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CALCULATION OF THE $p\mu$ + He^{++} REACTION

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The cross sections for μ^- meson capture by helium isotope nuclei from the ground state of pμ and dμ mesic atoms are calculated. The calculation results are compared with theoretical and experimental estimates. The perturbed stationary state method is employed in the calculations. The self-consistency of the method and its accuracy in practical calculations have been discussed in previous papers of the authors.

1. Calculation of the charge-exchange reaction

$$(p\mu)_{1s} + \mathrm{He}^{++} \to (\mathrm{He}\,\mu)_{1s}^{+} + p, \tag{1}$$

i.e., of the transfer of a μ^- meson from the ground state of the pμ atom into the ground state of the (He μ)$^+$ atom is of interest for two reasons. First, the experimental studies of this process[1] do not lead as yet to unambiguous conclusions with respect to its velocity. From the theoretical point of view, the reaction (1) is of interest as an example of scattering in a system of three charged particles without degeneracy, which is characteristic of the symmetrical charge exchange of the pμ + p type. A feature of the process (1) is Coulomb repulsion in the final state.

2. The two-level approximation of the method of perturbed stationary states (PSS)[2] in the case of reaction (1) leads to a system of Schrödinger equations for the relative motion of the nuclei (in mesic-atom units, e = h = M_μ = 1):

$$\frac{d^2\chi_i}{dR^2} + \left[k_i^2 - \frac{2M}{m}\left(V_i(R) + \frac{2}{R}\right) - \frac{L(L+1)}{R^2}\right]\chi_i = K_{ij}(R)\chi_j + 2Q_{ij}(R)\frac{d\chi_j}{dR}, \tag{2}$$

where

$$1/M = 1/M_1 + 1/M_2, \quad 1/m = 1/M_\mu + 1/(M_1 + M_2).$$

M_1, M_2, and M_μ are the respective masses of the proton, helium nucleus, and meson,

$$V_i(R) = E_i(R) - E_i(\infty) + (m/2M)[K_{ii}(R) - K_{ii}(\infty)]. \tag{3}$$

The effective potentials $E_i(R)$, $K_{ij}(R)$ and $Q_{ij}(R)$ are calculated by solving the two-center problem, i.e., the problem of meson motion in the field of two immobile nuclei[3]. They are illustrated in Figs. 1 and 2.

As $R \to \infty$, the function $\chi_1(R)$ represents the left-hand side of the reaction (1) and the function $\chi_2(R)$ its right-hand side. The momenta k_i in the two reaction channels are defined by

$$k_1^2 = (2M/m)\varepsilon, \quad k_2^2 = k_1^2 + k_0^2, \quad k_0^2 = (2M/m)\Delta E, \tag{4}$$

where

$$\Delta E = E_1 - E_2 = m\{{}^3/_2 - (m/2M)[(\varkappa-1)^2 - {}^1/_4(\varkappa+1)^2]\},$$
$$\varkappa = (M_2 - M_1)/(M_2 + M_1)$$

and ϵ is the initial collision energy.

3. The system (2) is solved by the phase-function method[5], which makes it possible to determine directly the elements t_{ij} of the reaction matrix T, in terms of which the partial cross sections σ_{ij} of different channels of the reaction in the system pμ + He^{++} are expressed in the following manner[6,7] 1)

$$\sigma_{ij}^L = \frac{4\pi}{k_i^2}(2L+1)\frac{D^2\delta_{ij} + (t_{ij}^L)^2}{(D-1)^2 + (t_{11}^L + t_{22}^L)^2} \tag{5}$$

$$D = \det T^L = t_{11}^L t_{22}^L - t_{12}^L t_{21}^L.$$

In the notation employed, the cross section σ_{11} describes the process of elastic scattering in the system pμ + He^{++}, and the cross section σ_{12} the reaction (1). As $k_1 \to 0$ we get $t_{12} = t_{21} \sim k_1$, $t_{11} \sim k_1$, and accordingly

$$\sigma_{11} \sim \text{const}, \; \sigma_{12} \sim 1/k_1, \; \sigma_{21} \sim k_1. \tag{6}$$

In the analysis of the experimental data on the reaction (1), it is convenient to use in place of the cross section σ_{12} the transfer rate

$$\lambda = \sigma_{12} v_1 N_p^0 \; [\mathrm{sec}^{-1}], \tag{7}$$

where v_1 is the initial relative velocity of pμ and He^{++}, and $N_p^0 = 4.2 \times 10^{22}$ cm^{-3} is the density of liquid hydrogen.

The cross sections were calculated for collision energies 10^{-2} eV $\le \epsilon \le 1$ eV. At $\epsilon \le 0.1$ eV, the rates of the transfer of the μ^- meson from the proton (deuteron) to He^3 and He^4 are constant. Their values together with the experimental[1] and theoretical[8] estimates are listed in the table. As expected, the calculated rates are small. With increasing collision energy, the values of λ begin to decrease. For estimates it may be useful to have the dependence of λ on the particle masses:

$$\lambda \sim (m/M)^{1/2}, \tag{8}$$

which follows from (4), (6), and (7). Such an estimate is particularly well satisfied when the hydrogen-isotope mass is constant.

4. In view of the smallness of the transfer constant λ, we estimated the probability of the transfer of the meson from the 1s level of the pμ atom to the excited 2s level of the (He μ)$^+$ atom:

$$(p\mu)_{1s} + \mathrm{He}^{++} \to (\mathrm{He}\,\mu)_{2s}^{+} + p. \tag{9}$$

It turns out that the probability of this process at thermal collision energies is ~ 10^{-10} of the probability of the process (1).

1) The scattering matrix is connected with the T matrix by the relation $S = (1 + iT)(1 - iT)^{-1}$.

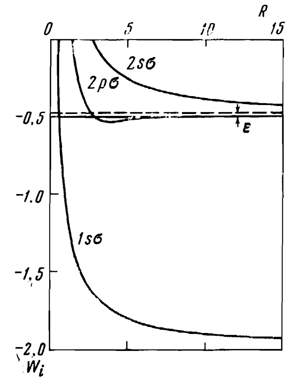

FIG. 1. Eigenvalues (terms) $W_i(R) = E_i(R) + 2/R$ of the $p\mu He^{++}$ system. The term $2p\sigma$ corresponds asymptotically to the system $p\mu + He^{++}$, and the terms $1s\sigma$ and $2s\sigma$ to the states of the system $p + (He\mu)^{+}$. The asymptotic form of the tems as $R \to \infty$ is calculated in [4] accurate to terms $\sim R^{-11}$.

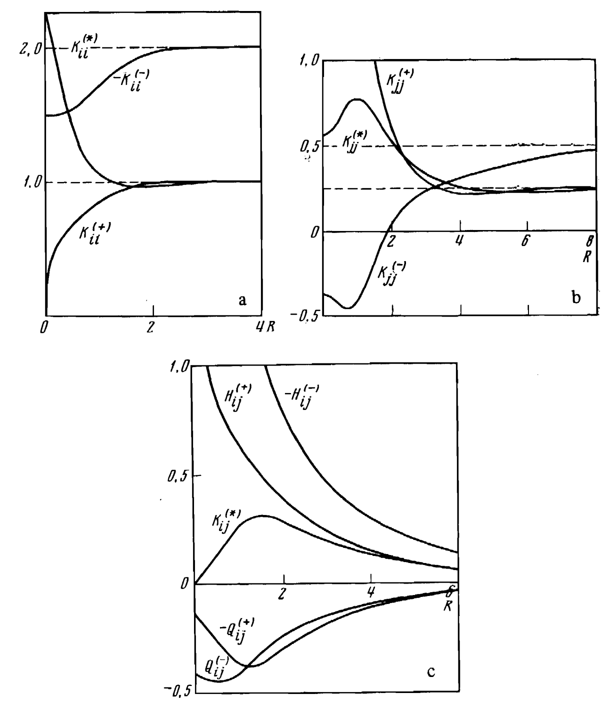

FIG. 2. Adiabatic corrections to the terms $W_i(R)$ in the reaction (1):
$K_{ij} = K_{ij}^{(+)} + \varkappa K_{ij}^{(-)} + \varkappa^2 K_{ij}^{(*)}$, $Q_{ij} = Q_{ij}^{(+)} + \varkappa Q_{ij}^{(-)}$, $K_{ij}^{(\pm)} = H_{ij}^{(\pm)} + dQ_{ij}^{(\pm)}/dR$,
$Q_{ij}^{(\pm)} = -Q_{ji}^{(\pm)}$, $H_{ij}^{(\pm)} = H_{ji}^{(\pm)}$,
$\varkappa = (M_2 - M_1)/(M_2 + M_1)$, $i \equiv \{1s\sigma\}$, $j \equiv \{2p\sigma\}$

5. Figure 3 shows the cross section σ_{11} of the elastic scattering process $p\mu$ + He^3, for which there is a clearly pronounced Ramsauer-Townsend effect. With the exception of the region where the cross section has a minimum ($\epsilon \approx 0.2$ eV), the main contribution to the cross section σ_{11} is made by the s wave. It should be noted that the scattering-length approximation for σ_{11} is reached only at $\epsilon \leq 10^{-4}$ eV. A special investigation has shown that the energy dependence of the cross section σ_{11} is practically insensitive to the second channel of the system (2) and is determined entirely by the effective potential $V_1(R)$ in the equation for $\chi_1(R)$. The minimum in the cross section σ_{11} is due to the characteristic long-range asymptotic form of the potential $V_1(R) \approx -9/R^4$ at $R \to \infty$[4].

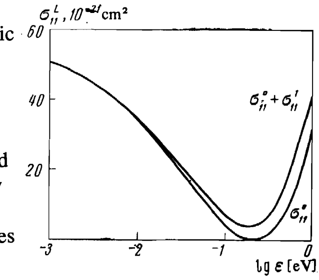

FIG. 3. Cross section for the elastic scattering $(p\mu)_{1s} + He^{++}$. The lower curve σ_{11}^0 is the contribution of the s-wave, and the upper curve is the sum of the partial cross sections of the s- and p-waves. The Ramsauer-Townsend effect is observed at a collision energy $\epsilon \approx 0.2$ eV. Owing to the coupling of the channels, the cross section σ_{11}^0 does not vanish [10].

6. The Coulomb repulsion in the second channel of the reaction (1) leads in the phase-function method to the following expression for the T-matrix:

$$T = \frac{1}{\cos\delta - \bar{t}_{22}\sin\delta}\begin{pmatrix} \bar{t}_{11}(\cos\delta - \bar{t}_{22}\sin\delta) - \bar{t}_{12}^2\sin\delta & \bar{t}_{12} \\ \bar{t}_{21} & \bar{t}_{22}\cos\delta + \sin\delta \end{pmatrix} \tag{10}$$

where $\delta = \delta_L = \arg\Gamma(L + 1 - i\eta)$, $\eta = M/mk_2$, and $\bar{t}_{ij}$ are the matrix elements of the auxiliary matrix $\overline{T}$ defined in[7]. Formulas (10) take into account the interference of pure Coulomb scattering and scattering by the potentials $V_1(R)$. In the elastic channel, this effect prevents the vanishing of the cross section σ_{11}, which would occur in the case of single-channel scattering (Fig. 3).

7. The small rates of reactions of the type (1) is due to the absence of term crossings and pseudocrossings in the $p\mu$ He^{++} system[8,9]. This conclusion, however, is valid only for transitions from the ground state of the $p\mu$ atom (Fig. 1). One of us has shown[9] that pseudocrossings take place in meson transfers from the levels

$$n \geqslant [1 - (2^{1/2} - 1)(1 + 2^{3/2})^{1/2}]^{-1} \approx 5$$

of the mesic atom $p\mu$, and consequently the corresponding transfer rates λ may turn out to be appreciable. An experimental study of the π^--meson transfer processes in accordance with the reaction $p\pi^- + He^{++} \to (He\,\pi)^+ + p$[10] confirms this conclusion.

The present study is a continuation of the authors' research on mesic-atom processes[6]. The features of the employed methods were discussed by us in detail earlier[7,11]. One of the authors (A. V. M.) is grateful to H. Schultz for kind collaboration with the numerical calculations.

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<table>
<tr><th rowspan="2">Source</th><th colspan="4">Transfer rate λ, 10^6 sec^{-1}</th><th rowspan="2">Method</th></tr>
<tr><th>$p\mu + He^3$</th><th>$p\mu + He^4$</th><th>$d\mu + He^3$</th><th>$d\mu + He^4$</th></tr>
<tr><td>Schiff (1961)</td><td colspan="4">$<10^3$</td><td>Experiment</td></tr>
<tr><td>Gershteĭn (1962)</td><td colspan="4">~ 0.1</td><td>Quisiclassical estimate</td></tr>
<tr><td>Zaĭmidoroga et al. (1963)</td><td colspan="2">$<10^2$</td><td colspan="2">—</td><td>Experiment</td></tr>
<tr><td>Placci et al. (1967)</td><td colspan="2">—</td><td colspan="2"><10</td><td>Experiment</td></tr>
<tr><td>Present paper</td><td>6.3</td><td>5.5</td><td>1.3</td><td>1.0</td><td>Calculation by the PSS method</td></tr>
</table>

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5