their increments is

$$\frac{\gamma_l}{\gamma_{tr}} \sim \frac{\omega_l\omega_{tr}}{\Omega_0^2}\left(\frac{\omega_{tr}}{\omega_l}\right)^{2/3}. \tag{27}$$

The increment of the longitudinal waves exceeds that of the transverse ones, since $\omega_{tr} > \omega_l$ and both exceed Ω_0. It should be noted, however, that the criterion (19) is more stringent than (26). Therefore, at not too large a ratio of the energies of the relativistic particles and

5)In an isotropic plasma ($H_0 = 0$), the plasma waves are damped by relativistic particles with spectrum $\delta(E - E_0)$; their decrement can be easily obtained for an arbitrary refractive index $n > 1$:

$$-\gamma_l = \omega_p\frac{\pi}{2}\frac{N_0}{n_0}\frac{m_0c^2}{E}\frac{1}{n^3}.$$

In Eq. (23), Landau damping for waves with $n \approx 1 (n > 1)$ in an isotropic plasma corresponds to the part of γ_e which does not depend on y_0. This expression is obtained by taking the limit in $\mathrm{Im}\,\omega_{zz}$[Eq. (10)] as $\omega\tilde{\omega}_H \to 0$; then $z \to -\infty$ and for any (finite) width of the energy spectrum the oscillations of the function $\Phi(z)$ are averaged out by integration with respect to the energy.

6)Sazonov[5] cites in place of (26) a more stringent inequality, $W_e \gg W_H(E_0/m_0c^2)^4$. It was obtained from the condition $1-n^2 \gg (m_0c^2/E)^2$ under the assumption that the synchrotron instability sets in at frequencies $\omega \sim \omega_H(E/m_0c^2)^2$. The criterion (26) follows from the condition $1-n^2 \ll 1$ with allowance for the fact that the frequency of the unstable waves is $\omega \sim \Omega_0(\Omega_0\omega\tilde{\omega}_H)^{1/2}$. The condition $1-n^2 \gg (m_0c^2/E)^2$ for this frequency leads to the readily satisfied inequality $W_H/W_e \gg (m_0c^2/E)^4$.

of the magnetic field (say $W_e/W_H \sim 100$), when transverse-wave instability is present, the question of the existence of longitudinal waves with $v_{ph} \approx c$ is not quite clear.

So far we have referred throughout to a temporal gain. The spatial plasma-wave gain $-\mathrm{Im}\, k$, which is connected with the increment by the relation $\gamma = -v_{gr}\, \mathrm{Im}\, k$ (gain corresponds to $\gamma > 0$ and $\mathrm{Im}\, k < 0$), also exceeds the gain of the transverse waves, since v_{gr} of unstable plasma waves is much smaller (in a "cold" plasma) or is of the order of (in a relativistic plasma) the value of v_{gr} of electromagnetic waves, and $\gamma_l \gg \gamma_{tr}$.

The following should be noted when estimating the feasibility of using the coherent synchrotron mechanism of radiation of plasma and electromagnetic waves for the interpretation of radiation of cosmic sources. In the presence of a "cold" plasma and in a purely relativistic plasma (with the possible exception of not too large W_e/W_H), the plasma-wave synchrotron instability is realized at the same parameters of the medium as the transverse-wave instability. The increments of the plasma waves greatly exceed the increments of the transverse waves. However, the transverse waves can freely leave the radiation source, but the plasma waves can do so only after being transformed into transverse waves, with a coefficient that is as a rule much smaller than unity. The relative effectiveness of the two coherent mechanisms must therefore be estimated in each concrete case, with account taken of the difference between the bands of the generated frequencies. There is no doubt, however, that the nonlinear relaxation of the relativistic-particle distribution function in the absence of an effective stabilization mechanism is determined by the synchrotron instability on the plasma waves.

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