Stabilization of Drift Instabilities by a Magnetosonic Wave

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We consider the influence of a magnetosonic wave propagating in a plasma across an external constant field on the drift instabilities of the plasma. We show that the use of a magnetosonic wave that produces transverse high-frequency currents in the plasma makes it possible to suppress fully the drift and drift-temperature instabilities of the plasma. Criteria are obtained for the influence of the magnetosonic wave on the drift instability, and the described stabilization method is compared with those proposed earlier.

1. INFLUENCE OF MAGNETOSONIC WAVE ON LOW-FREQUENCY PLASMA INSTABILITY

VARIOUS low-frequency instabilities (for example drift, flute, and ion-cyclotron) can arise in a plasma placed in a magnetic field. It was shown recently that instabilities of this type can be stabilized by producing a high-frequency transverse displacement of the electrons and ions, since the perturbations of the potential in such instabilities are elongated along the constant magnetic field. The produced high-frequency transverse currents short-circuit the regions where the potential is perturbed and thereby stabilize the instabilities. High-frequency displacement of the charges was produced by oscillations of the magnetic force lines[1] or by a wave of the helicon type[2]. It is known that one of the methods of heating a plasma is to produce magnetosonic resonance[3]. It is therefore of interest to consider the effect of a magnetosonic wave on the indicated low-frequency instabilities. In the present paper, using the most dangerous instabilities as examples, we show that stabilization with a magnetosonic wave is more effective than the previously proposed methods.

The problem was solved for the frequency region $\omega_{Hi} < \Omega < (\omega_{Hi}\omega_{He})^{1/2}$. The constant magnetic field was directed along the z axis; it was assumed that all the plasma parameters depend only on x. A magnetosonic wave propagating along the x axis produces a transverse displacement of the electrons in the y direction, inasmuch the electron and ion concentrations in the plasma depend on y as a result of the presence of a drift wave in the plasma. If the potential resulting from the separation of the charges is sufficient to distort the drift-wave potential, the drift instability can become stabilized. Let us estimate the values of the amplitude and frequency of the magnetic field of the magnetosonic wave satisfying the condition that the magnetosonic wave be able to influence the drift instabilities.

The electric field of the magnetosonic wave is directed along the x axis, and therefore the maximum electron velocity at $\Omega > \omega_{Hi}$ is determined by the relation

$$\mathbf{V}_e = \mathbf{e}_y V_{ye}, \quad V_{ye} = \frac{c}{H_0} E_{x1} = \frac{H_1}{H_0}\frac{\Omega}{\omega_{Hi}}\frac{\Omega}{k_0},$$

where H_1, Ω, and k_0 are the amplitude, frequency, and wave number of the magnetosonic wave. The electrons in the drift waves have a Boltzmann distribution ($\omega \ll k_z v_{Te}$), and therefore the maximum number of electrons displaced by the magnetosonic wave is $n_0 e\Phi_{max}/T_e$, where Φ_{max} is the maximum potential of the perturbation in the drift wave. When displaced a distance $\delta = H_1\Omega/H_0\omega_{Hi}k_0$, they produce a potential

$$\Phi' = \frac{4\pi e^2 n_0 \Phi_{max}}{T_e \varepsilon(\Omega)}\delta^2, \tag{1}$$

where $\epsilon(\Omega)$ is the dielectric constant of the medium at the frequency Ω. The criterion for the influence of the magnetosonic wave on the drift instabilities can be obtained from the condition $\Phi' > \Phi_{max}$ and takes the form

$$\delta^2 > r_D^2 \varepsilon(\Omega),$$

where $r_D = (T_e/4\pi e^2 n_0)^{1/2}$ is the Debye radius for the electrons.

2. CALCULATION OF THE CORRECTIONS TO THE ELECTRON AND ION DISTRIBUTION FUNCTIONS

The equations of motion for the electrons and ions in the magnetosonic wave, under the condition that $\omega_{He} > \Omega > \omega_{Hi}$, yield the following expression for the electron and ion velocities, respectively:

$$\mathbf{V}_e = \frac{c}{H_0^2}[\mathbf{EH}], \quad \mathbf{V}_i = -\frac{e\mathbf{E}}{i\Omega M}. \tag{2*}$$

Using Maxwell's equations, we obtain

$$-\mathbf{k}(\mathbf{kE}) + k^2\mathbf{E} = -\frac{\omega_{pi}^2}{c^2}\mathbf{E} - i\frac{\omega_{pi}^2}{\omega_{Hi}}\frac{\Omega}{c^2}[\mathbf{Eh}] + \frac{\Omega^2}{c^2}\mathbf{E}. \tag{3}$$

The magnetosonic wave under consideration has a wave vector $k = \{k_0, 0, 0\}$; the components of the electric field are given by the relations

$$E_x = \frac{\Omega}{\omega_{Hi}}\frac{\Omega}{k_0 c} H_1 \sin(k_0 x - \Omega t), \tag{4}$$

$$E_y = \frac{\Omega}{k_0 c} H_1 \cos(k_0 x - \Omega t). \tag{5}$$

We assume here that

$$\mathbf{H}_1(t) = \mathbf{e}_z H_1 \cos(k_0 x - \Omega t).$$

Determining the trajectories of the charged particles in such fields, we obtain for electrons

$$x = -\frac{H_1}{H_0 k_0}\sin(k_0 x - \Omega t) + x_0,$$
$$y = -\frac{H_1\Omega}{H_0\omega_{Hi}k_0}\cos(k_0 x - \Omega t) + y_0,$$
$$z = V_{0z}t + z_0 \tag{6}$$

and for the ions

$$x = -\frac{H_1}{H_0 k_0}\sin(k_0 x - \Omega t) + x_0,$$
$$y = \frac{H_1\omega_{Hi}}{H_0\Omega k_0}\cos(k_0 x - \Omega t) + y_0,$$
$$z = V_{0z}t + z_0. \tag{7}$$

*[EH] ≡ E × H.

It is seen from (6) and (7) that it is necessary to take into account only the transverse displacement of the electrons, since the remaining displacements are small in comparison with them.

We consider low-frequency instabilities in a plasma $\omega \ll \omega_{Hi}$, and therefore describe the behavior of the electrons and the ions with the aid of a drift-kinetic equation in the form

$$\frac{\partial f_\alpha}{\partial t}+\dot{\mathbf{R}}\,\mathrm{grad}_r f_\alpha+\frac{\partial f_\alpha}{\partial p_\perp}\dot{p}_\perp+\frac{\partial f_\alpha}{\partial p_\parallel}\dot{p}_\parallel=0, \qquad (8)$$

where α = i or e. The equilibrium distribution function depends on the constants of the motion. For electrons these constants are

$$x_0=x+\frac{H_1}{H_0k_0}\sin(k_0x-\Omega t), \qquad V_{0z}=V_z.$$

Since we are considering the case $\Omega \ll \omega_{He}$, the quantity $\mu = p_\perp^2/2H_0$ will also be conserved. The second term in the expression for x_0 can be neglected, since $x \gg H_1/k_0H_0 \sim xH_1/H_0$. Thus, the equilibrium distribution function is

$$f_\alpha{}^0=f_\alpha{}^0(x_0,\ V_z,\ \mu).$$

The small correction to the equilibrium distribution function, corresponding to the low-frequency instabilities, satisfies the equation

$$f_\alpha^{(1)}(t)=i\int_{-\infty}^{\infty}\left(\frac{e_\alpha}{m_\alpha}k_z\frac{\partial f_\alpha{}^0}{\partial V_z}+\frac{c}{H_0}k_y\frac{\partial f_\alpha{}^0}{\partial x_0}\right)\Phi(t')\,dt', \qquad (9)$$

where $\Phi(t')$ is the potential of the perturbations in the drift wave. Replacing f and Φ in (9) by their Floquet expansions

$$f=\sum_n\int_{-\infty}^{\infty}f_n\exp[-i\omega t+i\mathbf{kr}+i(k_0x+\Omega t)n]\,d\mathbf{k},$$

$$\Phi=\sum_m\int_{-\infty}^{\infty}\Phi_m\exp[-i\omega t+i\mathbf{kr}+i(k_0x+\Omega t)m]\,d\mathbf{k},$$

we obtain

$$f_{ne}^{(1)}=\left(\frac{e}{m}k_z\frac{\partial f_e{}^0}{\partial V_z}-\frac{c}{H_0}k_y\frac{\partial f_e{}^0}{\partial x_0}\right)\sum_{p,s}\frac{\Phi_pJ_s(a)J_{n+s-p}(a)}{(p-s)\Omega+\omega-k_zV_z}, \qquad (10)$$

$$f_{ni}^{(1)}=-\left(\frac{e}{M}k_z\frac{\partial f_i{}^0}{\partial V_z}+\frac{c}{H_0}k_y\frac{\partial f_i{}^0}{\partial x_0}\right)\frac{1}{\omega-k_zV_z+n\Omega}, \qquad (11)$$

where $a = k_yH_1\Omega/k_0H_0\omega_{Hi}$. Using the Poisson equation

$$\mathrm{div}\,\mathbf{E}=4\pi e(n_i-n_e), \qquad (12)$$

we obtain an infinite system of equations relative to Φ_n:

$$-\Phi_n=\delta\varepsilon_i(\omega+n\Omega)\Phi_n+\sum_{p,s}\delta\varepsilon_e[\omega+(p-s)\Omega]\Phi_sJ_pJ_{s+n-p}, \qquad (13)$$

where $\delta\epsilon_i$ and $\delta\epsilon_e$ are the partial contributions made to the dielectric constant by the ions and electrons, respectively.

Since $\omega < \omega_{Hi}$, the expressions for $\delta\epsilon_i$ and $\delta\epsilon_e$ in the frequency region ($\Omega \gg \omega$, k_zV_{Te}; $\omega_{Hi} < \Omega < (\omega_{Hi}\omega_{He})^{1/2}$ are

$$\delta\varepsilon_i(\omega)=\frac{4\pi e}{k^2}\int_{-\infty}^{\infty}\left(-\frac{e}{M}k_z\frac{\partial f_i{}^0}{\partial V_z}-\frac{c}{H_0}k_y\frac{\partial f_i{}^0}{\partial x_0}\right)\frac{dV_z}{\omega-k_zV_z}, \qquad (14)$$

$$\delta\varepsilon_e(\omega+(p-s)\Omega)=\sum_n\frac{4\pi e}{k^2}\int_{-\infty}^{\infty}\left(\frac{e}{m}k_z\frac{\partial f_e{}^0}{\partial V_z}-\frac{c}{H_0}k_y\frac{\partial f_e{}^0}{\partial x_0}\right)$$
$$\times I_n(Z_e)\exp(-Z_e)\frac{dV_z}{\omega-k_zV_z+(p-s)\Omega+n\omega_{He}}, \qquad (15)$$

where $Z_e = k_\perp^2\rho_{Le}^2/2$, ρ_{Le} is the Larmor radius of the electrons, and I is a Bessel function of imaginary argument. To determine $\delta\epsilon_i(\omega + n\Omega)$ at $n \neq 0$ we can no longer use the drift approximation, since the ions are not magnetized, by virtue of the condition $\Omega > \omega_{Hi}$. To determine the contribution made to the dielectric constant by the ions at the magnetic-sound frequency, we must use the formula

$$\delta\varepsilon_i(\omega+n\Omega)=\frac{4\pi e}{Mk^2}\int_{-\infty}^{\infty}\mathbf{k}\frac{\partial f_i{}^0}{\partial\mathbf{V}}\frac{d\mathbf{V}}{\omega-\mathbf{kV}+n\Omega}, \qquad n\neq 0. \qquad (16)$$

Multiplying both halves of the system (13) by J_{n-q} and summing over n, we obtain

$$-\Phi_n=\delta\varepsilon_i(\omega+n\Omega)\Phi_n+\sum_{q,r}\frac{\delta\varepsilon_e(\omega+q\Omega)}{1-\delta\varepsilon_e(\omega+q\Omega)}J_{n-q}[\delta\varepsilon_i(\omega)J_{-q}J_0\Phi_n$$
$$+\delta\varepsilon_i(\omega+r\Omega)\Phi_rJ_{r-q}+\delta\varepsilon_i(\omega-r\Omega)\Phi_{-r}J_{-r-q}]. \qquad (17)$$

When solving this system for the frequency region under consideration and under the condition $kr_D \ll 1$, we take into account only the harmonics with frequencies $\omega + n\Omega$ with n = 0 and ± 1, and take the expansion of the Bessel functions up to terms of second order. From the conditions that the system (17) have a solution under these conditions, we obtain the following dispersion relation (see the Appendix):

$$\frac{\varepsilon(\omega)}{[1+\delta\varepsilon_e(\omega)]\delta\varepsilon_i(\omega)}+\frac{1}{2}a^2\varphi(\Omega)=0; \qquad (18)$$

$$\varepsilon(\omega)=1+\delta\varepsilon_e(\omega)+\delta\varepsilon_i(\omega), \qquad \varphi(\Omega)=\frac{1}{2}\left[\frac{1}{\varepsilon(\omega+\Omega)}+\frac{1}{\varepsilon(\omega-\Omega)}\right]$$

where $a = H_1\Omega k_y/H_0\omega_{Hi}k_0$ is the argument of the Bessel functions.

3. UNIVERSAL DRIFT INSTABILITY

We consider the case of the universal drift instability[4], when

$$k_zV_{Ti}<\omega<k_zV_{Te}, \qquad n_0=n_0(x),$$
$$T_e=T_e(x), \qquad T_i=T_i(x), \qquad kr_D\ll 1.$$

Calculating $\delta\epsilon_i$ and $\delta\epsilon_e$ under these conditions and substituting the obtained relations in (18), we obtain the following dispersion equation for the universal drift instability

$$-\frac{\omega^*}{\omega}+1+\frac{i\pi^{1/2}}{|k_z|V_{Te}}\left[\omega-\omega^*\left(1-\frac{1}{2}\eta\right)\right]$$
$$-\frac{\omega^*}{\omega}\frac{a^2}{2}\frac{\varphi(\Omega)}{k^2r_D{}^2}\left[1+\frac{i\pi^{1/2}}{|k_z|V_{Te}}\left\{\omega-\omega^*\left(1-\frac{1}{2}\eta\right)\right\}\right]=0,$$
$$\omega^*=\frac{cT}{eH_0}k_y\frac{d\ln n_0}{dx}, \qquad \eta=\frac{d\ln T}{d\ln n_0}. \qquad (19)$$

Let us investigate the obtained dispersion equation. In the absence of a stabilizing field, i.e., when $H_1 = 0$, the argument of the Bessel function is a = 0, $J_0^2 = 1$, and the dispersion equation has the usual form

$$\omega=\omega^*-\frac{i\pi^{1/2}}{2|k_z|V_{Te}}\omega^{*2}\eta,$$

i.e., the instability takes place when $\eta < 0$. When the amplitude of the stabilizing field is sufficiently large, complete stabilization of the universal drift stability is observed. The stabilization threshold can be determined by assuming that ω is real (Im ω = 0). Equating separately the real and imaginary parts of (19) to zero, we obtain the threshold values ω_{thr} and $k_{z.thr}$. Simple calculations show that the universal drift instability becomes stabilized when

$$a^2 > k^2 r_D^2 |\eta| / \varphi(\Omega). \tag{20}$$

This result coincides formally with that obtained by Fainberg and Shapiro[5]. It will be shown below, however, that the criterion obtained in the present paper is much less stringent.

4. DRIFT-TEMPERATURE INSTABILITY

We consider one more important case of drift-temperature instability, which leads to an anomalously large escape of heat in a direction perpendicular to the magnetic field. As is well known, this instability is due to Landau damping by the ions at $\eta = d \ln T_i / d \ln n_0 > 2$. The spectrum of the unstable oscillations turns out in this case to lie in the frequency region $k_z V_{Te} > \Omega \gtrsim k_z V_{Ti}$. Expressions for $\delta\epsilon_i$ and $\delta\epsilon_e$ of instabilities of this type can be obtained by recognizing that the residue on the electrons is negligible, and therefore in the limit $\Omega \ll k_z V_{Te}$ the electrons in the field of the wave have a Botlzmann distribution $n_e = n_0 e\Phi/T_e$, and consequently

$$\delta\varepsilon_e = -4\pi e^2 n_0 / T_e k^2. \tag{21}$$

To determine $\delta\epsilon_i$, we use the drift-kinetic equation (8), from which it follows that

$$\begin{gathered}\frac{n_i}{n_0} = \int \frac{\omega^*(1 - {}^1/_2\eta + MV_z^2\eta/2T_i) - k_z V_z}{-\omega + k_z V_z} \frac{f_i^0}{n_0} dV_z \\ + i\pi \frac{f_i^0(\omega/k_z)}{n_0|k_z|}\left[\omega^*\left(1 - \frac{1}{2}\eta + M\left(\frac{\omega}{k_z}\right)^2 \frac{\eta}{2T_i}\right) - \omega\right], \\ \delta\varepsilon_i = \frac{4\pi e}{k^2} n_i.\end{gathered} \tag{22}$$

Substituting the obtained expressions for $\delta\epsilon_e$ and $\delta\epsilon_i$ in (18), we obtain the following dispersion relation:

$$\begin{gathered}1 = -\frac{1}{k^2 r_D^2} + A\frac{1}{k^2 r_D^2}\left\{\int \frac{\omega^*(1 - {}^1/_2\eta + MV_z^2\eta/2T_i) - k_z V_z}{-\omega + k_z V_z} \frac{f_i^0}{n_0} dV_z \right. \\ \left. + i\pi \frac{f_i^0(\omega/k_z)}{n_0|k_z|}\left[\omega^*\left(1 - \frac{1}{2}\eta + \frac{\omega^2}{k_z^2 V_{Ti}^2}\right) - \omega\right]\right\}, \\ A = 1 + a^2\varphi(\Omega)/2k^2 r_D^2.\end{gathered} \tag{23}$$

Equating the real and imaginary parts to zero separately, we obtain the instability criterion with respect to η and the values of $k_{z.\text{thr}}$ and ω_{thr} at the instability boundary

$$\omega_{\text{thr}} = \frac{k_z^2 V_{Ti}^2}{\eta\omega_i^*}\left(1 + \frac{B}{A}\right), \tag{24}$$

$$\begin{gathered}k_{z.\text{thr}}^2 = \frac{A}{B}\frac{(\eta/2 - 1)}{(1 + B/A)}\eta\frac{\omega_i^{*2}}{V_{Ti}^2}, \\ B = k^2 r_{Di}^2 (1 + 1/k^2 r_{De}^2).\end{gathered} \tag{25}$$

The instability condition is $k_z^2 > 0$. If $H = 0$, then $A = 1$ and $B = k^2 r_{Di}^2 + T_i/T_e \cong 1$. Therefore the instability condition is satisfied either for $\eta > 2$ or for $\eta < 0$. We note that $\omega/k_z V_{Ti} \sim A$, and consequently the phase velocity can be large in comparison with V_{Ti} when $A \gg 1$.

The dispersion relation (23) was obtained under the assumption that the electronic residue is small compared with the ionic one. It can be shown that this assumption is valid if the following inequality is satisfied:

$$\begin{gathered}\frac{1}{5A^2}\left(\frac{m}{M}\frac{T_i^3}{T_e^3}\right)^{1/2}\left\{\left[\frac{A^2}{2B(A+B)}\right]^{1/2} - \frac{T_e}{T_i}\left(\frac{B}{A+B}\right)^{1/2}\right\} \\ \times \exp\left\{(A+B)\frac{\eta - 2}{2B\eta}\right\} < \frac{A+B}{2B}\left(\frac{\eta-2}{\eta}\right)^{1/2}.\end{gathered} \tag{26}$$

When A increases, the left-hand side of the inequality can exceed the right-hand side, and the inequality is violated. This means that at large A the drift-temperature instability goes over into the universal drift instability, which, as shown above, is completely suppressed when $A > 1$. Estimating the value of A at which the inequality (26) is violated, we obtain a criterion for the stabilization of the drift-temperature instability:

$$\frac{H_1}{H_0} \geqslant 5\frac{V_{Ti}}{c}\varphi^{-1/2}(\Omega). \tag{27}$$

5. DISCUSSION OF RESULTS

We have shown that if the displacement of the electrons across the constant magnetic field is several times larger than $r_D\varphi^{-1/2}(\Omega)$, then the drift instabilities can be stabilized. We have assumed here that $\varphi(\Omega) > 0$. The value of $\varphi(\Omega)$ can be easily obtained with the aid of expressions (14)—(16) for the partial contributions of the electrons and ions to the dielectric constant. Neglecting ω and $k_z V_{Te}$ in comparison with the frequency of the magnetosonic wave, we obtain

$$\varepsilon(\Omega) = 1 + \frac{\omega_{pe}^2}{\omega_{He}^2}\frac{k_\perp^2}{k^2} - \frac{\omega_{pe}^2}{\Omega\omega_{He}}\frac{k_y\varkappa}{k^2} - \frac{\omega_{pe}^2}{\Omega^2}\frac{k_z^2}{k^2} - \frac{\omega_{pi}^2}{\Omega^2}, \tag{28}$$

where $k_\perp$ is the wave-vector component in the direction perpendicular to the external magnetic field and $\kappa = |d \ln n/dx|$. Thus, $\varphi(\Omega)$ takes the form[5]:

$$\varphi(\Omega) = \frac{\Omega^2[N\Omega^2 - \omega_{pe}^2 k_z^2/k^2 - \omega_{pi}^2]}{[N\Omega^2 - \omega_{pe}^2 k_z^2/k^2 - \omega_{pi}^2]^2 - \Omega^2(\omega_{pe}^4/\omega_{He}^2)(k_y^2\varkappa^2/k^4)} \tag{29}$$

where $N = 1 + \omega_{pe}^2 k_\perp^2/\omega_{He}^2 k^2$. It follows from (29) that $\varphi(\Omega) > 0$ for the frequency region[5]

$$\begin{gathered}\Omega_- < \Omega < N^{-1/2}(\omega_{pe}^2 k_z^2/k^2 + \omega_{pi}^2)^{1/2}, \\ \Omega_-^2 = \frac{1}{N^2}\left[N\left(\omega_{pe}^2\frac{k_z^2}{k^2} + \omega_{pi}^2\right) + \frac{\omega_{pe}^4}{2\omega_{He}^2}\frac{k_y^2\varkappa^2}{k^4} \right. \\ \left. - \left\{\left[N\left(\omega_{pe}^2\frac{k_z^2}{k^2} + \omega_{pi}^2\right) + \frac{\omega_{pe}^4}{2\omega_{He}^2}\frac{k_y^2\varkappa^2}{k^4}\right]^2 - N^2\left(\omega_{pe}^2\frac{k_z^2}{k^2} + \omega_{pi}^2\right)^2\right\}^{1/2}\right].\end{gathered} \tag{30}$$

In the case under consideration, stabilization takes place when the condition $a \gtrsim kr_D\varphi^{-1/2}(\Omega)$ is satisfied. Let us compare this criterion with the stabilization condition obtained by Faĭnberg and Shapiro[5], who obtained stabilization with the aid of a high-frequency electric field directed along the constant magnetic field. The criteria coincide if the quantity a in our expression is replaced by

$$a^* = \frac{\tilde{u}}{\Omega}k_z = k_z\frac{c}{\Omega}\frac{H_1}{H_0}\frac{\omega_{He}}{\omega_{pe}}\frac{c}{\omega_{pe}\delta}$$

where δ is the thickness of the skin layer, which generally speaking is larger than c/ω_{pe} and can reach a value c/ω_{pi}[6]. The thickness of the skin layer can increase as a result of the onset of ion-acoustic instability, since the oscillatory velocity is larger than the thermal velocity of the electrons, $\tilde{u} \geq V_{Te}$[5]. We note that such an increase is necessary, for otherwise the stabilization occurs, in accordance with[5], only in a very narrow layer c/ω_{pe}. For the same values of the frequencies and amplitudes of the high-frequency fields we have

$$\frac{a}{a^*} = \left(\frac{\omega_{pe}\delta}{c}\right)\frac{k}{k_z}\frac{\Omega}{(\omega_{He}\omega_{Hi})^{1/2}}. \tag{31}$$

This ratio is much larger than unity. In addition, as already noted, the magnetic sound is not concentrated in the skin layer as in the case of[5], and stabilizes the instability in the entire volume.

It would also be of interest to compare the obtained criterion with the condition obtained by Rudakov and one of the authors[1] for the stabilization of drift instabilities by a high-frequency magnetic field. For example, for the universal instability, the criterion $k_y V_{Te} H_1/\Omega H_0 \gg 1$ under the condition $k_y H_1/k_z H_0 \gtrsim B \gg 1$, and the increment is decreased in this case by a factor B. The ratio of the magnetic-sound amplitude to the amplitude of the stabilizing high-frequency magnetic field H_1^* can be written in the form

$$\frac{H_1}{H_1^*} = \frac{V_{Te}}{c}\left(\frac{m}{M}\right)^{1/2}\frac{k_y}{k_z}B^{-1}\varphi^{-1/2}(\Omega). \qquad (32)$$

As shown by Faĭnberg and Shapiro[5], by a suitable choice of the frequency it is possible to make $\varphi(\Omega)$ larger than unity or of the order of unity when the frequency Ω is approached. Thus, even for the case $\varphi^{1/2}(\Omega) \sim B^{-1}$, if we recognize that $k_y/k_z \lesssim (M/m)^{1/2}$ we find that the amplitude ratio is $H_1/H_1^* \ll 1$. It should be noted that the instability is completely suppressed in this case. The corresponding estimates made for the drift-temperature instability at the same temperature of the electrons and ions show that this ratio decreases by another factor $(M/m)^{1/2}$.

APPENDIX

By virtue of the assumptions $\Omega \gg \omega, k_z V_{Te}$; $k^2 r_D^2 \ll 1$, the infinite system of equations (13) reduces to a system of three equations for n = 0 and ±1, in the form

$$\Phi_0\left[-1-\delta\varepsilon_i(\omega)+\delta\varepsilon_i(\omega)\sum_q\frac{\delta\varepsilon_e(\omega-q\Omega)}{1+\delta\varepsilon_e(\omega-q\Omega)}J_q^2\right] + \Phi_1\delta\varepsilon_i(\omega-\Omega)\sum_q\frac{\delta\varepsilon_e(\omega-q\Omega)}{1+\delta\varepsilon_e(\omega-q\Omega)}J_qJ_{q-1} + \Phi_{-1}\delta\varepsilon_i(\omega+\Omega)\sum_q\frac{\delta\varepsilon_e(\omega-q\Omega)}{1+\delta\varepsilon_e(\omega-q\Omega)}J_qJ_{q+1}=0, \qquad (A.1)$$

$$\Phi_0\delta\varepsilon_i(\omega)\sum_q\frac{\delta\varepsilon_e(\omega-q\Omega)}{1+\delta\varepsilon_e(\omega-q\Omega)}J_{q-1}J_q+\Phi_1\left[-1-\delta\varepsilon_i(\omega-\Omega)\times\sum_q\frac{\delta\varepsilon_e(\omega-q\Omega)}{1+\delta\varepsilon_e(\omega-q\Omega)}J_{q-1}^2\right]+\Phi_{-1}\delta\varepsilon_i(\omega+\Omega)\times\sum_q\frac{\delta\varepsilon_e(\omega-q\Omega)}{1+\delta\varepsilon_e(\omega-q\Omega)}J_{q+1}J_{q-1}=0, \qquad (A.2)$$

$$\Phi_0\delta\varepsilon_i(\omega)\sum_q\frac{\delta\varepsilon_e(\omega-q\Omega)}{1+\delta\varepsilon_e(\omega-q\Omega)}J_qJ_{q+1}+\Phi_1\delta\varepsilon_i(\omega-\Omega)\times\sum_q\frac{\delta\varepsilon_e(\omega-q\Omega)}{1+\delta\varepsilon_e(\omega-q\Omega)}J_{q+1}J_{q-1}+\Phi_{-1}\left[-1-\delta\varepsilon_i(\omega+\Omega)+\delta\varepsilon_i(\omega+\Omega)\sum_q\frac{\delta\varepsilon_e(\omega-q\Omega)}{1+\delta\varepsilon_e(\omega-q\Omega)}J_{q+1}^2\right]=0. \qquad (A.3)$$

To solve this system of equations, we use the condition $\delta\epsilon_e(\omega) \gg 1$, and also the expansion of the Bessel functions, since we are interested in small values of the argument. Then the system (A.1)—(A.3) can be written as follows:

$$\begin{aligned}A_{11}\Phi_1+A_{10}\Phi_0+A_{1,-1}\Phi_{-1}&=0,\\ A_{01}\Phi_1+A_{00}\Phi_0+A_{0,-1}\Phi_{-1}&=0,\\ A_{-1,1}\Phi_1+A_{-1,0}\Phi_0+A_{-1,-1}\Phi_{-1}&=0,\end{aligned} \qquad (A.4)$$

where the coefficients $A_{i,j}$ are determined from the equations (A.1)—(A.3). From the condition for the solvability of the system (A.4)

$$A_{11}A_{00}A_{-1,-1}-A_{01}A_{10}A_{-1,-1}-A_{-1,0}A_{0,-1}A_{11}=0$$

we obtain the dispersion relation (18). This equation is accurate to $a^2/2$. It follows from the stabilization criteria (20) and (27) obtained by us for the universal-drift and drift-temperature instabilities that $a < 1$, and consequently the chosen accuracy with which we solved the system (13) is sufficient.

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