DOPPLER-SHIFTED SPIN RESONANCE IN THE ELECTRON FLUID OF METALS

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Submitted January 25, 1972

Zh. Eksp. Teor. Fiz. 63, 335—344 (July, 1972)

The spatial distribution of the radio-frequency magnetization of an electron fluid is investigated. It is shown that the magnetization produced in the bulk metal by Doppler-shifted spin resonance is distributed harmonically with a period determined by the parameters of the Fermi surface. The role of quasi-particle spin correlation in such a resonance is considered. The static paramagnetic structure of metals is discussed.

1. Spin waves in nonferromagnetic metals,[1] the possibility of propagation of which has been confirmed experimentally,[2] are the characteristic oscillations of the spin system of the quasiparticles of an electron fluid. Similar collective excitations are capable of leading to the anomalous penetration of the electromagnetic field into the bulk of the metal. The spatial distribution of this field depends on the energy spectrum of weakly attenuated waves of magnetization,[3,4] which is determined both by the spin correlation between the electrons and by the topology of their Fermi surface. However, in a number of cases, the region of propagation of the spin waves is limited by the requirement of the absence of Cerenkov absorption of their energy by the spins of the individual quasiparticles, which leads to strong damping. We note that similar absorption, accompanied by the spin reversal of the quasiparticle, can, in the case of its motion along the direction of propagation of the wave, be treated as a spin resonance at a frequency shifted by virtue of the Doppler effect.[5] In the present research, we shall show that although the Cerenkov spin absorption also leads to strong damping of the collective excitations of the magnetization, it contributes at the same time to the creation in the bulk of the metal of a harmonic distribution of the magnetization of another origin, due to the individual motion of the charged particles.

2. We consider the quasiparticles of an electron fluid, drifting in the field of constant magnetic induction $\mathbf{B}_0$ with a nonzero velocity v_{B_0}. In addition to the cyclotron rotation of the electrons about $\mathbf{B}_0$, precession takes place with a frequency ω, a precession of the spin magnetic moments directed parallel or antiparallel to the field $\mathbf{B}_0$ (naturally, the parallel orientations are energetically more favorable). In a variable magnetic field of frequency equal to the spin precession frequency, the probability develops of a reorientation of the magnetic moments. From the quasiclassical viewpoint, it is possible to represent it as a reversal of the magnetic moment, which continues to precess about $\mathbf{B}_0$, taking place within the time of the spin relaxation T_1. Such reversals lead to the appearance of a precessing transverse magnetization. However, as a consequence of the drift motion of the electrons, a Doppler effect takes place, and the quasiparticles experience a radio-frequency (RF) field at the frequency $\omega' = \omega - qv_{B_0}$, which differs from the field frequency ω. Here q is the length of the wave vector of the spin wave propagating along $\mathbf{B}_0$ the spin wave is given by the dispersion equation for the magnetic excitations of the electron fluid. Since the velocity v_{B_0} can be large, then in the case of waves with sufficiently large q, the electron spins can experience the resonant frequency $\omega' = \omega_0$, when the frequency of the RF field is far from ω. It is clear that the magnetic moment of the quasiparticle will resonantly remove energy from that harmonic of the spin wave packet whose wavelength λ is equal to d—the displacement of the given quasiparticle along the constant magnetic field within one period of precession with frequency $|\omega - \omega_0|$, i.e., $\lambda = d = 2\pi v_{B_0}/|\omega - \omega_0|$. The group of electrons which drift with velocity v_{B_0}, for which the condition of the Doppler-shifted spin resonance is satisfied for the same q, creates a transverse magnetization that precesses with frequency ω. As a consequence of the systematic motion of the quasiparticles, this is distributed harmonically over the volume of the metal with period d. Since the dif different groups of electrons drift with different velocities, the spatial oscillations of the magnetization of a single group of electrons will be smeared out in the total observed effect because of the presence of a group of electrons with slightly different velocities. An exception will be groups of those electrons which undergo extremal displacement d along the direction of propagation of the wave. The oscillations of the magnetization created by them are not neutralized and consequently appear in the total effect.

It should be noted that drift of the electrons along the direction of propagation of the spin wave is also possible when the constant magnetic field $\mathbf{B}_0$ is perpendicular to this direction. In that case, it is required that the Fermi surface be open and the field $\mathbf{B}_0$ be orthogonal to the mean direction of openness $\mathbf{n}$.

It follows from the properties of the different trajectories of the quasiparticles in a constant magnetic field[6] that there exist three cases of extremal displacement of the quasiparticles along the direction of propagation of the spin wave: a) The electrons move along helical trajectories which correspond to orbits of finite radius on the Fermi surface with extremal values of the velocity. Here

$$d = d_1 = \frac{c\varkappa}{eB_0}\left(\frac{\partial A}{\partial p_{B0}}\right)_{extr}, \tag{1}$$

where $A(p_{B_0}, \epsilon)$ is the cross section area of the energy surface of the plane p_{B_0} = const, p_{B_0} is the component of the momentum of the quasiparticle $\mathbf{p}$ along $\mathbf{B}_0$,

$\kappa = \omega_c/|\omega - \omega_0|$, and ω_c is the cyclotron frequency, which we shall assume to be constant. b) The electron orbits lie inside the limiting points on the Fermi surface. In this case

$$d = d_2 = 2\pi c\varkappa K^{-1/2} / eB_0, \tag{2}$$

where K is the Gaussian curvature of the energy surface at the limiting point. c) The quasiparticles move along open trajectories which are characterized by the same displacement

$$d = d_3 = cp_0\varkappa \cos\vartheta / eB_0, \quad \mathbf{B}_0 \perp \mathbf{q}, \quad \mathbf{B}_0 \perp \mathbf{n}, \tag{3}$$

where ϑ is the angle between $\mathbf{q}$ and $\mathbf{n}$, and p_0 is the period of the open orbit. Thus, it is clear that the period of the resultant harmonic distribution of the magnetization in the metal is determined by the different parameters of the Fermi surface. The characteristic damping time of magnetization in the given case is t_0—the time of relaxation of the momentum of the quasiparticle; therefore, for the existence of oscillations, it is necessary that the condition $\omega_0 t_0 \gg 1$ be satisfied.

We note that the phenomenon considered by us is outwardly similar to the radio-frequency size effect, which is due to the focusing of ineffective electrons by a magnetic field,[7] but appears as a result of some other (spin) mechanism.

3. We investigate the nonequilibrium magnetization $\mathbf{M}^1$, excited by the RF field $\widetilde{\mathbf{B}}(\sim e^{-i\omega t})$ in a semi-infinite metallic sample found in the field of constant magnetic induction $\mathbf{B}_0$. In the calculations, we shall use both the set of coordinates xyz with the z axis directed along $\mathbf{B}_0$, and the ζ axis which passes through the initial coordinate system and is directed into the interior of the metal, normally to the surface of the specimen. We note that, by virtue of the symmetry of the problem, all the quantities in the metallic half-space $\zeta > 0$ considered, which are functions of the coordinates, depend only on ζ. The transverse nonequilibrium magnetization $M_+^1 = M_x^1 + iM_y^1$ is determined by the expression

$$M_+^1(\zeta) = \mu_0 \int d\tau_p \delta g_+(\mathbf{p}, \zeta), \tag{4}$$

where $d\tau_p = (2\pi\hbar)^{-3} d^3p$ and $\delta g_+(\mathbf{p}, \zeta)$ is the nonequilibrium distribution function of the spin density of the quasiparticles, which, in the linear approximation, obeys the equation[3,4] 1)

$$\frac{\partial}{\partial t}\delta g_+ + \left(v_\zeta \frac{\partial}{\partial \zeta} + \omega_c \frac{\partial}{\partial \varphi} + i\omega_0 + \frac{1}{T_1} + \frac{1}{t_0}\right)\left(\delta g_+ + \frac{\partial n_0}{\partial \varepsilon}\beta B_+^1\right) \tag{5}$$
$$= \frac{1}{t_0}\frac{\partial n_0}{\partial \varepsilon}\left(\int d\tau_p \frac{\partial n_0}{\partial \varepsilon}\right)^{-1} \int d\tau_p \left(\delta g_+ + \frac{\partial n_0}{\partial \varepsilon}\beta B_+^1\right).$$

Here B_+^1 is the total nonequilibrium magnetic induction:

$$\beta B_+^1 = 2\mu_0 B_+^1 - 2\int d\tau_p' \Phi(\mathbf{p}, \mathbf{p}')\delta g_+(\mathbf{p}', \zeta),$$

$\Phi(\mathbf{p}, \mathbf{p}')$ characterizes the spin correlation of the quasiparticles of the electron fluid, ϵ and n_0 are respectively the energy and Fermi distribution function of the quasiparticles, and φ is the phase of the electron which rotates about the orbit with frequency ω_c.

Equation (5) is solved jointly with the Maxwell equations. The boundary conditions which need to be considered here are that on the surface of the metal $\zeta = 0$, the tangential components of the magnetic and electric fields are continuous, and as $\zeta \to \infty$ all the functions vanish. We shall assume that the effects associated with the character of the reflection of the quasiparticles from the surface are unimportant.[8,9] This circumstance permits us to obtain a solution of the problem in closed form, by reducing it to the finding of the distribution of the magnetization in an unbounded medium. We continue the nonequilibrium magnetic induction B_1^+ in even fashion into the region $\zeta < 0$ and introduce its Fourier component

$$b_q^+ = 2\int_0^\infty d\zeta B_1^+(\zeta)\cos q\zeta, \tag{6}$$

where q is the component of the wave vector $\mathbf{q}$ along ζ. Similar expansions are valid for δg_+ and M_+^1. Moreover, let us assume the simple model proposed by Silin[10] of a spin correlation in which $\Phi(\mathbf{p}, \mathbf{p}')$ is approximated by the constant quantity

$$\Phi(\mathbf{p}, \mathbf{p}') = -\beta_0 \Big/ 2\int d\tau_p \frac{\partial n_0}{\partial \varepsilon} = \text{const.} \tag{7}$$

As a result of the solution of Eq. (5), making use of formulas (4) and (6), we obtain the following relation, which connects m_q^+ and b_q^+—the spatial Fourier components of the nonequilibrium magnetic induction[10]

$$m_q^+ = \chi \frac{1 - (1 + i/\omega t_0)X(q, \omega)}{1 - [i/\omega t_0 + \beta_0/(1+\beta_0)]X(q, \omega)} b_q^+. \tag{8}$$

Here

$$\chi = -\frac{2\mu_0^2}{1+\beta_0}\int d\tau_p \frac{\partial n_0}{\partial \varepsilon}$$

is the static paramagnetic susceptibility of the electron fluid, and the function $X(q, \omega)$ in the general case can be written in the form

$$X = -i\omega\left\langle \frac{1}{\omega_c}\int_{-\infty}^{\varphi} d\varphi' \exp\left\{\frac{i}{\omega_c}\left[\left(\omega - \omega_0 + \frac{i}{T_1} + \frac{i}{t_0}\right)(\varphi - \varphi') - \int_{\varphi'}^{\varphi} qv_\zeta\, d\varphi''\right]\right\}\right\rangle, \tag{9}$$

where $\langle \ldots \rangle$ denotes averaging both over the closed and the open orbits on the Fermi surface. Strictly speaking, in the case of open trajectories, the phase variable φ must be replaced by a variable which changes along the orbit of the electron in p space, and the characteristic time of the motion must be the time within which the momentum of the quasiparticle changes by an amount equal to the period of the orbit in the direction of openness.

It is well known[3,16] that the field equations for not too high frequencies reduce to

$$\operatorname{rot}(\mathbf{B}^1 - 4\pi\mathbf{M}^1) = 0, \qquad \operatorname{div}\mathbf{B}^1 = 0. \tag{10}$$

It then follows that the spectrum of the magnetic excitations of the electron fluid is determined by the denominator of (8), i.e., it corresponds to the poles of the magnetization m_q^+. We note that averaging over the Fermi surface can lead to the appearance in (8) of additional singularities of the type of branch points. We find the sought magnetization M_+^1 after the inverse Fourier transformation in (8); first, one must carry out averaging in Eq. (9). In the case of an arbitrary dispersion law, this can only be done approximately. Here it is difficult to compare the relative value of the contributions to the integral over q from the different singularities m_q^+, each

1) Equation (5) differs inessentially from that obtained by Silin. [3,4]

of which corresponds to a definite mechanism of formation of the magnetization. Therefore we consider one problem further, furnishing specific models of the energy surface which are so selected that in each of them there would be one of the cases a), b), and c) enumerated above of extremal displacement of the quasiparticles.

4. In this section, we consider the situation when the constant magnetic field directed along the axis of symmetry of the energy surface is perpendicular to the surface of the metal.

A. We shall assume that the dispersion law has the form[11]

$$\varepsilon = \frac{p_\perp^2}{2m^*} - \frac{2p_1 v_1}{\pi}\sin^2\frac{\pi p_z}{2p_1}, \qquad (11)$$

where m* is the effective mass of the quasipatricle, $p_\perp$ is the component of its momentum perpendicular to the z axis, and p_1 and v_1 are parameters with the dimensions of momentum and velocity. The corresponding Fermi surface $\epsilon = \epsilon_F$ is a corrugated cylinder with necks at $p_z = (2n + 1)p_1$ (we assume $2p_1v_1/\pi < \epsilon_F$) and as a boundary of the Brillouin zone, we take $p_z = \pm p_1$. We note that, in spite of the fact that this surface is open, for the orientation of $\mathbf{B}_0$ chosen by us, only closed electron orbits in p space are possible. It follows from Eq. (11) that the velocity of motion of the electrons along $\mathbf{B}_0$

$$v_z(p_z) = \partial\varepsilon / \partial p_z = v_1 \sin(\pi p_z / p_1) \qquad (12)$$

has the extrema $\pm v_1$ for $p_z = \pm p_{1/2}$. This means that the extremal values of v_z are achieved on the helical trajectories. The expression (9) has the form

$$X = \Delta[(\Delta - 1 + i\eta)^2 - s^2]^{-1/2}, \qquad (13)$$

where $\Delta = \omega/\omega_0$, $s = qv_1/\omega_0$, $\eta = 1/\omega_0 t_0 + 1/\omega_0 T_1$, while the square root is determined in the complex s plane with cuts going off to infinity (see the figure), the points of which correspond to the conditions of Doppler-shifted spin resonance of the individual electrons moving with the velocities $v_z(p_z)$, and are given by the equation

$$\Delta - 1 - sv_z / v_1 + i\eta = 0. \qquad (14)$$

Choice of the analytic branch of the function $[1 - s^2/(\Delta - 1 + i\eta)^2]^{1/2}$ is understood from the drawing: the values of this function are purely imaginary on the cuts and the imaginary part is positive on the right side of the upper cut and on the left of the lower cut.

Setting the denominator of (8) equal to zero, we obtain the dispersion equation for collective excitations of the magnetization

$$F_1^{-1}(s) = [(\Delta - 1 + i\eta)^2 - s^2]^{1/2} - ih = 0, \qquad (15)$$

where $h = a - i\Delta f$, $f = \beta_0/(1 + \beta_0)$, $a = 1/\omega_0 t_0$. We note that the propagation of spin waves without Cerenkov spin absorption is possible only for

$$|s| < s_{\text{lim}} = |\Delta - 1|, \qquad (16)$$

i.e., the limiting values of s lie near the root branch points $s_b = \pm(\Delta = 1 + i\eta)$, from which the cuts are also constructed.

Having in mind an investigation of effects connected with Doppler-shifted spin resonance, which are the greatest for $\omega \neq \omega_0$, we eliminate from consideration the range of frequencies which correspond to $\Delta \sim 1$. Then, neglecting damping, we get from Eq. (15)

$$s_{1,2} \approx \pm[\Delta - 1 - (\Delta f)^2 / 2(\Delta - 1)], \quad |\Delta f| \ll |\Delta - 1|. \qquad (17)$$

Nonzero solutions of (15) for $\Delta = 0$ indicate a possible existence of a static paramagnetic structure of the metal[12,13] with period $2\pi v_1/\omega_0$. It is seen from the relation (17) that exchange interaction between the spins of quasiparticles leads to a frequency-dependent shift of the wave vectors of the spin waves from their limiting values to smaller |s|. In what follows, it is convenient to divide the frequency region considered into two parts: the low frequency, in which the imaginary part of s, which is responsible for the wave damping, exceeds the exchange shift ($\eta \gg (\Delta f)^2/2s_{\text{lim}}$) and the high frequency, in which the latter term prevails ($(\Delta f)^2/2s_{\text{lim}} \gg \eta$).

Since the wavelengths of the magnetic excitations, which make the principal contribution to M_+^1, are much greater than the thickness of the skin layer, then, using the even parity of X(s) in s, with the aid of Eqs. (6), (8) and (13), we get the following expression for the magnetization in the depth of the metal:

$$M_+^1(\zeta) \approx \frac{\chi\omega_0}{2\pi v_1}\delta\Delta(f - 1)\tilde{B}_+(0)\int_{-\infty}^{\infty} ds\, e^{isu}F_1(s), \quad \zeta \gg |\delta|, \qquad (18)$$

where the effective penetration depth of the exciting RF field in the sample is

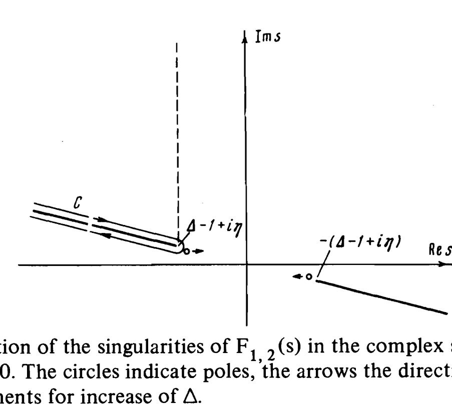
$$\delta = \tilde{B}_+^{-1}(0)\int_0^\infty d\zeta\, B_+^1(\zeta),$$

and $u = \zeta\omega_0/v_1$ has the meaning of the number of spin reversals in the time of passage of the quasiparticle through the distance ζ. We close the contour of integration in the upper half-plane of the complex variable s. Then, in the integral over s, which appears in (18), contributions are made both by the residues of the pole and also by I_C—the integral over the contour C which goes around the cut. Taking into account the jump in $F_1(s)$ in the transition from one side of the cut to the other, we reduce I_C to an integral around the line of the cut, which in turn is equal to an integral over the dotted line parallel to the imaginary axis (see the drawing). As a result, after calculations, we have ($u \gg 1$, $a \approx \eta \ll 1$)

$$I_c \approx \begin{cases} [(2\pi)^{1/2} i^{3/2} u^{-1/2}(\Delta - 1)^{-1/2} + \pi h(\Delta - 1)^{-1}]\exp[iu(\Delta - 1 + i\eta)], \\ \qquad (\Delta f)^2/2s_{\text{lim}} \ll \eta, \\ -2[2\pi i(\Delta - 1)]^{1/2} u^{-3/2}(\Delta f)^{-2}\exp[iu(\Delta - 1 + i\eta)], \qquad (19a) \end{cases}$$

$$(\Delta f)^2/2s_{\text{np}} \gg \eta. \qquad (19b)$$

The contribution to the integral over s from the residue



Location of the singularities of $F_{1,2}(s)$ in the complex s plane for $(\Delta - 1) < 0$. The circles indicate poles, the arrows the directions of their displacements for increase of Δ.

at the pole of $F_1(s)$, which lies in the upper half-plane, is equal to

$$I_p = 2\pi h[(\Delta - 1 + i\eta)^2 + h^2]^{-1/2}\exp\{iu[(\Delta - 1 + i\eta)^2 + h^2]^{1/2}\}. \quad (20)$$

We compare the relative values of the contributions to (18) made by the different singularities of $F_1(s)$. It is not difficult to establish the fact that at low frequencies, the first component of the integral over the contour C dominates. Therefore, in accord with (1) and (18), we can assume that

$$M_+^1(z) \approx \frac{\chi\delta\Delta(1-f)\tilde{B}_+(0)}{(zd_1)^{1/2}|\Delta-1|}\exp\left[i\frac{2\pi z}{d_1}\operatorname{sign}(\Delta-1) - i\psi - \frac{z}{l_1}\right], \quad (21)$$

where $l_1 = v_1t_0$ is the free path length of the quasiparticle, and $\psi = \arg[i(\Delta - 1)]^{1/2}$. It is known[7] that the action of the exciting RF field in the low-frequency case reduces to the formation of a skin layer near the surface of the metal. Therefore, as $\Delta \to 0$, the expressions (19a), (19b) and (20) can be used for consideration of the static paramagnetic structure of the metal[12], which arises in a strong constant magnetic field perpendicular to the surface in the presence of a weak, constant, transverse magnetic field incident at the skin layer depth. It is evident from the formulas mentioned that for $\Delta = 0$, within the framework of our considerations, the static structure disappears in the bulk of the metal, although the corresponding solution (17) of the dispersion equation also indicates the possibility of its existence. So far as the quasistatic structure is concerned, which arises for small $\Delta \neq 0$, it is due to the presence of the root branch point and not to the polar singularity.

The situation is different in the high frequency region. The principal contribution to the integration in (18) is made by the residue and the corresponding asymptote of the magnetization has the form

$$M_+^1(z) = \frac{\chi\delta\omega_0\Delta^2(f-1)f\tilde{B}_+(0)}{v_1 s_1}\exp\left(i\frac{z\omega_0 s_1}{v_1} - i\frac{\pi}{2} - \frac{z}{l_1}\right) \quad (22)$$

(we recall that s_1 was defined in Eq. (17)).

The results (21) and (22) indicate a rather curious fact; with change in frequency ω, the nature of the excitations which dominate in the creation of a nonequilibrium magnetization in the bulk of the metal also changes. Actually, if the magnetization at low frequencies is due principally to the motion of individual quasiparticles, then, in the transition to the high-frequency region, the decisive role belongs to the collective excitation—the spin wave. The reason for this is that, because of spin correlation between the quasiparticles, the condition of the Doppler-shifted spin resonance is violated with increase in frequency for electrons with extremal velocity v_z, and the spin waves are propagated almost without attenuation. From the mathematical point of view, this is due to the "departure" of the poles from the vicinity of the branch points (see the figure). Thus, even in the phenomenon considered by us, the correlation between quasiparticles leads to a new distinction in the behavior of nonequilibrium magnetization in comparison with that observed in a gas of noninteracting electrons.

B. In the case of a quadratic dispersion law, the extremal displacement along the direction of wave propagation is experienced by electrons of the reference points of the Fermi surface, at which $v_z = \pm v_2 = \pm v_F$ and Gaussian curvature $K = p_F^{-2}$. The nonequilibrium magnetization M_+^1 can be found from Eq. (18), where we replace v_1 by v_2 and $F_1(s)$ by

$$F_2(s) = \left\{2s\ln^{-1}\left(\frac{\Delta-1+s+i\eta}{\Delta-1-s+i\eta}\right) - ih\right\}^{-1}, \quad (23)$$

and the function $F_2(s)$ is determined in the complex s plane with cuts similar to those considered above, such that on the right edge of the upper cut, $\ln(\ldots) = \ln|\ldots| + i\pi$.

We also note that the quantity $|f| \ll 1$ which usually enters in $h = a - i\Delta f$ is negative.[2,3] Under these conditions, it follows from the dispersion equation $F_2^{-1}(s) = 0$ that weakly attenuated collective excitations of the magnetization are possible only for $|\Delta f| \gg a$, $\Delta < 1$ (as before, we do not consider the region of frequencies $\Delta \sim 1$) and correspond to

$$s_{1,2} \approx \mp[s_{\lim} - i\eta - 2s_{\lim}\exp(2s_{\lim}/\Delta f)], \quad s_{\lim} = |\Delta - 1| \gg |\Delta f|. \quad (24)$$

The presence of a pole in the upper half-plane determines the component appearing in the integral in (18):

$$I_p \approx -8\pi i s_{\lim}^2(\Delta f)^{-2}\exp(2s_{\rm np}/\Delta f + ius_1). \quad (25)$$

However, integration over the contour C, which goes around the cut from the branch point $s_b = \Delta - 1 + i\eta$, gives a contribution that is many times greater to the integral over s. This is connected with the fact that the exchange shift in the dimensionless wave vectors s from their limiting values cannot surpass Im s by very much and prevents damping of the waves. Therefore, the radio-frequency magnetization in the bulk metal is produced almost entirely as a result of Doppler-shifted spin resonance, due to the motion of the individual quasiparticles and has the form

$$M_+^1(z) \approx \frac{2\chi\delta|\Delta-1|\Delta(1-f)\tilde{B}_+(0)}{h^2 z} \times G(z,\Delta)\exp\left[i\frac{2\pi z}{d_2}\operatorname{sign}(\Delta-1) + i\arg(\Delta-1) - \frac{z}{l_2}\right], \quad (26)$$

$$G^{-1}(z,\Delta) = [2(\Delta-1)/h - i\ln(4\pi z/d_2) + (\pi/2)\operatorname{sign}(\Delta-1)]^2 - \pi^2 \quad (27)$$

In deriving (26), we used the saddle-point method. We note that the first component in square brackets is dominant in (27).

5. We now turn to the case in which the quasiparticles move along open trajectories, which tells a great deal about the character of the effect considered. Let the magnetic field $\mathbf{B}_0$ be parallel to the surface of the sample and the y axis coincide with the ζ axis, along which the spin waves propagate that are excited by the RF field $\tilde{B}_x$. For simplicity, we shall assume that all the electron trajectories are open. A similar situation is possible in the case in which the Fermi surface is modeled by a right circular cylinder whose axis is directed along the x axis. Under these conditions, the movement of the quasiparticle in momentum space takes place along the p_x axis, and in coordinate space, along the y axis; half of them move with the same mean velocity $\bar{v}_y = v_3 = p_0c/eB_0T$ and half with the velocity $-v_3$.[6,9] Averaging is carried out over T—the time of transit by the electron of the period of the orbit p_0 in p space. It should be clear that one means $2\pi/T$ by the cyclotron frequency ω_c in the expression (3). It follows from the

relations (6), (8) and (9) that the magnetization $M^1_+(\zeta)$ in the example considered is given by the formula (18) with the replacement in it of v_1 by v_3 and $F_1(s)$ by

$$F_3(s) = [(\Delta - 1 + i\eta)^2 - s^2 - ih(\Delta - 1 + i\eta)]^{-1}, \quad (28)$$

the only singularities of which are the poles

$$s_{1,2} \approx \pm(\Delta - 1 - \Delta f/2 + ia/2), \quad |\Delta f| \ll |\Delta - 1|. \quad (29)$$

As a result, we get the relation

$$M_1^+ \approx \frac{\chi\omega_0\delta|\Delta - 1|\Delta(1-f)B_z(0)}{2v_3 s_1} \exp\{is_1 u + i\arg[i(\Delta - 1)]\}. \quad (30)$$

In spite of the fact that $F_3(s)$ does have a characteristic branch, peculiar to the model considered, the fact that for $\Delta \ll 1$ the Eqs. (29) go over into the condition of resonance absorption indicates that these singularities stem from the presence of a threshold of spin absorption at the Doppler-shifted frequency. The phase change of the magnetization (30) on going through $\Delta = 1$, brought about by the fact that electrons with velocities v_ζ of opposite sign participate in the resonance on the two sides of $\Delta = 1$, supports this view. The spin waves (29) are analogous to dopplerons—waves that appear near the absorption thresholds for Doppler-shifted cyclotron resonance.[11,13]

Analyzing the problem of static paramagnetic structure (touched on by us in Sec. 4) with the help of (30), we note that such an effect disappears in the bulk of the metal at $\Delta = 0$ and the quasistatic structure, which arises for small $\Delta \neq 0$, is periodic in the direction $\mathbf{y} \perp B_0$ and is described by the poles of $F_3(s)$.

6. The expressions (21), (26), and (30), obtained for specific models, are also applicable for the qualitative description of the distribution of magnetization in metals with very complicated energy surfaces. Actually, we shall consider that the principal contribution to the oscillation of the magnetization is made by electrons which achieve the extremal displacement d, and we expand v_ζ in (9) in a series near the points of the Fermi surface corresponding to extremal d. It is clear from formulas (8) and (9) that the character of the singularities of m^+_q, which are connected with the Doppler-shifted spin resonance, is determined by the first (after zero) nonvanishing term of the expansion, i.e., it depends on the number of electron orbits for which the displacement d is close to extremal. In fact, a small number of limiting-point electrons determines the logarithmic branch and the quasiparticles on the extremal spiral trajectories the root branch. In the case of open trajectories, when the electrons drift along ζ with the same velocity, the singularities of m^+_q are poles. In accord with this, the amplitudes of the oscillations (26), (21) and (30) increase steadily with time. It is of interest that the first two of them are damped with the distance ζ in power fashion, as ζ^{-1} and $\zeta^{-1/2}$, respectively. We note that the dependence of v_s on φ in the general case leads to the appearance in the spatial oscillations of higher harmonics of the type $e^{2\pi in\zeta/d}$. So far as formula (30) is concerned, in real situations, in addition to the open trajectories, one must also take the closed trajectories into account, and also the trajectories which pass through the saddle point of the Fermi surface, near which the period T goes to infinity logarithmically. However, in the geometry considered, this corresponds to $\bar{v}_\zeta = 0$; therefore, their account only brings about a re-evaluation of the poles and does not change the picture qualitatively.

The harmonic distribution of the magnetization that has been discussed produces oscillations both of the spin contribution to the impedance and also of the RF field passing through the thick metal plane in a change of the constant field $\mathbf{B}_0$ or frequency ω. For $\Delta < 1$, this field is given by the expressions (21), (26) and (30), where one must put the thickness of the plate L in place of ζ. We recall that, near ordinary spin resonance $\Delta \sim 1$, it is described by formulas of "selective transparency."[2,8] In the experimental observation of the oscillations of the magnetization, it should be kept in mind that for $\Delta < 1$ they coexist with Gantmakher-Kaner oscillations,[6,7] the amplitude of which is generally greater. However, the fact that the harmonic distribution of the magnetization is damped out slowly with distance and has another period allows us to expect that in some case (large value of the free path), one can separate these two types of oscillations. In the high frequency region $\Delta > 1$, cyclotron waves can be propagated in the electron fluid even in the geometry considered.[3] For weak correlation between the quasiparticles, their minimal length[15] is large in comparison with the period of the harmonic distribution of magnetization and therefore the RF field, passing through the plate due to these waves, can be separated from the field of interest to us. We note that the most favorable situation for the observation of Doppler-shifted spin resonance is possible in metals where the electronic g factor is a little larger than two, since the condition of nondiffuse resonance $\omega_0 t_0 \gg 1$ in them can be satisfied in magnetic fields for which $\omega_c t_0 \lesssim 1$, and, consequently the Gantmakher-Kaner oscillations and the cyclotron waves are not observed.

The presence of Doppler-shifted spin resonance in metals can give information not only on the various parameters of the Fermi surface, but also on the value of the spin correlation of the quasiparticles, their effective mass and their g factor.

The author is deeply grateful to E. A. Kaner for useful discussion of problems touched on in the research.

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Translated by R. T. Beyer
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