Light-Electric Effect Near the Plasma Frequency

L. E. Gurevich and O. A. Mezrin
A. F. Ioffe Physico-technical Institute, USSR Academy of Sciences
Submitted November 29, 1971; Resubmitted January 29, 1972
Zh. Eksp. Teor. Fiz. 62, 2255–2264 (June, 1972)

The light-electric field created by electromagnetic wave pressure on the conduction electrons of a medium is investigated for the case when the plasma frequency ω_0 exceeds the collision frequency. It is found that if the wave frequency ω is such that $\omega < 2^{1/2}\omega_0$, the light-electric field is much stronger than at higher frequencies. In this case it increases with decreasing frequency until the latter becomes lower than the plasma frequency; the field becomes independent of the frequency at lower frequencies.

KHAĬKIN and Yakubovskiĭ [1] have measured the potential difference produced in a bismuth crystal irradiated by a monochromatic wave of frequency $\omega = 2\pi \times 21.5$ GHz $\approx 1.35 \times 10^{11}$ sec^{-1}. They have observed that the potential difference exceeds by several orders the value predicted in[2] for the case of the high frequency "light-electric" effect, as this phenomenon was named. A comparison of the experiment with the theoretical prediction, however, is not valid in this case, since the experimental conditions differ greatly from those under which the theory is valid. First, the theory of the high-frequency light-electric effect, when $\omega\tau > 1$ (τ is the relaxation time) is valid only when $\omega \gg \omega_0$ ($\omega_0 = (4\pi e^2 N/m)^{1/2}$ is the plasma-oscillation frequency and N is the free-carrier density); second, the theory[2] deals with the case of normal skin effect, when the wave damping depth exceeds the carrier mean free path l and their mean free path $l = \bar{v}/\omega$ per period of the wave ($\bar{v}$ is the average carrier velocity), whereas under the experimental condition the wave damping depth greatly exceeds the mean free path; third, finally, the theory did not deal with a quantizing external magnetic field, since a field of 1 kOe in bismuth is quantizing at helium temperature. Deferring the general question of the theory of the light-electric field in the presence of a strong magnetic field (and particularly a quantizing one) to a separate article, we shall show in the present paper that the first of the indicated circumstances enhances the light-electric field in comparison with the predictions of[2,3] by several orders of magnitude. We consider both the normal and the anomalous skin effect. For the latter we assume satisfaction of the condition

$$\nu < \omega_0\bar{v}/c < \omega < \omega_0, \tag{1}$$

where $\bar{v}$ is the average carrier velocity and $\nu = 1/\tau$ is the collision frequency.

Electromagnetic waves propagating in a conducting medium transfer their momentum to free carriers and produce, when the circuit is open, a constant light-electric field. The case of waves of high frequency, $\omega > 1/\tau$, was investigated also in[3], where it was shown that for waves propagating with a velocity on the order of c (c is the speed of light), spatial dispersion produces an additional contribution to the light-electric field. This contribution differs from that obtained earlier[2] by a numerical factor on the order of unity. It turns out, however, that interesting features of the light-electric field arise when the plasma frequency ω_0 also exceeds the collision frequency $\nu = 1/\tau$. We shall show that in this case when

$$\tau^{-1} < \omega < \omega_0(2/\varepsilon_0)^{1/2}$$

the light-electric field E increases when the frequency ω decreases to $\omega_0/\epsilon_0^{1/2}$, and at frequencies satisfying the inequality

$$\omega_0^2/\omega^2 - \varepsilon_0 \lesssim \omega_0^2/\omega^3\tau,$$

it ceases to depend on the frequency. In this latter range the field is much stronger than previously obtained[2,3] (by an approximate factor $\omega^2\tau^2 \gg 1$). We confine ourselves to the case $\hbar\omega \ll \bar{\mathscr{E}}$, where $\bar{\mathscr{E}}$ is the average energy of the free carriers, and for semiconductors also to the case $\hbar\omega \ll \mathscr{E}_0$, where $\mathscr{E}_0$ is the width of the forbidden band.

1. NORMAL SKIN EFFECT

In a conducting medium, the magnetic field of a plane wave always lags in phase the electric field. We denote the electric and magnetic field of the wave by $\mathbf{E}_1$ and $\mathbf{H}_1$, $\mathbf{E}_1 \sim e^{i\mathbf{kr}-i\omega t}$, $\mathbf{H}_1 \sim e^{i\mathbf{kr}-i\omega t+i\varphi}$, where φ is the phase difference between $\mathbf{E}_1$ and $\mathbf{H}_1$. In the approximation linear in these fields, the correction f_1 to the equilibrium carrier distribution function f_0 satisfies the equation

$$\frac{\partial f_1}{\partial t} - \left[\frac{\partial f_1}{\partial t}\right]_{st} + \mathbf{v}\nabla f_1 = -\frac{e}{m}\mathbf{E}_1\frac{\partial f_0}{\partial \mathbf{v}}, \tag{2}$$

the solution of which is (at $k\bar{v} < \omega$)

$$f_1 = -\frac{\partial f_0}{\partial \mathscr{E}}\frac{e\tau}{1-i\omega\tau}\left\{1 - i\frac{\tau_1}{1-i\omega\tau_1}(\mathbf{kv})\right\}\mathbf{E}_1\mathbf{v}.$$

The relaxation times $\tau(\mathscr{E})$ and $\tau_1(\mathscr{E})$ are connected with the "transport" cross section and with the "deviation" cross section[4] by

$$\tau^{-1} = \int W(\mathbf{p},\mathbf{p}_1)(1-\cos\theta)\,d\Omega, \qquad \tau_1^{-1} = {}^3/_2\int W(\mathbf{p},\mathbf{p}_1)\sin^2\theta\,d\Omega;$$

these times coincide for scattering mechanisms in which the transition probability $W(\mathbf{p}, \mathbf{p}_1)$ does not depend on the angle between the quasimomenta $\mathbf{p}$ and $\mathbf{p}_1$ of the free carriers before and after the scattering (this is precisely the case for scattering by acoustic and nonpolar optical phonons). In a conducting medium the vector k is in general complex. At $\omega\tau > 1$ and $\mathbf{k}' \parallel \mathbf{k}''$ we have

$$k = k' + ik'' = \frac{\omega}{c}\left[\left(\varepsilon_0 - \frac{\omega_0^2}{\omega^2}\right) + i\frac{\omega_0^2\langle\tau^{-1}\rangle}{\omega^3}\right]^{1/2}$$

$$\langle\tau^{-1}\rangle = \int_0^\infty \tau^{-1}\frac{\partial f_0}{\partial \mathscr{E}}d\mathscr{E}\Big/\int_0^\infty\frac{\partial f_0}{\partial \mathscr{E}}d\mathscr{E},$$

where ϵ_0 is the dielectric constant at $\omega \gg \omega_0$) (we are considering "generalized" plane waves). Separating the real and imaginary parts, we have

$$k',''=\frac{\omega}{c}\left\{\frac{1}{2}\left[\pm\left(\varepsilon_0-\frac{\omega_0^2}{\omega^2}\right)+\left\{\left(\varepsilon_0-\frac{\omega_0^2}{\omega^2}\right)^2+\left(\frac{\omega_0^2\langle\tau^{-1}\rangle}{\omega^3}\right)^2\right\}^{1/2}\right]\right\}^{1/2}. \tag{3}$$

The distribution-function increment that is quadratic in the wave fields and independent of the time, satisfies the equation

$$-\left[\frac{\partial f_2}{\partial t}\right]_{st}+\mathbf{v}\nabla f_2=-\frac{e}{m}\mathbf{E}\frac{\partial f_0}{\partial \mathbf{v}}-\frac{1}{2}\frac{e}{m}\operatorname{Re}\left\{\left(\mathbf{E}_1^*+\frac{1}{c}[\mathbf{v}\mathbf{H}_1^*]\right)\frac{\partial f_1}{\partial \mathbf{v}}\right\}, \tag{4*}$$

the solution of which (for a short-circuited sample) is

$$f_2=-\frac{1}{2}\frac{e^2\tau}{m}\left[\frac{1}{c}\frac{\partial f_0}{\partial \mathscr{E}}\frac{\tau(1+\omega\tau\operatorname{tg}\varphi)}{1+\omega^2\tau^2}\operatorname{Re}[\mathbf{E}_1\mathbf{H}_1^*]\mathbf{v}\right.$$
$$-\frac{\partial f_0}{\partial \mathscr{E}}\frac{\tau\tau_1[\omega(\tau+\tau_1)-\operatorname{tg}\varphi(\omega^2\tau\tau_1-1)]}{(1+\omega^2\tau^2)(1+\omega^2\tau_1^2)}(\mathbf{E}_1\mathbf{E}_1^*)(\mathbf{k}'\mathbf{v})$$
$$-m\frac{\tau_2}{\tau}\frac{\partial}{\partial \mathscr{E}}\left\{\frac{\partial f_0}{\partial \mathscr{E}}\frac{\tau\tau_1[\omega(\tau+\tau_1)-\operatorname{tg}\varphi(\omega^2\tau\tau_1-1)]}{(1+\omega^2\tau^2)(1+\omega^2\tau_1^2)}\right\}$$
$$\left.\times\left\{(\mathbf{E}_1\mathbf{v})(\mathbf{E}_1^*\mathbf{v})+\frac{\tau}{\tau_3}v^2(\mathbf{E}_1\mathbf{E}_1^*)\right\}(\mathbf{k}\mathbf{v})\right],$$

where the relaxation times τ_2 and τ_3

$$\tau_2^{-1}=\int W(\mathbf{p},\mathbf{p}_1)\left(1+{}^3/_2\cos\theta-{}^5/_2\cos^3\theta\right)d\Omega,$$
$$\tau_3^{-1}={}^1/_2\int W(\mathbf{p},\mathbf{p}_1)\cos\theta\sin^2\theta\,d\Omega,$$

are given in the Appendix for several scattering mechanisms. The asterisk denotes the complex conjugate, and $\tan\varphi=k''/k'$. The current density of the constant light-electric field is therefore $\mathbf{j}=\chi\mathbf{I}$, where

$$\chi=-A\int_0^\infty\frac{\tau^2}{1+\omega^2\tau^2}\frac{\partial f_0}{\partial \mathscr{E}}\left\{(1+\omega\tau\operatorname{tg}\varphi)+\frac{2}{5}\frac{\omega\tau_1}{1+\omega^2\tau_1^2}\right.$$
$$\left.\times B(\mathscr{E})[\omega(\tau+\tau_1)+(1-\omega^2\tau\tau_1)\operatorname{tg}\varphi]\right\}\mathscr{E}^{3/2}d\mathscr{E}, \tag{5}$$

$$B(\mathscr{E})=\frac{5}{2}\left[\frac{\tau_2}{\tau}\left(1+5\frac{\tau}{\tau_3}\right)\left(1+\frac{2}{5}\frac{\partial\ln[\tau_2(1+5\tau/\tau_3)]}{\partial\ln\mathscr{E}}\right)-1\right]$$

(at $\tau_2=\tau$ and $\tau_3^{-1}=0$, $B=\partial\ln\tau/\partial\ln\mathscr{E}$), $\mathscr{E}$ is the carrier energy,

$$A=\frac{8\cdot 2^{1/2}e^3}{3\pi\hbar^3m^{1/2}c^2},\quad \mathbf{I}=\mathbf{I}(0)\exp(-2k''z),$$

$\mathbf{I}$ is the energy flux density in the medium and depends on the coordinate z in the direction of wave propagation, and $I(0)$ is the same quantity for $z\to+0$. In the case of an open circuit, there is no current and a constant light-electric field is produced

$$\mathbf{E}=\gamma\mathbf{I}=-\chi\mathbf{I}/\sigma,$$

where σ is the static conductivity. The wave reflected from the real surface of the crystal can be disregarded if the crystal thickness exceeds the damping length of the wave.

At $\omega\tau\gg 1$, the coefficient χ is equal to

$$\chi=-\frac{A}{\omega^2}\int_0^\infty\frac{\partial f_0}{\partial \mathscr{E}_0}\left\{1+\omega\tau\operatorname{tg}\varphi+\frac{2}{5}B(\mathscr{E})\left[\frac{\tau+\tau_1}{\tau_1}-\omega\tau\operatorname{tg}\varphi\right]\right\}\mathscr{E}^{3/2}d\mathscr{E}, \tag{6}$$

where $\tan\varphi$ is given by (3). The phase shift depends on the ratio ω_0/ω in the high-frequency case.

We consider a number of limiting cases:

1. $\omega\gg\omega_0(2/\epsilon_0)^{1/2}$. Then $\tan\varphi\ll\langle\tau^{-1}\rangle/\omega$ and we can neglect the corresponding terms in (6). This case was in fact considered in[3], in which case χ is equal to

$$\chi^{(1)}=-\frac{A}{\omega^2}\int_0^\infty\frac{\partial f_0}{\partial \mathscr{E}}\left[1+\frac{2}{5}\frac{\tau+\tau_1}{\tau_1}B(\mathscr{E})\right]\mathscr{E}^{3/2}d\mathscr{E}\sim\frac{1}{\omega^2}, \tag{7}$$

and the condition for the normal skin effect

$$\varepsilon_0^{-1/2}\frac{\omega_0}{c}\frac{\omega_0}{\omega}\frac{\langle\tau^{-1}\rangle}{\omega}l\approx\varepsilon_0^{-1/2}\frac{\bar v}{c}\left(\frac{\omega_0}{\omega}\right)^2<1 \tag{8}$$

is satisfied in this region.

2. $\omega_0^2\langle\tau^{-1}\rangle/\omega^3<\epsilon_0-\omega_0^2/\omega^2\le{}^1/_2$. Here

$$\operatorname{tg}\varphi=\frac{1}{2}\frac{\omega_0^2\langle\tau^{-1}\rangle}{\omega^3}\left[\varepsilon_0-\frac{\omega_0^2}{\omega^2}\right]^{-1},$$

$$\chi^{(2)}=-\frac{1}{2}\frac{A}{\omega^2}\frac{\omega_0^2\langle\tau^{-1}\rangle}{\omega^2}\left[\varepsilon_0-\frac{\omega_0^2}{\omega^2}\right]^{-1}\int_0^\infty\tau\frac{\partial f_0}{\partial \mathscr{E}}\left[1-\frac{2}{5}B(\mathscr{E})\right]\mathscr{E}^{3/2}d\mathscr{E}. \tag{9}$$

The normal skin-effect condition

$$\frac{1}{2}\frac{\langle\tau^{-1}\rangle}{c}\frac{\omega_0^2}{\omega^2}\left(\varepsilon_0-\frac{\omega_0^2}{\omega^2}\right)^{-1/2}l\approx\frac{1}{2}\frac{\bar v}{c}\frac{\omega_0^2}{\omega^2}\left(\varepsilon_0-\frac{\omega_0^2}{\omega^2}\right)^{-1/2}<1$$

is also satisfied in this region. The coefficient $\chi^{(2)}$ is of the order of $\omega\tau\chi^{(1)}$ so long as ω satisfies (8), but is less than $\omega_0(2/\epsilon_0)^{1/2}$; when $\omega\approx\omega_0(2/\epsilon_0)^{1/2}$ is reached we get $\chi^{(2)}\approx\chi^{(1)}$, and in this case $\chi=\chi^{(1)}+\chi^{(2)}$.

3. $|\epsilon_0-\omega_0^2/\omega^2|<\omega_0^2\langle\tau^{-1}\rangle/\omega^3$. Here $\tan\varphi={}^1/_2$ and

$$\chi^{(3)}=-\frac{A}{\omega}\int_0\tau\frac{\partial f_0}{\partial \mathscr{E}}\left[1-\frac{2}{5}B(\mathscr{E})\right]\mathscr{E}^{3/2}d\mathscr{E}. \tag{10}$$

The skin effect is normal when

$$\frac{1}{2}\frac{\omega_0}{c}\left(\frac{\langle\tau^{-1}\rangle}{\omega}\right)^{1/2}l<1.$$

Comparison of (7), (9), and (10) shows that $\chi^{(3)}$ increases in comparison with $\chi^{(1)}$ by a factor $\omega\tau\gg 1$ and $\chi^{(2)}$ increases in the same ratio when ω satisfies (8), while $\chi^{(2)}\to\chi^{(1)}$ when $\omega\approx\omega_0(2/\epsilon_0)^{1/2}$.

4. $\omega_0^2/\omega^2-\epsilon_0>\omega_0^2\langle\tau^{-1}\rangle/\omega^3$. We have

$$\operatorname{tg}\varphi=2\left(\frac{\omega_0^2}{\omega^2}-\varepsilon_0\right)\frac{\omega^3}{\omega_0^2\langle\tau^{-1}\rangle},$$

$$\chi^{(4)}=-2A\left(1-\varepsilon_0\frac{\omega^2}{\omega_0^2}\right)\frac{1}{\langle\tau^{-1}\rangle}\int_0^\infty\tau\frac{\partial f_0}{\partial \mathscr{E}}\left[1-\frac{2}{5}B(\mathscr{E})\right]\mathscr{E}^{3/2}d\mathscr{E}. \tag{11}$$

In this region, the condition of normal skin effect is

$$\frac{\omega}{c}\left(\frac{\omega_0^2}{\omega^2}-\varepsilon_0\right)^{1/2}l\approx\frac{\omega_0}{c}l<1.$$

This formula is valid in the entire "cutoff" region, and the coefficient $\chi^{(4)}$ is $\omega^2\tau^2\gg 1$ times larger than $\chi^{(1)}$ when $\omega<\omega_0/\epsilon_0^{1/2}$, and does not depend on ω.

None of these formulas, however, yield the real value of the light-electric current (or of the field when the circuit is open) for a specified radiation flux I_0 incident on the crystal from the outside, since different fractions of this radiation penetrate inside the crystal in the four considered frequency regions. We are interested in the ratio

$$j/I_0=\chi I/I_0=\chi Q=\chi_0,$$

where

$$Q=1-\left|\frac{1-\varepsilon^{1/2}}{1+\varepsilon^{1/2}}\right|^2$$

is the transmission coefficient, $\epsilon=(ck/\omega)^2$ is the dielectric constant (we are considering normal incidence of the wave on the crystal from vacuum). In the frequency region o, the transmission coefficient

*$[\nu\mathbf{H}_1^*]\equiv\nu\times\mathbf{H}_1^*$.

$Q = 4\epsilon_0^{1/2}/(\epsilon_0^{1/2}+1)^2$ does not depend on the frequency and

$$\chi_0^{(1)} = \chi^{(1)} Q \sim \omega^{-2}.$$

In region 2, the quantity

$$Q \approx \frac{4(\varepsilon_0 - \omega_0^2/\omega^2)^{1/2}}{[(\varepsilon_0 - \omega_0^2/\omega^2)^{1/2} + 1]^2}$$

changes when the frequency is increased from $4(\langle\tau^{-1}\rangle/\omega)^{1/2}$ to $4\epsilon_0^{1/2}/(\epsilon_0^{1/2}+1)$, and consequently the coefficient

$$\chi_0^{(2)} = -2\frac{A}{\omega^2}\frac{\omega_0^2\langle\tau^{-1}\rangle}{\omega^2}\frac{(\varepsilon_0 - \omega_0^2/\omega^2)^{1/2}}{[(\varepsilon_0 - \omega_0^2/\omega^2)^{1/2}+1]^2} \times \int_0^\infty \tau\frac{\partial f_0}{\partial \mathscr{E}}\left[1 - \frac{2}{5}B(\mathscr{E})\right]\mathscr{E}^{3/2}d\mathscr{E}$$

changes from $4\chi^{(2)}/(\omega\tau)^{1/2} \approx \chi_0^{(1)}(\omega\tau)^{1/2}$ to $\chi_0^{(2)} \approx \chi_0^{(1)}$ at $\omega \approx \omega_0(2/\epsilon_0)^{1/2}$. In region 3, when $\omega \approx \omega_0/\epsilon_0^{1/2}$, we have

$$Q = 2^{3/2}\frac{\omega_0}{\omega}\left(\frac{\langle\tau^{-1}\rangle}{\omega}\right)^{1/2} \approx 2^{3/2}\left(\frac{\langle\tau^{-1}\rangle}{\omega}\right)^{1/2},$$

and χ_0 is equal to

$$\chi_0^{(3)} = -2\sqrt{2}\frac{A}{\omega}\left(\frac{\langle\tau^{-1}\rangle}{\omega}\right)^{1/2}\int_0^\infty \tau\frac{\partial f_0}{\partial \mathscr{E}}\left[1 - \frac{2}{5}B(\mathscr{E})\right]\mathscr{E}^{3/2}d\mathscr{E} \approx \chi_0^{(1)}(\omega\tau)^{1/2}.$$

Finally, in the fourth region

$$Q = 2\frac{\omega_0^2}{\omega^3}\langle\tau^{-1}\rangle\left[\frac{\omega_0^2}{\omega^2} - \varepsilon_0\right]^{-3/2} \approx 2\frac{\langle\tau^{-1}\rangle}{\omega_0}, \tag{12}$$

and χ_0 takes the form

$$\chi_0^{(4)} = -4\frac{A}{\omega}\left[\frac{\omega_0^2}{\omega^2} - \varepsilon_0\right]^{1/2}\int_0^\infty \tau\frac{\partial f_0}{\partial \mathscr{E}}\left[1 - \frac{2}{5}B(\mathscr{E})\right]\mathscr{E}^{3/2}d\mathscr{E} \approx -4\frac{A}{\omega_0}\int_0^\infty \tau\frac{\partial f_0}{\partial \mathscr{E}}\left[1 - \frac{2}{5}B(\mathscr{E})\right]\mathscr{E}^{3/2}d\mathscr{E}.$$

Then $\chi_0^{(4)} \approx \chi_0^{(1)}\omega^2\tau/\omega_0$ does not depend on the frequency.

Thus, the light-electric coefficient χ increases with decreasing frequency in the high-frequency region ($\omega\tau > 1$) when $\omega < \omega_0(2/\epsilon_0)^{1/2}$, and reaches a maximum at

$$\omega_0^2/\omega^2 - \varepsilon_0 \approx \omega_0^2\langle\tau^{-1}\rangle/\omega^3,$$

at lower frequencies it remains constant in the entire region

$$1/\tau < \omega < \omega_0/\varepsilon_0^{1/2}$$

if the conditions for the normal skin effect are satisfied.

2. ANOMALOUS SKIN EFFECT

We assume that the electromagnetic flux $\mathbf{I}$ is directed along the z axis, and the electric and magnetic fields $\mathbf{E}_1(z)$ and $\mathbf{H}_1(z)$ are parallel to x and y, respectively. Then the solution (2) for deviations, linear in the wave field, of the distribution function from the equilibrium function $f_1^{(-)}$ describing the carriers moving towards the surface ($v_z < 0$), or the function $f_1^{(+)}$ describing carriers moving from the surface into the interior of the sample ($v_z > 0$), assuming P specularly and 1 − P diffusely reflected carriers from the surface, is given by[5,6]

$$f_1^{(-)} = \frac{ev_x}{v_z}\frac{\partial f_0}{\partial \mathscr{E}}\int_z^\infty \exp\left[-\frac{z-z'}{v_z}(-i\omega+\nu)\right]E_1(z')dz',$$

$$f_1^{(+)} = -\frac{ev_x}{v_z}\frac{\partial f_0}{\partial \mathscr{E}}\left\{\int_0^z \exp\left[-\frac{z-z'}{v_z}(-i\omega+\nu)\right]E_1(z')dz' + P\int_0^\infty \exp\left[-\frac{z+z'}{v_z}(-i\omega+\nu)\right]E_1(z')dz'\right\}. \tag{13}$$

Taking similar boundary conditions into account, we obtain for $f_2^{(-)}$ and $f_2^{(+)}$

$$f_2^{(-)} = e\frac{\partial f_0}{\partial \mathscr{E}}\int_z^\infty \exp\left(-\frac{z-z'}{v_z}\nu\right)E(z')dz' + \frac{1}{2}\frac{e}{mv_z}\cdot \tag{14}$$
$$\times\int_z^\infty \exp\left(-\frac{z-z'}{v_z}\nu\right)\mathrm{Re}\left\{E_1^*(z')\frac{\partial f_1^{(-)}}{\partial v_x} - \frac{v_z}{c}H_1^*(z')\frac{\partial f_1^{(-)}}{\partial v_z} + \frac{v_x}{c}H_1^*(z')\frac{\partial f_1^{(-)}}{\partial v_z}\right\}dz',$$

$$f_2^{(+)} = -e\frac{\partial f_0}{\partial \mathscr{E}}\left\{\int_0^z \exp\left(-\frac{z-z'}{v_z}\nu\right)E(z')dz' - P\int_0^\infty \exp\left(-\frac{z+z'}{v_z}\nu\right)E(z')dz'\right\} - \frac{e}{2mv_z}\left\{\int_0^z \exp\left(-\frac{z-z'}{v_z}\nu\right)\right.$$
$$\times\mathrm{Re}\left[E_1^*(z')\frac{\partial f_1^{(+)}}{\partial v_x} - \frac{v_z}{c}H_1^*(z')\frac{\partial f_1^{(+)}}{\partial v_x} + \frac{v_x}{c}H_1^*(z')\right.$$
$$\times\left.\frac{\partial f_1^{(+)}}{\partial v_z}\right]dz' + P\int_0^\infty \exp\left(-\frac{z+z'}{v_z}\nu\right)\mathrm{Re}\left[E_1^*(z')\cdot\right.$$
$$\left.\left.\times\frac{\partial f_1^{(-)}}{\partial v_x}(-v_z) + \frac{v_z}{c}H_1^*(z')\frac{\partial f_1^{(-)}}{\partial v_x}(-v_z) + \frac{v_x}{c}H_1^*(z')\frac{\partial f_1^{(-)}}{\partial v_z}(-v_z)\right]dz'\right\}$$

(it is assumed here that the circuit is not closed in the z direction).

Under the conditions (1), the electric field of the wave is given in first approximation by $\mathbf{E}_1 \sim \exp(-\omega_0 z/c)$, so that (13) and (14) can be easily calculated:

$$f_2^{(-)} = e\frac{\partial f_0}{\partial \mathscr{E}}\int_z^\infty \exp\left(-\frac{z-z'}{v_z}\nu\right)E(z')dz' + \frac{1}{2}\frac{e^2}{m\omega^2}E_1(0)E_1^*(0)\cdot\left[\frac{\partial f_0}{\partial \mathscr{E}} + \frac{mv_x^2}{2}\frac{\partial^2 f_0}{\partial \mathscr{E}^2}\right]\exp\left(-2\frac{\omega_0}{c}z\right),$$

$$f_2^{(+)} = -e\frac{\partial f_0}{\partial \mathscr{E}}\left\{\int_0^z \exp\left(-\frac{z-z'}{v_z}\nu\right)E(z')dz' - P\int_0^\infty \exp\left(-\frac{z+z'}{v_z}\nu\right)\right.$$
$$\left.\times E(z')dz'\right\} + \frac{1}{2}\frac{e^2}{m\omega^2}E_1(0)E_1^*(0)\left\{\left[\frac{\partial f_0}{\partial \mathscr{E}} + \frac{mv_x^2}{2}\frac{\partial^2 f_0}{\partial \mathscr{E}^2}\right]\right.$$
$$\times\exp\left(-2\frac{\omega_0}{c}z\right) + \frac{mv_x^2}{2}\frac{\partial^2 f_0}{\partial \mathscr{E}^2}\exp\left(-\frac{z}{v_z}\nu\right) -$$
$$-\cos\left(\frac{\omega}{v_z}z\right)\exp\left[-\left(\frac{\omega_0}{c} + \frac{\nu}{v_z}\right)z\right]\frac{\partial}{\partial v_x}\left[v_x\frac{\partial f_0}{\partial \mathscr{E}}\right]$$
$$+2\frac{\omega_0}{\omega}\frac{v_z}{c}\left[\left(1 - \frac{v_x^2}{2v_z^2}\right)\frac{\partial f_0}{\partial \mathscr{E}} + \frac{mv_x^2}{2}\frac{\partial^2 f_0}{\partial \mathscr{E}^2}\right]\sin\left(\frac{\omega}{v_z}z\right)$$
$$\times\exp\left[-\left(\frac{\omega_0}{c} + \frac{\nu}{v_z}\right)z\right] + \frac{\omega_0}{c}\frac{v_x^2}{v_z^2}z\frac{\partial f_0}{\partial \mathscr{E}}\cos\left(\frac{\omega}{v_z}z\right)$$
$$\left.\times\exp\left[-\left(\frac{\omega_0}{c} + \frac{\nu}{v_z}\right)z\right]\right\};$$

these calculations yield the density of the constant light-electric current $j = j_I + j_E$, where j_I and j_E are respectively the currents produced by the electromagnetic flux and by the light-electric field $\mathbf{E}$:

$$j_E=-\frac{e^2}{\pi^2}\frac{m}{\hbar^3}\int_0^\infty \mathscr{E}\frac{\partial f_0}{\partial\mathscr{E}}\left\{\int_0^\infty\left[\int_0^1\exp\left(\frac{\nu}{v}\frac{z-z'}{\alpha}\right)\alpha\,d\alpha\right]\right.$$
$$\times E(z')\,dz'+\int_0^z\left[\int_0^1\exp\left(-\frac{\nu}{v}\frac{z-z'}{\alpha}\right)\alpha\,d\alpha\right]E(z')\,dz'$$
$$\left.-P\int_0^\infty\left[\int_0^1\exp\left(-\frac{\nu}{v}\frac{z+z'}{\alpha}\right)\alpha\,d\alpha\right]E(z')\,dz'\right\}d\mathscr{E},$$
$$j_I=-\frac{e^3}{2\pi^2\hbar^3\omega^2}E_1(0)E_1^*(0)\left\{\int_0^\infty\mathscr{E}\frac{\partial f_0}{\partial\mathscr{E}}\int_0^1\left[2\alpha+\frac{\nu}{v}z\left(\frac{1}{2}\right.\right.\right.$$
$$\left.\left.+\frac{\partial\ln\tau}{\partial\ln\mathscr{E}}\right)\right](1-\alpha^2)\exp\left(-\frac{\nu}{v}\frac{z}{\alpha}\right)d\alpha\,d\mathscr{E} \qquad (15)$$
$$+2\int_0^\infty\mathscr{E}\frac{\partial f_0}{\partial\mathscr{E}}\int_0^1\left[(2\alpha^2-1)\cos\left(\frac{\omega}{v}\frac{z}{\alpha}\right)-\frac{1}{2}\frac{\omega}{v}\frac{z}{\alpha}(1-\alpha^2)\right.$$
$$\left.\times\sin\left(\frac{\omega}{v}\frac{z}{\alpha}\right)\right]\alpha\exp\left[-\left(\frac{\omega_0}{c}+\frac{\nu}{v}\frac{1}{\alpha}\right)z\right]d\alpha\,d\mathscr{E}$$
$$+\frac{\omega_0}{\omega}\frac{1}{c}\left(\frac{2}{m}\right)^{1/2}\int_0^\infty\mathscr{E}^{3/2}\frac{\partial f_0}{\partial\mathscr{E}}\int_0^1\left[\sin\left(\frac{\omega}{v}\frac{z}{\alpha}\right)-\frac{\omega}{v}\frac{z}{\alpha}(1+\alpha^2)\right.$$
$$\left.\left.\times\cos\left(\frac{\omega}{v}\frac{z}{\alpha}\right)\right]\exp\left[-\left(\frac{\omega_0}{c}+\frac{\nu}{v}\frac{1}{\alpha}\right)z\right]d\alpha\,d\mathscr{E}\right\}. \qquad (16)$$

Assuming

$$U(z)=\int_0^z E(z')\,dz',$$

where $U(z)$ is the potential difference between the point z and the surface of the crystal, we can express j_E in terms of $U(z)$:

$$j_E=-\frac{e^2}{\pi^2}\frac{m}{\hbar^3}\int_0^\infty\frac{\partial f_0}{\partial\mathscr{E}}\mathscr{E}\frac{\nu}{v}\left\{\int_z^\infty\left[\int_0^1\exp\left(\frac{\nu}{v}\frac{z-z'}{\alpha}\right)d\alpha\right]\right.$$
$$\times U(z')\,dz'-\int_0^z\left[\int_0^1\exp\left(-\frac{\nu}{v}\frac{z-z'}{\alpha}\right)d\alpha\right]U(z')\,dz'$$
$$\left.+P\int_0^\infty\left[\int_0^1\exp\left(-\frac{\nu}{v}\frac{z+z'}{\alpha}\right)d\alpha\right]U(z')\,dz'\right\}d\mathscr{E}. \qquad (17)$$

When the circuit is open, the total current density is $j=0$. Assuming for simplicity that the carriers are degenerate and taking the Laplace transforms of (15) and (17), we obtain for fully diffuse reflection of the carriers the Laplace transform of the function $U(z)$:

$$U(s)=-\frac{e}{2m\omega^2}\frac{s^2}{a^2}\left\{\int_0^1\frac{\alpha^2(1-\alpha^2)\,d\alpha}{a+s\alpha}+\frac{a}{2}\left[\frac{1}{2}+\frac{\partial\ln\tau}{\partial\ln\zeta}\right]\right.$$
$$\times\int_0^1\frac{\alpha^2(1-\alpha^2)\,d\alpha}{(a+s\alpha)^2}+\int_0^1\frac{[(s+\omega_0/c)\alpha+a](2\alpha^2-1)\alpha^2\,d\alpha}{[(s+\omega_0/c)\alpha+a]^2+(\omega/v_0)^2}$$
$$+\left(\frac{\omega}{v_0}\right)^2\int_0^1\frac{[(s+\omega_0/c)\alpha+a](1-\alpha^2)\alpha^2\,d\alpha}{\{[(s+\omega_0/c)\alpha+a]^2+(\omega/v_0)^2\}^2}$$
$$+\frac{\omega_0}{c}\int_0^1\frac{\alpha\,d\alpha}{[(s+\omega_0/c)\alpha+a]^2+(\omega/v_0)^2}+\frac{\omega_0}{c}\left(\frac{\omega}{v_0}\right)^2\cdot$$
$$\left.\times\int_0^1\frac{(1+\alpha^2)\alpha\,d\alpha}{\{[(s+\omega_0/c)\alpha+a]^2+(\omega/v_0)^2\}^2}\right\}\left[\frac{1}{2}\ln\frac{a+s}{a-s}-\frac{s}{a}\right]^{-1},$$

($0<\mathrm{Re}\,s<a$), where v_0 and ζ are the Fermi velocity and energy, and $a=\nu/v_0$. When taking the inverse Laplace transform, account must be taken of the presence of poles at $s=0$, $s\approx-1$, $2a$, $s=-\omega_0/c\pm i\omega/v_0$, and of branch points at $s=\pm a$, $s=-\omega_0/c\pm i\omega/v_0$. It can be shown that the contributions of all the poles, with the exception of $s=0$, and of all the branch-cut integrals to $U(z)$ tend exponentially to zero as $z\to\infty$. Discarding them, we obtain the total potential difference $U=U(z)$ as $z\to\infty$:

$$U=-\frac{16\pi}{3}\frac{e}{m\omega_0^2}\frac{1}{v_0}\left[1+\frac{2}{5}\frac{\partial\ln\tau}{\partial\ln\zeta}\right]I(0). \qquad (18)$$

Let us compare this result with the result of the "cutoff" in the normal skin effect (case 4 of the first part), for which the potential difference can be easily calculated and differs from (18) by a factor $4\nu c/3\omega_0 v_0<1$ and in the sign of the second term. If we recognize that in the case of the anomalous skin effect (1) we have

$$I(0)=QI_0,\quad Q=3v_0/4c,$$

then we can obtain from (18) U as a function of the external flux I_0 (it is assumed here, just as in the first part, that the flux is normally incident on the sample surface):

$$U=-4\pi\frac{e}{m\omega_0^2}\frac{1}{c}\left[1+\frac{2}{5}\frac{\partial\ln\tau}{\partial\ln\zeta}\right]I_0. \qquad (19)$$

This result differs from the calculation of U in the case of "cutoff" in the normal skin effect and when the transmission coefficient (12) is taken into account, in that the sign of the second term of (19), which is smaller than unity, is reversed.

APPENDIX

We calculate τ_1, τ_2, and τ_3 for several scattering mechanisms:

1) For scattering by impurities producing a potential $V=V_0\delta(r)$ we have

$$\tau_1=\tau_2=\tau,\quad \tau_3^{-1}=0;$$

2) for scattering by charged centers (in the Born approximation)

$$\tau_1=\frac{1}{3}\tau,\quad \tau_2=\tau\frac{\lambda}{6\lambda-7}\approx\frac{1}{6}\tau,\quad \tau_3=\tau\frac{\lambda}{\lambda-2}\approx\tau,$$

where λ is the Coulomb logarithm;

3) for (high-temperature) scattering by acoustic phonons

$$\tau_1=\tau,\quad \tau_2=\tau,\quad \tau_3^{-1}=0;$$

4) for scattering by nonpolar optical phonons we get

$$\tau_1=\tau,\quad \tau_2=\tau,\quad \tau_3^{-1}=0;$$

5) for scattering by polar optical phonons at $\hbar\omega_l\ll T$

$$\tau_1=2\tau/3,\quad \tau_2=6\tau/11,\quad \tau_3=6\tau,$$

if $\hbar\omega_l\gg T$ we have

$$\tau_1=10\tau/9,\quad \tau_2=\tfrac{1}{2}\tau,\quad \tau_3=\tfrac{1}{3}\tau;$$

6) for piezoelectric (high-temperature) scattering

$$\tau_1=2\tau/3,\quad \tau_2=6\tau/11,\quad \tau_3=6\tau.$$

[1]M. S. Khaikin and A. Yu. Yakubovskii, Zh. Eksp. Teor. Fiz. **60**, 2214 (1971) [Sov. Phys.-JETP **33**, 1189 (1971)].
[2]L. E. Gurevich and A. A. Rumyantsev, Zh. Eksp. Teor. Fiz. **53**, 1144

(1967) [Sov. Phys.-JETP **26**, 675 (1968)].
[3]N. A. Brynskiĭ, A. A. Grinberg, and E. Z. Imamov, Fiz. Tekh. Poluprovodn. **5**, 1735 (1971) [Sov. Phys.-Semicond. **5**, 1516 (1972)].
[4]V. I. Perel' and Ya. M. Pinskii, Zh. Eksp. Teor. Fiz. **54**, 1889 (1968) [Sov. Phys.-JETP **27**, 1014 (1968)].
[5]G. E. H. Reuter and E. H. Sondheimer, Proc. R. Soc. A **195**, 336 (1948).
[6]V. P. Silin and A. A. Rukhadze, Élektromagnitnye svoĭstva plazmy i plazmopodobnykh sred (Electromagnetic Properties of Plasma and Plasma-like Media), Atomizdat, 1961.

Translated by J. G. Adashko
262