EFFECT OF LATTICE OSCILLATIONS ON THE TUNNEL CURRENT IN NORMAL METALS

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Incoherent inelastic scattering of tunneling electrons in an N – I – N system by local and nonlocal oscillations is taken into account. It is shown that the singularities of the tunnel current second derivative make it possible to determine the local and quasilocal oscillation spectrum and yield information on the impurity charge.

OF all the methods of investigation which allow us to obtain data on the object studied, the most detailed information is given by the tunnel effect (see, for example, the review[1]). Combining theory and experiment on tunneling, Schrieffer, Scalapino and Wilkins[2] investigated the structure of the density of states of superconductors with strong electron-phonon interaction. In contrast to the BCS theory, the Green's function of the phonons enters into the initial equations for the characteristic energy part $\Sigma_2(\omega)$ of anomalous superconductors, which describes the Cooper pairing. This circumstance permitted the authors of[2] to obtain, from $\Sigma_2(\omega)$ dependence obtained experimentally from the tunnel effect, information on the function $g(\omega)$ characterizing the energy of the lattice oscillation density. Recently, Helmut and Schmid[3] discussed the possibility of determining the phonon state density from the volt-ampere characteristic of a tunnel junction of two normal metals. It follows from the calculation of the authors[3] that weak current bursts will appear on the tunnel characteristics, due to account of the momentum dependence of the tunnel matrix element. The current I, as is known, is expressed in terms of the electron Green's function and is thus connected with the electron mass operator. Therefore, the appearance of bursts is due to the singularities in the phonon densities of states.

For a junction one of whose electrodes is in the superconducting state, calculation of the tunnel current of the "dressed" electrons was carried out by Duke and Kleiman,[4] who used the method of the tunnel Hamiltonian in the calculation of I(V). Generalization of the model approach with the tunnel Hamiltonian, which allows us to take into account the interaction of the tunneling electrons with phonons and impurities in the region of the barrier, was suggested in the work of B Bennet, Duke and Silverstein.[5] In that paper, the authors found the contributions to the tunnel current associated with elastic scattering on impurities of tunneling electrons and the inelastic scattering with excitation of a single phonon in the lattice of an ideal crystal. However, the role of the impurity is not reduced solely to the excitation of oscillations in the unperturbed lattice. The presence of the impurity atoms leads to the appearance of oscillations that differ essentially from the oscillations of the ideal lattice. Here the electrons undergo significant inelastic scattering not only from the impurity atom itself, but also from an appreciable number of atoms around it, the oscillations of which are perturbed. The latter leads to a radical change in the inelastic contribution to the tunnel current. It is interesting to note that the characteristic energy effects and effects associated with the inelastic scattering of tunneling electrons can be separated experimentally by a study of the odd and even parts of the differential conductivity of the contact (see, for example,[6]). In this connection, as will be shown below, tunneling in normal metals can be used as an effective tool of investigation of local and quasilocal oscillations of the impurity atoms.

In the present research, we took into account incoherent inelastic scattering of the tunneling electrons by local and quasilocal oscillations. We have made use here of the theory of the electrical conductivity of metals with impurities, developed by Kagan and Zhernov.[7] The assumptions made by these authors remain in force. To be precise, we shall assume the lattice of the crystal forming the junction to be monatomic and cubic, neglect the change in the electron spectrum, which can arise in the introduction of impurity atoms, use the model of a "rigid ion" for the electron-ion interaction, and limit ourselves to the Born approximation in the consideration of electron scattering.

We write the Hamiltonian of interaction of electrons with ions in the form

$$H_{int} = \sum_{i,n} U_n(\mathbf{r}_i - \mathbf{R}_n). \tag{1}$$

Here the summation is carried out over all possible configurations of electrons and ions, $\mathbf{R}_n = \mathbf{R}_n^0 + \mathbf{u}_n$ is the radius vector of the n-th ion, $\mathbf{u}_n$ the displacement of the n-th ion from the equilibrium position $\mathbf{R}_n^0$, and $\mathbf{r}_i$ the coordinates of the electrons.

Separating from this sum that part of the operator which is responsible for the interaction of electrons with quantum oscillations of the lattice, we write this part of the Hamiltonian in the following fashion:

$$H = i\sum_{n,\mathbf{g}} (\mathbf{g}\mathbf{u}_n)\mathscr{V}_n(\mathbf{g}) \int d\mathbf{r}\, \hat{\rho}(\mathbf{r}) \exp[i\mathbf{g}(\mathbf{r} - \mathbf{R}_n^0)], \tag{2}$$

where $\mathscr{V}_n(\mathbf{g})$ is the Fourier component of the potential $U_n(\mathbf{r})$ of electron-ion interaction and $\hat{\rho}(\mathbf{r})$ is the electron gas density operator.

The electron density operator $\hat{\rho}(\mathbf{r})$ can be written in the form of a sum $\hat{\rho} = \hat{\rho}_{1,1} + \hat{\rho}_{2,2} + \hat{\rho}_{1,2}$, where $\hat{\rho}_{1,1}$ and $\hat{\rho}_{2,2}$ represent the electron density operators in the first and second metals, expressed in the second-quantization representation in terms of the creation and annihilation

operators of the particles $\alpha^+\alpha$ and $\beta^+\beta$, respectively in right and left metals, and $\hat{\rho}_{1,2}$ is the cross term, containing the operator products $\alpha^+\beta$ and $\beta^+\alpha$. The operators α and β are introduced by expansion of the electron field operator, $\Psi_\sigma(\mathbf{r})$ in the complete set of functions $\{f_p^\pm\}$:

$$\Psi_\sigma(\mathbf{r}) = \sum_p [f_p^+(\mathbf{r})\alpha_{p\sigma} + f_p^-(\mathbf{r})\beta_{p\sigma}].$$

The separation of the Hamiltonian of the tunnel junction into the Hamiltonian of the left and right electrons in the one-dimensional interaction Hamiltonian is achieved in[8] by means of the functions $\{f_p^\pm\}$.

Separating from the relation (2) the terms leading to tunneling, we get the correction T to the one-particle tunnel Hamiltonian:

$$T = i\sum_{\substack{p,g\\k,n}} T_{pg}^{n}(\mathbf{k})(\mathbf{ku}_n)\exp(-i\mathbf{kR}_n^0)\alpha_p^+{}_g + \text{h.c.} \qquad (3)$$

where

$$T_{pg}^{n}(\mathbf{k}) = \mathscr{V}_n(\mathbf{k})\int d\mathbf{r}\, f_p^{+*}(\mathbf{r}) f_g^-(\mathbf{r})\exp(i\mathbf{kr}). \qquad (4)$$

The tunnel Hamiltonian (3) describes processes of inelastic tunneling of the electrons.

Using the relation (3), we get the following expression for the inelastic contribution I to the tunnel current:

$$I = 2e\,\mathrm{Re}\sum T_{pg}^{n}(\mathbf{k})T_{p'g'}^{n'}(\mathbf{k}')\exp[-i(\mathbf{kR}_n^0 - \mathbf{k}'\mathbf{R}_{n'}^0)]\int_{-\infty}^{t} dt'\exp[ieV(t'-t)]\langle[(\mathbf{k}'\mathbf{u}_{n'}(t'))\times\beta_{g'}^+(t')\alpha_{p'}(t')\,|\,(\mathbf{ku}_n(t))\alpha_p^+(t)\beta_g(t)]_-\rangle. \qquad (5)$$

$\alpha(t)$, $\beta(t)$, and $\mathbf{u}_n(t)$ in (5) are the operators α, β and $\mathbf{u}_n$ in the interaction representation; the angle brackets denote averaging over the equilibrium ensemble, e is the electron charge, and V the potential at the junction.

After carrying out the commutation operation and standard transformations, the formula for the current (5) can be expressed in terms of the displacement correlator

$$I = 2e\,\mathrm{Re}\sum T_{pg}^{n}(\mathbf{k})T_{pg}^{n'}(\mathbf{k}')\exp[-i(\mathbf{kR}_n^0 - \mathbf{k}'\mathbf{R}_{n'}^0)]\int_0^\infty d\tau\exp[i(\varepsilon_p - \varepsilon_g - eV)\tau][(1-n_p)n_g\times\langle(\mathbf{k}'\mathbf{u}_{n'}(\tau))(\mathbf{ku}_n(0))\rangle - (1-n_g)n_p\langle(\mathbf{ku}_n(0))(\mathbf{k}'\mathbf{u}_{n'}(\tau))\rangle]. \qquad (6)$$

Here n_p and n_g are the occupation numbers of the electrons.

For typical experiments, the thickness of the isolating layer between the electrodes is $d \sim 10^{-6}-10^{-7}$ cm. In this connection, consideration of two limiting regions is possible.

a) The long-wave region. This region corresponds to the case in which the wavelength of the phonon $\lambda \gg d$.

b) The short-wave region, corresponding to the inverse inequality $\lambda \ll d$.

One cannot assume for long-wave phonons that the barrier represents an obstacle with low transparency as is the case of electrons. To the contrary, the transparency of the barrier for such long waves will be of the order of unity and therefore the system of long-wave phonons at the junction must be considered as a single system, without separating it into left and right parts. In this limit, we can make the transformation in the relation for the current (6) to the continuum approximation, replacing the summation over n and n′ by integration over the entire region of the junction. Carrying out this operation, and also solving the equation of motion for the lattice displacement vector $\mathbf{P}$, we can show that changes in the volt-ampere characteristics of the tunnel junction will result from the emission (absorption) of quanta of lattice vibrations of the electrodes, and the properties of the dielectric layer will not have any effect on the character of the electron tunneling.

In the other limiting case ($\lambda \ll d$), only those phonons which are excited in the dielectric or in the immediate neighborhood of the electrodes directly in contact with insulating layer will take part in the inelastic process. We shall be interested below in the local and quasilocal oscillations, which pertain as a rule to just this limiting case.

We shall assume the concentration of impurity atoms to be small. Then, averaging the expression for the current (6) in the usual fashion[7] over all possible configurations of the impurities, we get the following current increment due to inelastic scattering by the regions of the perturbations:

$$I_1 = 4\pi e^2 N^2(0)\sum_{k,j}|T(\mathbf{k})|^2(\mathbf{kv}_{kj})^2 \times\int_{-\infty}^{\infty} d\omega\, F(\omega - eV)\frac{\varepsilon c\omega^2\gamma(\omega)}{[\omega^2(1-\varepsilon c\Delta'(\omega)) - \omega_{kj}^2]^2 + [\varepsilon c\omega^2\gamma(\omega)]^2} - I_0, \qquad (7)$$

$$I_2 = 4e^2N^2(0)c\left(\frac{Z_1 - Z_0}{Z_0}\right)^2\sum_{k,j}|T(\mathbf{k})|^2(\mathbf{kv}_{kj})^2\int_{-\infty}^{\infty} d\omega\frac{\gamma(\omega)F(\omega - eV)}{\varepsilon\omega^2}, \qquad (8)$$

$$I_3 = 8e^2N^2(0)c\frac{Z_1 - Z_0}{Z_0}\sum_{k,j}|T(\mathbf{k})|^2(\mathbf{kv}_{kj})^2 \times\int_{-\infty}^{\infty} d\omega\, F(\omega - eV)\left[P\frac{\gamma(\omega)}{\omega^2 - \omega_{kj}^2} + \pi\Delta(\omega)\delta(\omega^2 - \omega_{kj}^2)\right],$$

$$I_0 = 4\pi e^2N^2(0)\sum_{k,j}|T(\mathbf{k})|(\mathbf{kv}_{kj})^2\int_{-\infty}^{\infty} d\omega\, F(\omega - eV)\delta(\omega^2 - \omega_{kj}^2)\,\mathrm{sign}\,\omega. \qquad (9)$$

Here N(0) is the density of states on the Fermi surface, $|T(\mathbf{k})|^2$ the matrix element $|T_{pg}(\mathbf{k})|^2$ averaged over all the vectors $\mathbf{p}$ and $\mathbf{g}$ on the Fermi surface. The symbols ω_{kj} and $\mathbf{v}_{kj}$ denote the eigenfrequency and polarization vector an ideal lattice, corresponding to the wave vector $\mathbf{k}$ and branch number j, P is the symbol of the principal value; the functions

$$\gamma(\omega) = \frac{\pi\varepsilon\omega^2 g(\omega^2)}{R(\omega)}\,\mathrm{sign}\,\omega, \qquad (10)$$

$$\Delta(\omega) = \Delta'(\omega)\,\mathrm{sign}\,\omega = \frac{1 - \varepsilon\omega^2 J(\omega)}{R(\omega)}\,\mathrm{sign}\,\omega, \qquad (11)$$

where

$$R(\omega) = [1 - \varepsilon\omega^2 J(\omega)]^2 + (\pi\varepsilon\omega^2 g(\omega^2))^2,$$

$$J(\omega) = P\int_0^\infty \frac{dz\, g(z)}{\omega^2 - z}, \qquad (12)$$

$g(\omega^2)$ are the distribution functions of the squares of the frequencies of the phonon spectrum of the host crystal, Z_0 and Z_1 are the change on the ion of the host crystal

and the change of the impurity ion in the given junction; the functions

$$F(\omega - eV) = \frac{N}{MN^2(0)e}\sum_{p,g}\int_0^\infty d\tau \cos(\varepsilon_p - \varepsilon_g - eV)\tau \times [n_g(1-n_p)(1+\nu_\omega) - n_p(1-n_g)\nu_\omega], \quad (13)$$

where M is the mass of the atom of the ideal lattice, N the number of ions in the crystal and ν_ω the phonon occupation number, $\epsilon = (M - M_1)/M$ (M_1 is the mass of the impurity atom).

In the limit when the impurity concentration c tends to zero, we shall have only the tunnel-current increment due to inelastic scattering of the tunneling electrons by the lattice of the unperturbed crystal. For a concentration of impurity atoms different from zero, additional contributions appear in the volt-ampere characteristic; the contribution I_1 due to scattering on the deformed lattice spectrum, the current I_2 due to inelastic scattering directly on the impurity atoms, and the additional tunnel current I_S due to interference scattering.

Relations (7)—(9) contain quantities that require cumbersome calculations. In order to avoid this, we express the currents I_1, I_2, I_3 in terms of simpler quantities that are observed experimentally. For this purpose, we note that as $T \to 0°K$ the second derivative of the function $F(\omega - eV)$ with respect to the potential is proportional to a delta function. Using this property of the function $F(\omega - eV)$, we introduce the function G(eV, 0), which is determined by the following formula:

$$G(eV,0) = \frac{\pi e^3 N^2(0) N}{2M(2\pi)^6}\sum_{k,j}|T(k)|^2(\mathbf{kv}_{kj})^2\delta((eV)^2 - \omega_{kj}^2)\operatorname{sign} V. \quad (14)$$

The function G(eV, 0) corresponds to the value of the second derivative with respect to the potential of the contribution to the tunnel current due to inelastic scattering on the lattice of the base crystal at absolute zero temperature. An analytic expression for the function G(eV, 0) was obtained in [5] under the assumption that the electrons interact with the longitudinal acoustic phonons in the Debye model as a result of the deformation potential. In the general case, the function G(eV, 0) can be obtained by extrapolating the experimental curve of the second derivative of G(eV, T) to $T = 0°K$. We note that for $T \ll \omega_0$ (ω_0 are the characteristic frequencies in the density of states of the phonon spectrum) G(eV, t) is practically independent of the temperature, i.e., it coincides with the relation (14).

With the help of the expression (14) we get for the second derivatives of the relations (7)—(9) with respect to the potential, the equations (in the approximation linear in c):

$$G_1 = \frac{d^2I_1}{dV^2} = -\frac{c}{\pi}\varepsilon\int_{-\infty}^{\infty} d\omega\,\omega^2\gamma(\omega)F_1(\omega - eV)\frac{d}{d\omega^2}A(\omega), \quad (15)$$

$$G_2 = \frac{d^2I_2}{dV^2} = \frac{c}{\pi\varepsilon}\left(\frac{Z_1 - Z_0}{Z_0}\right)^2\int_{-\infty}^{\infty} d\omega\frac{\gamma(\omega)}{\omega^2}F_1(\omega - eV)\int_{-\infty}^{\infty} d\Omega\,\Omega G(\Omega,0), \quad (16)$$

$$G_3 = \frac{d^2I_3}{dV^2} = \frac{2c}{\pi}\frac{Z_1 - Z_0}{Z_0}\int_{-\infty}^{\infty} d\omega\,[\gamma(\omega)A(\omega) + \pi\Delta(\omega)G(\omega;0)]F_1(\omega - eV), \quad (17)$$

where

$$F_1(y) = \beta e^{\beta y}[\beta y + 2 + e^{\beta y}(\beta y - 2)]/(e^{\beta y} - 1)^3, \quad \beta = 1/T,$$

$$A(\omega) = P\int_0^\infty \frac{d\Omega^2 G(\Omega,0)}{\omega^2 - \Omega^2}. \quad (18)$$

The interaction of electrons with lattice vibrations is accompanied by the annihilation or creation of a phonon with energy ω. At high temperatures, when the inequality $\beta\omega \ll 1$ is satisfied, the inelasticity can be neglected. Here the second derivatives of the corrections to the current vanish, i.e., the singularities of the I—V characteristics of the tunnel junction are smoothed out at high temperatures. At low temperatures, the inelasticity of the scattering will appear strongly on the volt-ampere characteristics. This is of interest in connection with the possibility of the study of phonon spectra and the observation of local and quasilocal levels.

Let us analyze the obtained relations (15)—(17). The integrands of (15)—(17) contain the functions $\gamma(\omega)$ and $F_1(\omega - eV)$. The first of these has a sharply delineated character near the frequency $\omega = \omega_0$, determined by the equation

$$1 - \varepsilon\omega^2 J(\omega) = 0. \quad (19)$$

We write down the approximate formula for (10) near the maximum, for which we expand $\omega^2 J(\omega)$ in powers of $(\omega - \omega_0)^2$ ($\omega > 0$):

$$\gamma(\omega) = \frac{1}{\varepsilon|b|}\frac{\Gamma_0}{(\omega - \omega_0)^2 + \Gamma_0^2}, \quad (20)$$

where

$$\Gamma_0 = |\pi\omega_0^2 g(\omega_0^2)/b|, \quad b = [\omega_0^2 J(\omega_0)]'.$$

The expression (20) corresponds to a clearly expressed resonance if the quantity Γ_0 is much smaller than ω_D (ω_D is the Debye energy of the unperturbed crystal)

$$\Gamma_0 \ll \omega_D. \quad (21)$$

When the condition (21) is satisfied, the frequency ω_0 ($\omega_0 < \omega_D$) is called the frequency of resonant oscillation, as is well known. If the function $g(\omega^2)$ in this is equal to zero (corresponding to values of ω lying outside the permissible range of frequencies of the host crystal), the resonant oscillations are considered as local oscillations. However, in the region of allowed frequencies, the resonance oscillations, in contrast with the exact local oscillations, decay into oscillations of the continuous spectrum and have a finite width. Such oscillations are usually called quasilocal.

The function $F_1(\omega - eV)$ has a maximum at the point $\omega = eV$, which is characterized by a width $\sim T$. Figure 1 shows a plot of $F_1(y)$.

The resonance character of the behavior of the functions F_1 and $\gamma(\omega)$ make it possible to find immediately the dependence of the derivatives of (15)—(17) with respect to the potential in the two limiting cases.

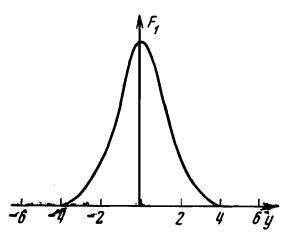

FIG. 1

1) The degree of diffuseness of the function F_1 is much less than the width of the diffuseness of the function $\gamma(\omega)$ ($T \ll \Gamma_0$).

In this limit, after integration of the expressions (15)–(17) with respect to ω we obtain the following results:

$$G_1(eV) = -\frac{c}{\pi} V^2 \varepsilon\gamma(eV) \frac{d}{dV^2} A(eV), \qquad (22)$$

$$G_2(eV) = \frac{c}{\pi}\left(\frac{Z_1 - Z_0}{Z_0}\right)^2 \frac{\gamma(eV)}{\varepsilon(eV)^2} \int_{-\infty}^{\infty} d\Omega\, \Omega G(\Omega, 0), \qquad (23)$$

$$G_3(eV) = \frac{2c}{\pi} \frac{Z_1 - Z_0}{Z_0} \gamma(eV)\left[A(eV) + G(eV, 0) \frac{1 - (eV)^2 eJ(eV)}{(eV)^2 \varepsilon g(e^2V^2)}\right], \qquad (24)$$

We shall assume that the amplitudes of the scattering by the impurity atoms and the unperturbed crystal are practically equal ($Z_1 = Z_0$). Then the correction to the current as a whole is determined by the relation (22). The quantity $A(\omega)$ behaves qualitatively in the same fashfashion as $J(\omega)$. In this connection, in the vicinity of a quasilocal or local level, $dA(\omega)/d\omega^2 < 0$ and, as seen from Eq. (20), a resonant positive (for $V > 0$) spike will appear on the second derivative for $eV \approx \omega_0$. If $\Gamma_0 \ll \omega_0$, then the form of the spike will be Lorentzian with a center at $eV = \omega_0$. The amplitude of the spike has the relative intensity $\alpha_1 \sim c/\pi^2\omega_0^2 g(\omega_0^2) \sim (\omega_D a/\omega_0 r_0)^2$, where a is a quantity of the order of the lattice constant of the crystal, r_0 the mean distance between the impurity ions, $\alpha_i = G_i(\omega_0)/G(\omega_0)$. The estimate of α_1 is carried out for the case of quasilocal oscillations ($\omega_0 < \omega_D$). As is seen, values of the order of unity are easily obtained experimentally. In the case of local oscillations, the intensity of interaction of the tunneling electrons with the phonons changes insignificantly; however, $G(\omega_0) \equiv 0$ and in this connection, the whole effect that is different from zero in the vicinity of $eV \approx \omega_0$ will be connected with this oscillation.

We now consider the case in which $Z_1 \neq Z_0$. In this case, in addition to the term G_1, which as a whole is due to the deformed phonon spectrum of the crystal with impurities, a contribution to the current will also be made by G_2, which arises from the excitation of the impurity by the tunneling electron and also by the interference term G_3. In order of magnitude, the relative intensity α_2 is equal to $\chi^2\alpha_1$, where $\chi = |(Z_1 - Z_0)\omega_D/\epsilon\omega_0|$. It is easy to see that χ in the real situation can change from zero to a value of the order of several units and consequently, for certain conditions, the term G_2 can significantly exceed G_1. However, the shape of the spike that arises in G_2 does not differ from G_1 and, from the experimental viewpoint, no more detailed information is contained in it in comparison with G_1. So far as the interference term G_3 is concerned, it can considerably change the shape of the resulting spike on the second derivative of the tunnel current for the case $T \ll \Gamma_0$. The relative intensity is of the order of $\chi\alpha_1$. The first term in G_3 in the vicinity of resonance can be both larger and smaller than zero, depending on the sign of the quantity $Z_1 - Z_0$ and, like G_2, practically does not change the shape of the spike. However, the second term has the form of an S-shaped curve with center at $eV \approx \omega_0$. The qualitative form of the resulting curve in the vicinity of a quasilocal oscillation is shown in Figs. 2 and 3. As is seen from

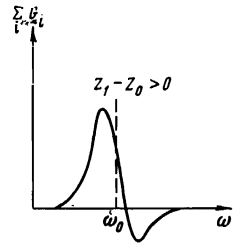

FIG. 2

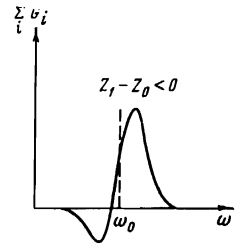

FIG. 3

these figures, information can be extracted from the shape of the resonance spike regarding the sign of the charge of the impurity ion in the given matrix in relation to the charge of the ion of the unperturbed matrix.

2) The other limiting case corresponds to the inequality $\Gamma_0 \ll T$. Now the function $\gamma(\omega)$ has a more sharply expressed maximum. After integration over ω, we get ($\omega_0 > 0$)

$$G_1(eV) = -\frac{c}{|b|} \omega_0^2 F_1(\omega_0 - eV) \frac{d}{d\omega_0^2} A(\omega_0), \qquad (25)$$

$$G_2(eV) = \frac{c}{|b|}\left(\frac{Z_1 - Z_0}{Z_0}\right)^2 \frac{F_1(\omega_0 - eV)}{\varepsilon^2\omega_0^2} \int_{-\infty}^{\infty} d\Omega\, \Omega G(\Omega, 0), \qquad (26)$$

$$G_3(eV) = \frac{c}{|b|} \frac{Z_1 - Z_0}{\varepsilon Z_0} F_1(\omega - eV) A(\omega_0). \qquad (27)$$

As is seen from these relations, there will be a positive resonance spike on the second derivative of the tunnel current even in this case. The shape of this spike is completely determined by the function F_1. Distortion of the symmetry of the spike, which is possible because of the interference term for the case in which $\Gamma_0 \gg T$, is now absent; more precisely, it has a smallness of the order of Γ_0/T and is omitted in the expression (28).

We note that the spikes on the second derivative of the tunnel current, which are associated with local oscillations, were recently observed experimentally by Schein and Compton.[9] In our opinion, the relatively large width of these spikes ($\Delta\omega/\omega_0 \sim 0.06$) indicates that the case in which $\Gamma_0 \ll T$ is achieved. However, if the functions F_1 are used for estimate of the temperature of the experiment, then it is easy to obtain the result that $T \sim 12°K$. An experiment was carried out in[9] at $T = 4.2°K$ and, as the authors observe, the width of the spike depends weakly on the temperature. Evidently, this is connected with the fact that local heating arises in this region of the junction contact, to which pressure is applied. The latter is also confirmed by the experiments of Voloshin et al.,[10] in which the population of the rotational levels of the HCl molecule injected into the tunnel contact was studied.

In this case, when the tunneling electrons interact with the molecules in the vicinity of the tunnel barrier in the same fashion was the relations (15)–(17) were obtained for the second derivative of the tunnel current G, it is easy to obtain the expression

$$G = \pi e^3 c N^2(0) \sum_{n,m} B_{nm}(f_n - f_m) F_1(E_n - E_m - eV). \qquad (28)$$

Here f_n is the population of the energy level with energy of the molecule E_n, B_{nm} is the square of the matrix element of interaction, averaged over the Fermi surface,

$$B_{nm} = \frac{1}{(4\pi)^2} \int d\mathbf{n}_1 \int d\mathbf{n}_2 |U_{nm}(p_0\mathbf{n}_1, p_0\mathbf{n}_2)|^2,$$

where p_0 is the Fermi momentum, n a unit vector, U_{nm} the matrix element of the interaction potential of electrons with the molecule.

It was shown in[10] that for a decrease in the isolating layer, the population of rotational molecular levels HCl for one and the same temperature of the thermostat (T = 1.6°K) increased anomalously. The latter correlates well with the assumption on the local heating. The fact is that for T = 1.6°K, the molecule of HCl should be found in the lowest state with j = 0 (j is the rotational quantum number) and, as is seen from the formula (29), only the contribution of transitions from the ground state into the excited enters into the tunnel current. However, in an experiment[10] on the contacts with sufficiently narrow dielectric layers, the intensities of the spikes that appear because of transitions with j = 1 and j = 2 achieve the same order as with j = 0. On wide dielectrics, the heating per unit volume decreases (since the total power of the heating falls off appreciably because of the exponential lessening of the transparency of the barrier) and only spikes associated with the excitations of molecules from the ground state are observed.

[1] Superconductivity, a collection of papers. Nauka, 1967.

[2] D. J. Scalapino, J. R. Schrieffer and J. W. Wilkins, Phys. Rev. **148**, 263 (1966).

[3] H. Helmut and A. Schmidt, Z. Physik **211**, 313 (1968).

[4] C. B. Duke and G. G. Kleiman, Phys. Rev. **2**, 1270 (1970).

[5] A. Bennet, C. B. Duke and S. D. Silverstein, Phys. Rev. **176**, 969 (1968).

[6] J. M. Rowell, W. L. McMillan and W. L. Feldmann, Phys. Rev. **180**, 658 (1969).

[7] Yu. Kagan and A. P. Zhernov, Zh. Eksp. Teor. Fiz. **50**, 1107 (1966) [Soviet Phys.-JETP **23**, 737 (1966)].

[8] Yu. M. Ivanchenko, Preprint, Donets Physico-technical Institute, Academy of Sciences, Ukrainian SSR No. 76, 1965.

[9] L. B. Schein and W. Dall Compton, Phys. Rev. **4B**, 1128 (1971).

[10] V. A. Voloshin, Yu. M. Ivanchenko and O. M. Ignat'ev, Plenum komissii po optike i spektroskopii (Plenary Session of the Commission on Optics and Spectroscopy) Academy of Sciences, Ukrainian SSR, Khar'kov, 1970.

Translated by R. T. Beyer
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