RESONANCE PHENOMENA IN A PARAMETRIC SPIN WAVE SYSTEM

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Resonant excitation of collective parametric spin wave oscillations induced by a weak variable magnetic field with a frequency near to the pumping frequency ω is detected in yttrium garnet single crystals subjected to parallel pumping. The resonance frequency Ω_0 of the oscillations is proportional to the magnitude of the interaction between the spin wave pairs whereas the damping is determined by the relaxation time of the spin waves. A theoretical model is proposed which explains the main features of the phenomenon and in particular the value of the eigenfrequency Ω_0 and its dependence on the pumping amplitude and also the high resonance susceptibility of the spin waves with respect to the weak signal and its pronounced asymmetry at frequencies $\omega \pm \Omega_0$.

WE investigate in the present paper resonance phenomena due to collective oscillations in a system of parametrically-excited spin-wave pairs (SWP). The parametric excitation conditions are satisfied as a rule simultaneously for a large number of waves, the interaction between which determines the established stationary state and small oscillations about this state. We are interested in the possibility of exciting collective SWP oscillations under the influence of an alternating external field. The basis for this is the observed ability of the SWP system to execute harmonic oscillations about a stationary state, with a natural frequency Ω_0 proportional to the pair interaction and with a damping determined by the spin-wave relaxation time.

The existence of natural oscillations is easiest to observe by investigating the stability of the SWP against small perturbations of their amplitudes n_k and phases ψ_k. In the simplest case, when the perturbations are the same for all pairs, $\delta\psi_k, \delta n_k \sim e^{\nu t}$ and the instability increment is[1]

$$\nu = -\gamma \pm [\gamma^2 - 4S(2T+S)N_0^2]^{1/2},$$

where γ^{-1} is the spin-wave relaxation time, S and T are the coefficients of the pair-interaction Hamiltonian, and $N_0 = n_k$ is the summary amplitude of the pairs. In a stable state that exists at $S(2T + S) > 0$, the system can execute harmonic oscillations $\delta\psi, \delta N \sim \exp(i\Omega_0 t)$ with natural frequency $\Omega_0 = \mathrm{Im}\,\nu$. These oscillations appear as low-frequency ($\Omega_0 \sim 10^6$ sec^{-1}) oscillations of the SWP amplitude and phase; they can be observed, in particular, in the form of damped oscillations in the transient regime[2].

Undamped collective oscillations of SWP can be excited with an alternating magnetic field of frequency Ω_0 or $\omega \pm \Omega_0$ (ω is the pump-field frequency). This produces easily spatially homogeneous oscillations in which all the pairs oscillate in phase with one another. In addition to the homogeneous oscillations, inhomogeneous oscillation types, and waves that are analogous in a certain sense to second-sound waves in ferromagnets[3], are possible in principle in an SWP system. The excitation and observation of inhomogeneous oscillations, however, entails methodological difficulties.

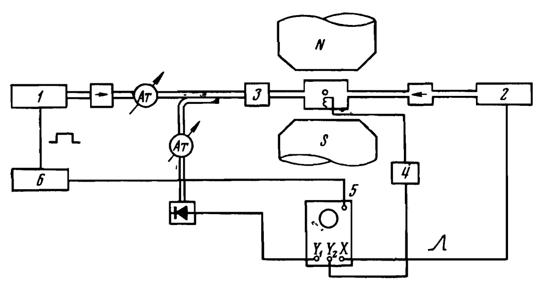

FIG. 1. Experimental setup for the measurement of the reaction of parametric spin waves to a weak microwave signal: 1–magnetron, 2–klystron, 3–impedance transformer, 4–amplifier, 5–oscilloscope, 6–modulator, AT–attenuator.

The present paper is devoted only to homogeneous collective SWP oscillations.

We have observed resonance produced in an SWP system by a weak alternating magnetic field at frequencies $\omega \pm \Omega_0$. The experiment was performed with parallel pumping in single-crystal yttrium garnets (YIG) and consisted of observing the reaction of SWP to a weak field parallel to the pump field. The resonance was detected in two ways: 1) by the appearance of peaks at the frequencies $\omega \pm \Omega_0$ in the absorption spectrum of a weak microwave signal and 2) by the excitation of longitudinal-magnetization polarization at the low frequency Ω_0.

EXPERIMENT

The experimental setup is shown in Fig. 1. The YIG sample was placed at the center of a rectangular TE_{102} resonator with two coupling ports, one of which was used for parallel pumping of the spin waves and the other to apply to the sample an additional weak field parallel to the pump field. Pumping at a frequency ω = 9.40 GHz was with a CW magnetron rated 15 W, modulated with square pulses of duration 10^2–10^3 μsec with a repetition frequency 25 Hz. The weak-signal source was a klystron, to the reflecting electrode of which there was applied the oscilloscope sweep voltage, as a result of which its frequency varied periodically in a narrow range ($|\Omega| < 10$ MHz) relative to the pump frequency. The oscilloscope sweep was triggered simultaneously with application of the pump pulse to the sample. The weak-signal power passed through the resonator and was detected and registered on the oscilloscope screen in the form of a curve describing the frequency dependence of the transmission coefficient $D^2_{\omega+\Omega}$ at a fixed pump power. Reflection from the resonator at the magnetron frequency was almost completely eliminated by matching the resonator with the aid of an impedance transformer. To ensure observation of the weak-signal absorption it suffices for the reflected power to be not more than 10% of the klystron power. For measurements at low frequency, a small coil, whose axis was parallel to the constant magnetic field $\mathbf{H}_0$, was placed near the sample in the resonator. The coil picked up the low-frequency oscillations of the longitudinal magnetization component m_z, which were then amplified with a low-noise amplifier and fed to the oscilloscope.

The measurements were performed on several YIG samples in the form of a sphere with a narrow ferromagnetic-resonance line, $2\Delta H_0 = 0.3-0.6$ Oe, at different orientations of the external field H_0 relative to the crystal axes. The main measurements were performed at an orientation $\mathbf{H}_0 \parallel [100]$, in which the low-frequency self-oscillations of the magnetization are significantly suppressed beyond the threshold and up to 8–10 dB above threshold[4].

Figure 2 shows oscillograms of the frequency dependences of $D^2_{\omega+\Omega}$ and $m_{z,\Omega}$, plotted at different pump powers h^2. Oscillogram a pertains to the regime below threshold ($h^2 < h_c^2$) and represents the resonance curve of the resonator. The null beats at the crest of the curve indicate that the frequencies of the klystron and magnetron are equal at this point ($\Omega = 0$). Oscillograms b–e pertain to the region beyond threshold. They show that, immediately beyond the threshold, a relatively narrow absorption peak appears on the crest of the resonance curve and increases with increasing pump power and shifts towards higher frequencies. In addition, excitation of low-frequency oscillations of m_z is observed, with a maximum amplitude reached simultaneously with the absorption peak. The difference in the behavior of $D^2_{\omega+\Omega}$ and $m_{z,\Omega}$ is that the oscillogram of $m_{z,\Omega}$ has a symmetrical maximum at the signal frequency $\omega - \Omega_0$, but no such maximum is observed on the absorption curve. At large excesses above threshold, $h^2 \gtrsim 8$ dB, corresponding to the second threshold[4,5], the resonance peak broadens and new absorption lines appear (oscillogram e); simultaneously there appear intense self-oscillations that modulate the pump power reflected from the resonator. Additional absorption lines are observed also at the orientation $\mathbf{H}_0 \parallel [111]$, where intense self-oscillations are excited immediately beyond the first threshold.

The value of the absorbed power is directly proportional to the weak-signal power, making it possible to introduce the susceptibility at the frequency $\omega + \Omega$. The

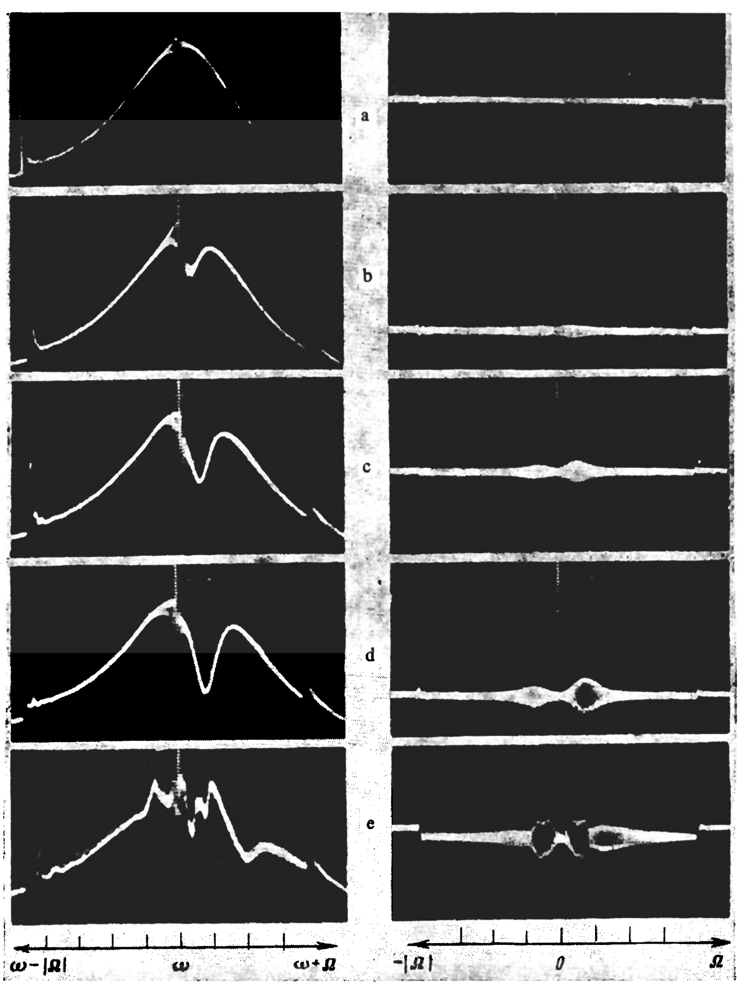

FIG. 2. Weak-signal absorption spectrum (left) and frequency dependence of longitudinal magnetization m_z(right) at different pump power-to-threshold ratios: a–$h^2/h_c^2 = 0$ dB, b–2 dB, c–4 dB, d–6 dB, 3–13 dB. Sweep rate 1.5 MHz/cm. Sample–YIG sphere, orientation $\mathbf{M} \parallel [100]$; $H_0 = 1570$ Oe ($k = 1.3 \times 10^5$ cm^{-1}), $\omega = 9.40$ GHz.

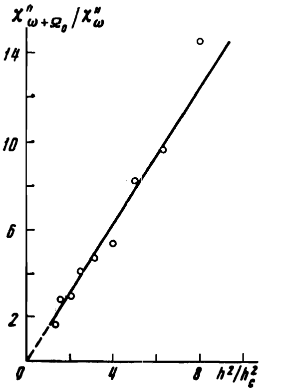

FIG. 3

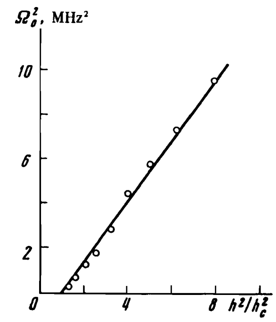

FIG. 4

FIG. 3. Dependence of the relative resonant susceptibility $\chi''_{\omega+\Omega_0}/\chi''_\omega$ on the pump power. Here χ''_ω is the susceptibility to the pump field.

FIG. 4. Dependence of the resonant frequency on the pump power.

dependence of the resonant susceptibility $\chi''_{\omega+\Omega_0}$ on the pump power is shown in Fig. 3. Figure 4 shows the dependence of the resonance frequency Ω_0 on the pump power.

THEORY

To calculate the reaction of the SWP to a weak signal, we used the equations of motion[1] for the correlation functions $n_{\mathbf{k}} = \langle a_{\mathbf{k}}a_{\mathbf{k}}^*\rangle$ and $\sigma_{\mathbf{k}} = \langle a_{\mathbf{k}}a_{-\mathbf{k}}e^{i\omega t}\rangle$, where $a_{\mathbf{k}}$ and $a_{-\mathbf{k}}$ are the complex amplitudes of the traveling spin waves. For the case of isotropic or cubic ferromagnets (with magnetization M ∥ [100] or [111]), it is convenient to write the equation of motion in the form

$$\frac{d\sigma_\theta}{dt} + \gamma_\theta\sigma_\theta + i\left(\omega_\theta - \frac{\omega}{2} + 2\sum_{\theta'} T_{\theta\theta'}n_{\theta'}\right)\sigma_\theta = -in_\theta\left(hV_\theta + \sum_{\theta'} S_{\theta\theta'}\sigma_{\theta'}\right). \tag{1}$$

Here

$$\sigma_\theta = \int_0^{2\pi} \sigma_{\mathbf{k}} e^{2i\varphi} d\varphi, \quad n_\theta = \int_0^{2\pi} n_{\mathbf{k}} d\varphi;$$

θ and φ are the polar and azimuthal angles of the wave vector **k**; ω_θ and γ_θ are the natural frequency and damping of the waves $a_{\mathbf{k}}$; $V_\theta = V\sin^2\theta$ is the coefficient of coupling with the pump field h;

$$S_{\theta\theta'} = \frac{1}{2\pi}\int_0^{2\pi} S_{\mathbf{k}\mathbf{k}'} \exp\{2i(\varphi_{\mathbf{k}} - \varphi_{\mathbf{k}'})\} d(\varphi_{\mathbf{k}} - \varphi_{\mathbf{k}'}),$$

$$T_{\theta\theta'} = \frac{1}{2\pi}\int_0^{2\pi} T_{\mathbf{k}\mathbf{k}'} d(\varphi_{\mathbf{k}} - \varphi_{\mathbf{k}'})$$

are the coefficients of the SWP interaction Hamiltonian averaged over φ.

The stationary solutions of (1), which were investigated earlier (see[1,4]), show that beyond the threshold, up to a certain excess above threshold, the only SWP that are excited are those with $\theta = \pi/2$ and with wave vector $|\mathbf{k}| = k_0$, defined by the condition

$$\omega_{\pi/2} - \omega/2 + 2TN_0 = 0, \tag{2}$$

$$N_0 \equiv n_{\pi/2} = \frac{[(hV)^2 - \gamma^2]^{1/2}}{|S|}, \tag{3}$$

where N_0 is the summary amplitude of the pairs with $\theta = \pi/2$, $T \equiv T_{\pi/2,\pi/2}$, $S \equiv S_{\pi/2,\pi/2}$ and $\gamma \equiv \gamma_{\pi/2}$. Here $\omega_M = 4\pi gM_0$, $N_z = 1/3$ is the demagnetization factor

Experimental proof of the existence in YIG of a beyond-threshold region $h^2 \lesssim 8$ dB, in which only pairs with $\theta = \pi/2$ are excited, is contained in our earlier paper[5]. A second group of pairs with $\theta \approx \pi/4$ is excited at high powers.

Our problem is to solve Eq. (2) in the case when the ferromagnet is acted upon, in addition to the external pump $he^{-i\omega t}$, also by a weak signal $h_1e^{-i(\omega+\Omega)t}$. We make the important assumption that the field h_1 does not alter the character of the pair distribution in k-space. We neglect here the effects connected with the broadening of the packet and with the oscillations of its center of gravity relative to k_0. It can be shown that the contribution of these effects is small relative to the parameter $\sqrt{\gamma/\omega}$ down to very low frequencies Ω of the order of $\Omega_0\gamma/\omega$. At still lower frequencies $\Omega < \Omega_0\gamma/\omega$ the center of gravity of the packet follows adiabatically the summary amplitude in accordance with the external stability condition (2). We introduce a new variable $c = \sqrt{\sigma_\theta}$, for which the initial equation (1) at $\theta = \pi/2$ takes the form

$$\left[\frac{d}{dt} + \gamma + i\left(\omega_{k_0} - \frac{\omega}{2} + 2T|c|^2\right)\right]c + i[h(t)V + Sc^2]c^* = 0. \tag{4}$$

Substituting $h(t) = h + h_1e^{-i\Omega t}$, we seek a solution in the form of an expansion in the small parameter h_1/h:

$$c = c_0 + c_\Omega e^{i\Omega t} + c_{-\Omega}e^{-i\Omega t} + \ldots$$

For $c_{\pm\Omega}$ we obtain the system of equations

$$[\Omega + 2(T+S)N_0 - i\gamma]c_\Omega + [hV + (2T+S)c_0^2]c_{-\Omega}^* = 0,$$

$$-[hV + (2T+S)c_0^{*2}]c_\Omega + (\Omega - 2(T+S)N_0 - i\gamma]c_{-\Omega}^* = h_1Vc_0,$$

whence

$$c_\Omega = -h_1Vc_0\frac{hV + (2T+S)c_0^2}{\Omega^2 - \Omega_0^2 - 2i\gamma\Omega}, \tag{5}$$

$$c_{-\Omega}^* = h_1Vc_0\frac{\Omega + 2(T+S)N_0 - i\gamma}{\Omega^2 - \Omega_0^2 - 2i\gamma\Omega}, \tag{6}$$

$$\Omega_0 = 2[S(2T+S)]^{1/2}N_0. \tag{7}$$

These expressions make it easy to calculate the weak-signal power absorbed by the SWP:

$$P_{\omega+\Omega} = 2(\omega+\Omega)h_1V\,\mathrm{Im}(c_0^*c_{-\Omega}^*) \tag{8}$$

and the amplitude of the longitudinal magnetization

$$m_{z,\Omega} = 2g|c_0^*c_\Omega + c_0c_{-\Omega}^*| \tag{9}$$

(here g is the magnetomechanical ratio). With the aid of (5) and (6) we obtain from (8) and (9) final expressions for the imaginary part of the susceptibility $\chi''_{\omega+\Omega}$ and $m_{z,\Omega}$:

$$\chi''_{\omega+\Omega} = \chi_\omega''\left(\frac{h}{h_c}\right)^2\frac{2\gamma^2[\Omega_0^2 + \Omega^2 + 4\Omega(T+S)N_0]}{(\Omega_0^2 - \Omega^2)^2 + 4\gamma^2\Omega^2}, \tag{10}$$

$$m_{z,\Omega} = \Delta m_{z0}\frac{h_1}{h_c}2\gamma\left[\frac{(\Omega + 2SN_0)^2 + 4\gamma^2}{(\Omega_0^2 - \Omega^2)^2 + 4\gamma^2\Omega^2}\right]^{1/2}. \tag{11}$$

Here χ''_ω is the susceptibility of the SWP to the pump field h, Δm_{z0} is the stationary change of M_z following excitation of the SWP, and $h_c = \gamma/V$ is the threshold pump amplitude.

DISCUSSION OF RESULTS

The theory of resonant interaction of a weak signal with an SWP wave makes use of the coefficients S and T of the SWP interaction Hamiltonian. The procedure for their calculation is given in[4]. For cubic ferromagnets, taking into account the energy of the dipole-dipole and exchange interactions, the Zeeman energy, and the crystallographic anisotropy energy, we obtain for **M** ∥ [100]

$$T = \frac{g}{2M_0}\frac{\omega^2+\omega_M^2}{\omega^2}\left[\omega_M(N_z-1)+\frac{\omega_M^2}{(\omega^2+\omega_M^2)^{1/2}}-9\omega_a\right],$$
$$S = \frac{g}{2M_0}\frac{\omega_M^2}{\omega^2}[\omega_M(N_z-1)+(\omega^2+\omega_M^2)^{1/2}-9\omega_a]. \qquad (12)$$

for a sphere, and ω_a/g is the anisotropy field in the [100] direction. Calculation by means of formula (12) for the concrete values ω = 9.40 GHz, ω_M = 4.9 GHz and ω_a/g = −80 Oe (for YIG at room temperature) yields T, S > 0 and a ratio T/S = 0.5. Using this ratio, let us compare quantitatively the resonant-interaction theory with experiment.

The resonance frequency Ω_0 is given by formula (7). Using (3), we obtain a linear dependence of Ω_0^2 on the pump power h^2:

$$\Omega_0^2 = (2\gamma)^2\frac{2T+S}{S}\left(\frac{h^2}{h_c^2}-1\right). \qquad (13)$$

The experimental dependence of Ω_0^2 on $(h/h_c)^2$ is shown in Fig. 4 in the form of a straight line with slope (1.5 ± 0.2) MHz^2. The theoretical value of the slope $(2\gamma)^2(2T + S)/S$ = 1.3 MHz^2 agrees well with experiment (we used the value γ = 0.4 MHz which is given below).

The weak-signal susceptibility $\chi''_{\omega+\Omega_0}$ has, in accordance with (10), a resonant character with a maximum near $\omega \pm \Omega_0$ and a resonance half-width γ. At the resonance point we have

$$\chi''_{\omega\pm\Omega_0} = \chi''_\omega\left(\frac{h}{h_c}\right)^2\frac{\sqrt{S(2T+S)}\pm(T+S)}{\sqrt{S(2T+S)}}. \qquad (14)$$

Thus, the resonant susceptibility $\chi''_{\omega+\Omega}$ exceeds the susceptibility χ''_ω to the pump by a order-of-magnitude factor $(h/h_c)^2$. It is seen from Fig. 3 that the experimental ratio $\chi''_{\omega+\Omega_0}/\chi''_\omega$ is ≈ 2 near the threshold, and reaches ~ 14 at the point of the second threshold at h^2/h_c^2 = 8 dB. The susceptibility χ''_ω was determined here from an independent experiment. Its dependence on the pump power was considered in detail earlier[4]. The experimental dependence of $\chi''_{\omega+\Omega_0}/\chi''_\omega$ on the pump power is, in accordance with (14), linear up to the second threshold; the theoretical value of the slope $1 + (T + S)/[S(2T + S)]^{1/2}$ ≈ 2 agrees with the experimental value 1.6 ± 0.5.

The asymmetry of the susceptibilities $\chi''_{\omega+|\Omega|}$ and $\chi''_{\omega-|\Omega|}$ is a characteristic feature of the SWP resonance. It is of fundamental importance that the signs of the susceptibilities at the frequencies $\omega + \Omega_0$ and $\omega_0 - \Omega_0$, as follows from (14), are different: an intensification of the weak signal at one of the frequencies is observed at the other. For YIG, the coefficients T and S are positive, i.e., the theory predicts absorption at the frequency $\omega + \Omega_0$, in agreement with experiment (see Fig. 2). The amplification of the frequency $\omega - \Omega_0$ in YIG is much smaller. From (14) we get $\chi''_{\omega-\Omega_0}/\chi''_{\omega+\Omega_0}$ = 0.03 (for T/S = 0.5), and this explains the absence of a peak at the frequency $\omega - \Omega_0$ in Fig. 2.

The frequency dependence of $m_{z,\Omega}$ has an entirely different character, namely, formula (11) leads at the frequencies ± |Ω| to a weak asymmetry that depends on the pump power, in agreement with experiment (Fig. 2). Measurement of $m_{z,\Omega}$ is a very sensitive method of determining the value of Δm_{z0}, which is proportional to the number of parametric magnons.

Damping of spin waves with different k in YIG

	$k\cdot 10^5$ cm^{-1}			
	0	1	2	4
2γ, MHz (from SWP resonance)	0,6	0,7	0,9	1.2
$2g\Delta H_k$, MHz (from the instability threshold).	0,4	0,6	0,9	1,3

It should be indicated that the phenomenon of resonant absorption of a weak signal can be used as a convenient illustrative method of measuring the spin-wave relaxation time. Measurements show that, in accordance with the theory, the width of the SWP resonance is independent of the pump power (up to the second threshold), within the limits of experimental accuracy, and agrees well with the value $2\Delta H_k$ obtained from the threshold of parallel pumping (see the table).

As is well known, low-frequency self oscillations of the longitudinal magnetization n_z (cf., e.g.,[6]), appear frequently and spontaneously in parametric excitation of spin waves. These self-oscillations are similar to the induced resonant SWP oscillations considered in the present paper. It can be assumed that the SWP oscillation modes that take part in these self-oscillations are analogous to those that interact with the weak external signal. The detailed mechanism (or mechanisms) for the excitation of the self-oscillations calls for a special study.

We note in conclusion that the phenomenon of resonant absorption of a weak signal by parametric spin waves was observed by us in different crystals not only for parallel but also for transverse pumping. A discussion of these questions, however, is beyond the scope of the present paper.

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