SPIN-WAVE RELAXATION IN ANTIFERROMAGNETS

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The spin-wave relaxation rate in antiferromagnets is calculated in the case when the antiferromagnetic vector is directed perpendicular to a constant magnetic field. Various relaxation mechanisms are investigated. It is shown that the main contribution to the relaxation is made by four-magnon scattering processes.

ONE of the interesting problems in the physics of magnetically ordered crystals is the study of spin-wave relaxation. In recent years, a large number of theoretical papers[1-7] has been devoted to the study of relaxation processes in antiferromagnets. At the same time, investigations of parametric phenomena[8-12] give a considerable quantity of experimental data concerning spin-wave relaxation. To interpret these results, and also, ultimately, to make further progress in the study of parametric phenomena,[13] it is necessary to calculate the possible relaxation mechanisms rigorously on the basis of a method possessing sufficient accuracy. For this purpose, at low temperatures, we can make use of Dyson's formalism.[14] In this paper, this method is applied to investigate the relaxation of spin waves in an antiferromagnetic insulator with anisotropy of the "easy-plane" type.

We write the Hamiltonian of an antiferromagnet with "easy-plane" anisotropy in a two-sublattice model with exchange interaction between nearest neighbors in the following form:[15]

$$\mathcal{H} = -\sum_{\mathbf{R}_1\Delta} J(\Delta)\mathbf{S}_{\mathbf{R}_1}\mathbf{S}_{\mathbf{R}_1+\Delta} + 2\sum_{\mathbf{R}_1\Delta} D(\Delta)(S^z_{\mathbf{R}_1}S^y_{\mathbf{R}_1+\Delta} - S^y_{\mathbf{R}_1}S^z_{\mathbf{R}_1+\Delta}) + \sum_{\mathbf{R}} P(S^x_{\mathbf{R}})^2 + \sum_{\mathbf{R}}\sum_{\mathbf{r}\neq\mathbf{R}} Q(\mathbf{R}-\mathbf{r})S^x_{\mathbf{R}}S_{\mathbf{r}}{}^x - \mu_B gH\sum_{\mathbf{R}} S^z_{\mathbf{R}},$$

where zy is the basal plane of the crystal, $J(\Delta) < 0$, and

$$P - \sum_{\mathbf{r}\neq\mathbf{R}} Q(\mathbf{R}-\mathbf{r}) > 0.$$

Going over to the idealized-spin operators $\tilde{S}$ and then to spin-deviation Bose operators using the formulas[16]

$$\tilde{S}_{\mathbf{R}}{}^{\zeta} = S - a_{\mathbf{R}}{}^+a_{\mathbf{R}}, \quad \tilde{S}_{\mathbf{R}}{}^+ = (2S)^{1/2}\left(1 - \frac{a_{\mathbf{R}}{}^+a_{\mathbf{R}}}{2S}\right)a_{\mathbf{R}}, \quad \tilde{S}_{\mathbf{R}}{}^- = (2S)^{1/2}a_{\mathbf{R}}{}^+,$$

we obtain for the operators of the idealized-spin components in the laboratory coordinate frame

$$\tilde{S}_{\mathbf{R}}{}^z = \sin\psi(S - a_{\mathbf{R}}{}^+a_{\mathbf{R}}) \mp i\cos\psi(S/2)^{1/2}\left(a_{\mathbf{R}}{}^+ - a_{\mathbf{R}} + \frac{a_{\mathbf{R}}{}^+a_{\mathbf{R}}a_{\mathbf{R}}}{2S}\right),$$

$$\tilde{S}_{\mathbf{R}}{}^y = \pm\cos\psi(S - a_{\mathbf{R}}{}^+a_{\mathbf{R}}) + i\sin\psi(S/2)^{1/2}\left(a_{\mathbf{R}}{}^+ - a_{\mathbf{R}} + \frac{a_{\mathbf{R}}{}^+a_{\mathbf{R}}a_{\mathbf{R}}}{2S}\right),$$

$$\tilde{S}_{\mathbf{R}}{}^x = (S/2)^{1/2}\left(a_{\mathbf{R}}{}^+ + a_{\mathbf{R}} - \frac{a_{\mathbf{R}}{}^+a_{\mathbf{R}}a_{\mathbf{R}}}{2S}\right),$$

where $\sin\psi \approx H + H_D/2H_E$ for $H \ll H_E$; $H_E = z|J|S/\mu_B g$, $H_D = 2z|D|S/\mu_B g$, and the upper sign corresponds to the sublattice in the direction of the positive z and y semi-axes. Expanding $a_{\mathbf{R}}$ in a Fourier series in **k** and diagonalizing the part of the Hamiltonian that is quadratic in the Fourier components of the spin-deviation operators by means of the relations[17,18]

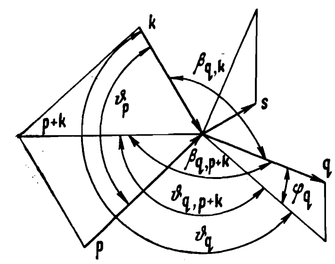

FIG. 1. Spherical system of coordinates in momentum space for the case of mutual scattering of two magnons.

$$a_{l\mathbf{k}} = \sum_{j=1}^{2}(u_{lj}c_{j\mathbf{k}} + v_{lj}c^+_{j-\mathbf{k}}),$$

where for $H \ll H_E$, $k\Delta \ll 1$,

$$u_{lj} \approx \frac{1}{2\varepsilon_{j\mathbf{k}}{}^{1/2}}\left(\Theta_c{}^{1/2} + \frac{\varepsilon_{j\mathbf{k}}}{2\Theta_c{}^{1/2}}\right)(-1)^{(1+l)\delta_{j,2}},$$

$$v_{lj} \approx -\frac{1}{2\varepsilon_{j\mathbf{k}}{}^{1/2}}\left(\Theta_c{}^{1/2} - \frac{\varepsilon_{j\mathbf{k}}}{2\Theta_c{}^{1/2}}\right)(-1)^{l\delta_{j,2}},$$

we find the spin-wave spectrum in the form[19,20]

$$\varepsilon_{j\mathbf{k}} \sim (\varepsilon_{j0}{}^2 + \Theta_c{}^2(k\Delta)^2)^{1/2}, \quad j = 1, 2; \ \Theta_c \sim \mu_B gH_E;$$

$$\varepsilon_{10} = \mu_B g[H(H+H_D)]^{1/2}, \quad \varepsilon_{20} = \mu_B g[2H_AH_E + H_D(H+H_D)]^{1/2}.$$

The purpose of the paper is to calculate the rate of relaxation of the spin waves of the low-frequency (LF—index 1) and high-frequency (HF—index 2) branches.

1. THE LOW-FREQUENCY BRANCH

In studying the relaxation of magnons of the LF branch, we shall discuss first of all the four-magnon scattering of magnons of the LF branch (11 → 11) (Fig. 1). To calculate the contribution of this process to the relaxation rate we make use of the lowest Born approximation[1)]

$$\Delta\omega_{1\mathbf{k}} = \sum_{\mathbf{pqs}}\frac{8\pi|\Psi^{\mathbf{q,s}}_{\mathbf{k,p}}|^2}{\hbar}[\bar{n}_{\mathbf{p}}(\bar{n}_{\mathbf{q}}+1) \times(\bar{n}_{\mathbf{s}}-1) - (\bar{n}_{\mathbf{p}}-1)\bar{n}_{\mathbf{q}}\bar{n}_{\mathbf{s}}]\delta(\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{q}} - \varepsilon_{\mathbf{s}}). \tag{1}$$

We shall find the amplitude of the process being studied.

[1)] In studying the scattering of magnons of the LF branch, we shall omit the branch index.

Transforming in the interaction Hamiltonian[2)]

$$\mathcal{H}^{(4)} = -\sum_{R_1\Delta} J(\Delta)\,[a^+_{R_1}a_{R_1}a^+_{R_1+\Delta}a_{R_1+\Delta} + \frac{1}{2}(a_{R_1}a^+_{R_1+\Delta}a_{R_1+\Delta}a_{R_1+\Delta} + a_{R_1+\Delta}a^+_{R_1}a_{R_1}a_{R_1})]$$

to the spin-wave operators c_{jk} and c^+_{jk}, after simple calculations we obtain the expression

$$\Psi_{k,p}^{q,s} \approx -\frac{\mu_B g H_E}{8SN(\varepsilon_k\varepsilon_p\varepsilon_q\varepsilon_s)^{1/2}}[(\varepsilon_k+\varepsilon_p)(\varepsilon_q+\varepsilon_s)-(\varepsilon_k\varepsilon_p+\varepsilon_q\varepsilon_s) - (\Theta_c\Delta)^2(\mathbf{k}+\mathbf{p})(\mathbf{q}+\mathbf{s})+(\Theta_c\Delta)^2(\mathbf{kp}+\mathbf{qs})-3\Theta_c(\varepsilon_k+\varepsilon_p-\varepsilon_q-\varepsilon_s)] \times\Delta(\mathbf{k}+\mathbf{p}-\mathbf{q}-\mathbf{s}),$$

which coincides in form (when the energy conservation law is taken into account) with the result of the calculation by the Holstein-Primakoff method (for $S \gg 1$).

When the energy and quasi-momentum conservation laws are taken into account, the expression for the amplitude is brought to the simpler form

$$\Psi_{k,p}^{q,k+p-q} \approx \frac{(\mu_B g)^2 H_E}{8M_0V(\varepsilon_k\varepsilon_p\varepsilon_q\,|\varepsilon_k+\varepsilon_p-\varepsilon_q|)^{1/2}}[3\varepsilon_0^2+(\varepsilon_k\varepsilon_p-\alpha_k\alpha_p\cos\vartheta_p) - (\varepsilon_k\varepsilon_q-\alpha_k\alpha_q\cos\beta_{qk})-(\varepsilon_p\varepsilon_q-\alpha_p\alpha_q\cos\beta_{pq})], \tag{2}$$

where $\alpha_k \equiv \Theta_c k\Delta$.

In the limit of infinite volume, going over to an integration in formula (1) and taking the quasi-momentum conservation law into account, we obtain

$$\Delta\omega_{1k} \approx (1-e^{-\varepsilon_k/T})\frac{8\pi V^2}{(2\pi\Delta)^6\hbar}\iint d\mathbf{p}d\mathbf{q}\,|\Psi_{k,p}^{q,k+p-q}|^2\bar{n}_p(1+\bar{n}_q)(1+\bar{n}_{k+p-q}) \times\delta(\varepsilon_k+\varepsilon_p-\varepsilon_q-\varepsilon_{k+p-q}), \tag{3}$$

with

$$s^2 = k^2+p^2+q^2+2pk\cos\vartheta_p-2q(k^2+p^2+2pk\cos\vartheta_p)^{1/2}\cos\beta_{q,p+k},$$
$$d\mathbf{p} = p^2\sin\vartheta_p\,dp\,d\vartheta_p\,d\varphi_p,\quad d\mathbf{q} = q^2\sin\vartheta_q\,dq\,d\vartheta_q\,d\varphi_q. \tag{4}$$

We shall express the angle $\beta_{q,p+k}$ in terms of the integration variable ϑ_q:

$$\cos\beta_{q,p+k} = \cos\vartheta_{q,p+k}\cos\varphi_q,$$
$$\cos\vartheta_{q,p+k} = \frac{k\cos\vartheta_q+p\cos(\vartheta_p-\vartheta_q)}{(p^2+k^2+2pk\cos\vartheta_p)^{1/2}}. \tag{5}$$

Below we shall consider several limiting cases.

A. $\alpha_k \gg T \gg \epsilon_0$.

Anticipating the result of integrating over ϑ_q, ϑ_p, φ_p, φ_q, q and p, we note that the most important p are those such that $\alpha_p \sim T$. Then, since $\alpha_k \gg T$, for $\cos\beta_{q,p+k}$, we find $\cos\beta_{q,p+k} \approx \cos\beta_{q,p+k} \approx \cos\vartheta_q\cos\varphi_q$.

We shall make use of the energy conservation law, namely, the presence of the δ-function in formula (3) for $\Delta\omega_{1k}$, in the integration over ϑ_q:

$$\Delta\omega_{1k} = \int_0^\pi F\delta(\varepsilon_k+\varepsilon_p-\varepsilon_q-\varepsilon_{p+k-q})\sin\vartheta_q\,d\vartheta_q$$
$$= \int_{-1}^{1} F\,|\varepsilon_k+\varepsilon_p-\varepsilon_q+\varepsilon_{p+k-q}|\,\delta[(\varepsilon_k+\varepsilon_p-\varepsilon_q)^2-\varepsilon_{p+k-q}^2]\,d\cos\vartheta_q,$$

where F incorporates all the other integrals and factors. If formulas (4) and (5) are taken into account, $\Delta\omega_{1k}$ is brought to the form

$$\Delta\omega_{1k} = \int_{-1}^{1} F\left|\frac{\varepsilon_k+\varepsilon_p-\varepsilon_q+\varepsilon_{p+k-q}}{2R_{kp}\alpha_q\cos\varphi_q}\right|\delta\Big\{\cos\vartheta_q - \Big[\frac{(\varepsilon_k+\varepsilon_p-\varepsilon_q)^2-(\varepsilon_0^2+\alpha_k^2+\alpha_p^2+\alpha_q^2+2\alpha_p\alpha_k\cos\vartheta_p)}{2R_{pk}\alpha_q\cos\varphi_q}\Big]\Big\}\,d\cos\vartheta_q$$

$$= F_0\left|\frac{\varepsilon_k+\varepsilon_p-\varepsilon_q}{R_{kp}\alpha_q\cos\varphi_q}\right| \times\theta\left\{1-\left|\frac{\varepsilon_k(\varepsilon_p-\varepsilon_q)-\varepsilon_p\varepsilon_q+\varepsilon_0^2-\alpha_k\alpha_p\cos\vartheta_p}{R_{kp}\alpha_q\cos\varphi_q}\right|\right\}, \tag{6}$$

where we must neglect ϵ_0 and α_p compared with α_k; F_0 is the value of F for $\epsilon_{p+k-q} = \epsilon_p + \epsilon_k - \epsilon_q$; $R_{kp} = (\alpha_k^2 + \alpha_p^2 + 2\alpha_p\alpha_k\cos\vartheta_p)^{1/2}$.

The subsequent calculations can be divided into two stages: from the θ-function, we find the limits of the integration over ϑ_p; from the condition $|\cos\vartheta_p| < 1$, we find the limits of the integration over φ_q; from the condition $|\cos\varphi_q| < 1$, we find the limits of the integration over q. The integration over φ_p gives a factor 2π. We can integrate over p between the limits 0 and ∞, since $\Theta_c \gg T \gg \epsilon_0$.

The ranges of the integrations over ϑ_p, φ_q and q have the following form:

$$1-qp^{-1}(1+|\cos\varphi_q|) < \cos\vartheta_p < 1-qp^{-1}(1-|\cos\varphi_q|),$$
$$0<|\cos\varphi_q|<1,\qquad 0<q<p;$$
$$1-qp^{-1}(1+|\cos\varphi_q|) < \cos\vartheta_p < 1-qp^{-1}(1-|\cos\varphi_q|),$$
$$0<|\cos\varphi_q|<2p/q-1,\qquad p<q<2p;$$
$$-1<\cos\vartheta_p<1-qp^{-1}(1-|\cos\varphi_q|),$$
$$1-2p/q<|\cos\varphi_q|<1,\qquad 2p<q<k.$$

We note immediately that the largest contribution to $\Delta\omega_{1k}$ is made by $q \sim k$. In this case, the formulas for the cosines of the angles β_{pq} and β_{kq} are simplified:

$$\cos\beta_{kq}\approx\cos\beta_{q,p+k},\qquad \cos\beta_{pq}\approx\cos\vartheta_p\cos\beta_{q,p+k},$$

with

$$\cos\beta_{q,p+k} = [\varepsilon_q\varepsilon_k+\varepsilon_p(\varepsilon_q-\varepsilon_k)-\varepsilon_0^2+\alpha_k\alpha_p\cos\vartheta_p]/\alpha_q\alpha_k$$

and the expression (2) for the amplitude is brought to the following form:

$$\Psi_{k,p}^{k+p-q} \approx -\frac{(\mu_B g)^2 H_E\varepsilon_q\alpha_p\cos\vartheta_p}{8M_0V(\varepsilon_k\varepsilon_p\varepsilon_q|\varepsilon_k+\varepsilon_p-\varepsilon_q|)^{1/2}}$$

Performing the integration, we find the relaxation rate

$$\Delta\omega_{1k} \sim 1{,}2\cdot10^{-4}\frac{(\mu_B g)^4}{\hbar(\Theta_c\Delta)^6}\left(\frac{H_E}{M_0}\right)^2(\alpha_k T)^{5/2}. \tag{7}$$

B. $\epsilon_k \ll T \ll \Theta_c$.

In this case, $\cos\beta_{q,p+k}$ is expressed in terms of $\cos\vartheta_q$ as follows:

$$\cos\beta_{q,p+k}\approx\cos\varphi_q\cos(\vartheta_p-\vartheta_q). \tag{8}$$

Changing from the integration over $\cos\vartheta_q$ to an integration over x by means of the relations

$$\cos\vartheta_q \approx x\cos^2\vartheta_p-\sin\vartheta_p(1-x^2\cos^2\vartheta_p)^{1/2},$$
$$x=\cos\beta_{q,p+k}/\cos\vartheta_p\cos\varphi_q$$

and otherwise performing transformations analogous to (6), we bring the formula for $\Delta\omega_{1k}$ to the form

$$\Delta\omega_{1k}\approx F_0\left(1+\frac{x_0\sin\vartheta_p}{(1-x_0^2\cos^2\vartheta_p)^{1/2}}\right) \times\left|\frac{(\varepsilon_k+\varepsilon_p-\varepsilon_q)\cos\vartheta_p}{R_{kp}\alpha_q\cos\varphi_q}\right|\theta\left\{1-\left|\frac{\varepsilon_k(\varepsilon_p-\varepsilon_q)-\varepsilon_p\varepsilon_q+\varepsilon_0^2-\alpha_k\alpha_p\cos\vartheta_p}{R_{kp}\alpha_q\cos\vartheta_p\cos\varphi_q}\right|\right\}$$

where F_0 and x_0 are the values of F and x for $\epsilon_{k+p-q} = \epsilon_k + \epsilon_p - \epsilon_q$.

Finally, we obtain several ranges of integration over ϑ_p, ϑ_q and q, of which we cite only the most important:

$$(\varepsilon_k-\varepsilon_q)/(\alpha_k+\alpha_q|\cos\varphi_q|) < \cos\vartheta_p < (\varepsilon_k-\varepsilon_q)/(\alpha_k-\alpha_q|\cos\varphi_q|),$$
$$0<|\cos\varphi_q|<1,\quad \varepsilon_0<\varepsilon_q<\varepsilon_k;$$

[2)]The part of the interaction Hamiltonian due to the anisotropy has been omitted, as it gives a negligibly small contribution to the relaxation.

$$-1 < \cos\vartheta_p < (\varepsilon_k - \varepsilon_p)/(\alpha_k + \alpha_p|\cos\varphi_q|),$$
$$(\varepsilon_q - \varepsilon_k - \alpha_k)/\alpha_q < |\cos\varphi_q| < 1, \quad \varepsilon_k + \alpha_k < \varepsilon_q < \varepsilon_p.$$

Taking into account that $\cos\beta_{q,p} \approx \cos\beta_{q,p+k}$ and using the expression for $\cos\beta_{q,p+k}$, we rewrite formula (2) for the amplitude in the form

$$\Psi_{k,p}^{q,p+k-q} \approx -\frac{(\mu_B g)^2 H_E (2\varepsilon_0^2 + \alpha_k\alpha_q\cos\beta_{qk})}{8M_0V(\varepsilon_k\varepsilon_p\varepsilon_q|\varepsilon_k+\varepsilon_p-\varepsilon_q|)^{1/2}}.$$

The absolute square of the amplitude consists of three terms. Hence, the formula for $\Delta\omega_{1k}$ can be represented in the form of a sum of three integrals. In the first, containing ϵ_0^4, $q \sim k$ are the most important, while in the other two, the most important q are $q \sim p$, and this gives $\cos\beta_{qk} \approx \cos\vartheta_p$. Performing the integration, we obtain for $\Delta\omega_{1k}$ the following expression:

$$\Delta\omega_{1k} \sim 6\cdot10^{-4}\frac{(\mu_B g)^4}{\hbar(\Theta_c\Delta)^6}\left(\frac{H_E}{M_0}\right)^2 T^2\Big[a(k)\varepsilon_0^3 - a'\varepsilon_0^{3/2}\alpha_k^{5/2}\frac{\varepsilon_k+3\alpha_k}{\varepsilon_k^2} + a''\alpha_k^{7/2}\frac{\varepsilon_k+12\alpha_k}{\varepsilon_k^2}T^{1/2}\Big], \qquad (9)$$

where $a(0) \approx 1$, $a' \approx 7\times10^{-2}$ and $a'' \approx 1.5\times10^{-2}$.

The calculation of a(k) involves considerable mathematical difficulties (see the Appendix). Since we have it in mind to compare $\Delta\omega_{1k}$ calculated from the magnitudes of the threshold fields of the parametric effects (ϵ_{1k} = const) with the experimental data, we cite the following fairly crude estimate for a(k):

$$a(k) \sim 1 - 0{,}5k/k_{max},$$

where $k_{max} = \epsilon_{1k}/\Theta_c\Delta$. The relation obtained is applicable for $k \leqslant 0.9k_{max}$. a(k) then increases as k^α ($\alpha \gg 1$) up to the value $a(k) \sim 2$ at $k = k_{max}$. The dependence $\Delta\omega_{1k}(k)$ for fixed ϵ_{1k} and with the above estimates taken into account is shown in Fig. 2.

In another limiting case, when $\Theta_c \gg \epsilon_0 \gg T \gg \alpha_k$ is fulfilled, we find for $\Delta\omega_{1k}$

$$\Delta\omega_{1k} \sim 8\cdot10^{-4}\frac{(\mu_B g)^4}{\hbar(\Theta_c\Delta)^6}\left(\frac{H_E}{M_0}\right)^2\varepsilon_0^4 T\exp\left(-\frac{\varepsilon_0}{T}\right). \qquad (10)$$

The formulas (7), (9) and (10) make it possible to trace the qualitative dependence of $\Delta\omega_{1k}$ on ϵ_{1k}/T. As ϵ_{1k} increases from values $\epsilon_{1k} \ll 10^{-2}T$, the minimum of $\Delta\omega_{1k}$ becomes deeper and shifts in the direction of the maximum k. For values of $\epsilon_{1k} \sim T$, the curve levels out. The temperature dependence of $\Delta\omega_{1k}$ becomes sharper. For $\epsilon_{1k} \gg T$ and $\alpha_k \ll \epsilon_{10}$, the quantity $\Delta\omega_{1k}$ is exponentially small; for $\alpha_k \gg T \gg \epsilon_{10}$, $\Delta\omega_{1k} \sim (\alpha_k T)^{5/2}$ (cf. formula (7)).

We turn to other possible mechanisms of relaxation of spin waves of the LF branch. A calculation shows that those processes are forbidden (the amplitudes Ψ are equal to zero) which correspond to the following terms in the interaction Hamiltonian:

$$c_{1k}c_{1p}c_{1q}^+c_{2s}^+; \qquad c_{1k}c_{2p}c_{2q}^+c_{2s}^+.$$

Also forbidden is the coalescence of three magnons into one. Of the four-magnon processes, the possible processes are those described in the Hamiltonian by the terms

$$c_{1k}c_{1p}c_{2q}^+c_{2s}^+ \ (11 \to 22); \quad c_{1k}c_{2p}c_{1q}^+c_{2s}^+ \ (12 \to 12)$$

We note that the relation $\epsilon_{20} \geqslant T$ is usually fulfilled

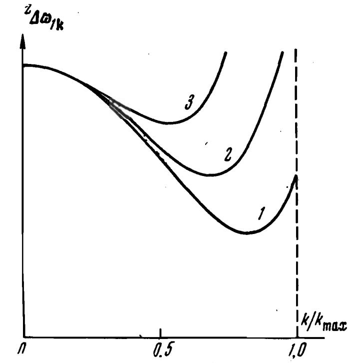

FIG. 2. Approximate dependence of $\Delta\omega_{1k}$ on k for $\epsilon_{20} \gg T$, ϵ_{1k} = const, $T \ll \Theta_c$. Curve 1) $t = 10\,\epsilon_{1k}$; 2) $T = 50\,\epsilon_{1k}$; 3) $T = 250\,\epsilon_{1k}$.

in easy-plane antiferromagnets for $T \ll \Theta_c$. Then, for $\epsilon_{1k} \ll T$, $\epsilon_{20}^2 \gg T\epsilon_{1k}$ holds. Under these conditions, the first of the above processes ($11 \to 22$) gives an exponentially small contribution to $\Delta\omega_{1k}$. The second process ($12 \to 12$) is less important, for $\epsilon_{20} \geqslant T$, than the scattering of magnons of the LF branch ($11 \to 11$) treated above, and, for $\epsilon_{20} \gg T$, its contribution to $\Delta\omega_{1k}$ is exponentially small.

But in the limiting case of small anisotropy, when $\epsilon_{20} \ll \epsilon_{10}$, the contribution of four-magnon processes with participation of magnons of the HF branch to the relaxation rate exceeds the contribution of the scattering of magnons of the LF branch ($11 \to 11$). Calculating the amplitudes of these processes, we find

$$\Psi_{1k,1p}^{2q,2s} = \frac{(\mu_B g)^2 H_E}{8M_0V(\varepsilon_{1k}\varepsilon_{1p}\varepsilon_{2q}\varepsilon_{2s})^{1/2}}[(\varepsilon_{1k}+\varepsilon_{1p})(\varepsilon_{2q}+\varepsilon_{2s}) + \varepsilon_{1k}\varepsilon_{1p} + \varepsilon_{2q}\varepsilon_{2s} - (\Theta_c\Delta)^2[(\mathbf{k}+\mathbf{p})(\mathbf{q}+\mathbf{s}) + \mathbf{kp} + \mathbf{qs}] - 2\Theta_c(\varepsilon_{1k}+\varepsilon_{1p}-\varepsilon_{2p}-\varepsilon_{2s})]\Delta(\mathbf{k}+\mathbf{p}-\mathbf{q}-\mathbf{s}), \qquad (11)$$

$$\Psi_{1k,2p}^{1q,2s} = \frac{(\mu_B g)^2 H_E}{8M_0V(\varepsilon_{1k}\varepsilon_{2p}\varepsilon_{1q}\varepsilon_{2s})^{1/2}}[(\varepsilon_{1k}-\varepsilon_{2p})(\varepsilon_{1q}-\varepsilon_{2s}) + \varepsilon_{1k}\varepsilon_{2p} + \varepsilon_{1q}\varepsilon_{2s} - (\Theta_c\Delta)^2[(\mathbf{k}-\mathbf{p})(\mathbf{q}-\mathbf{s}) + \mathbf{kp} + \mathbf{qs}] + \Theta_c(\varepsilon_{1k}+\varepsilon_{2p}-\varepsilon_{1q}-\varepsilon_{2s})]\Delta(\mathbf{k}+\mathbf{p}-\mathbf{q}-\mathbf{s}). \qquad (12)$$

After integrating over p and q for $\epsilon_{1k} \gg T$, we find

$$\Delta\omega_{1k} \sim 3{,}6\cdot10^{-3}\frac{(\mu_B g)^4}{\hbar(\Theta_c\Delta)^6}\left(\frac{H_E}{M_0}\right)^2[b(k)\varepsilon_{1k}^2T^3 + b'\alpha_k^{3/2}(\varepsilon_{1k}+12\alpha_k)T^{5/2}], \qquad (13)$$

where

$$b' \approx 0{,}02, \quad b(k) \sim \begin{cases} 1, & \alpha_k \ll \varepsilon_{20}, \\ 0{,}2, & \alpha_k \gg \varepsilon_{10}, \\ 1, & \alpha_k = \varepsilon_k \end{cases}$$

(see the Appendix).

For the case $\alpha_k \gg T \gg \epsilon_{j0}$ (j = 1, 2), we obtain an expression of the form (7), in which, however, the numerical coefficient is $\sim 6\times10^{-3}$.

At temperatures which are low compared with one or the other activation energy (ϵ_{10} or ϵ_{20}), the main role is played by the processes $11 \to 11$ and $12 \to 12$. For the case $\epsilon_{20} \gg \epsilon_{10} \gg T \gg \alpha_k$, formula (10) holds. In other cases, the process $12 \to 12$ makes the largest contribution to $\Delta\omega_{1k}$:

$$\Delta\omega_{1k} \sim 10^{-4}\frac{(\mu_B g)^4}{\hbar(\Theta_c\Delta)^6}\left(\frac{H_E}{M_0}\right)^2\begin{cases} 2\varepsilon_{10}^{3/2}\,T^{7/2}, & \varepsilon_{10} \gg T \gg \varepsilon_{20},\ \alpha_k; \\ 1.5\varepsilon_{20}^{3/2}\,\varepsilon_{10}^{3/2}\,T^2 e^{-\varepsilon_{20}/T}, & \varepsilon_{10} \gg \varepsilon_{20} \gg T \gg \alpha_k. \end{cases} \tag{14}$$

Finally, we estimate the role of the only possible three-magnon process: the process of coalescence of two magnons of the LF branch into one magnon of the HF branch ($11 \to 2$).[3)] In this case, the inequality $2\epsilon_{10} < \epsilon_{20}$ must be fulfilled. In the limits $\epsilon_{10} = \epsilon_{20}$, the contribution of this process to the relaxation rate is equal to zero. We shall consider the case $\epsilon_{20} \gg \epsilon_{10}$. With the assumptions, used here, that $T \ll \Theta_C$ and $k\Delta \ll 1$, and with the usual estimate of the activation energy of the HF branch in easy-plane antiferromagnets ($MnCO_3$, $CoCO_3$, $NiCO_3$, $CsMnF_3$), namely, $\epsilon_{20} \gtrsim 0.1\Theta_C$, the contribution of this process to the relaxation rate is exponentially small. This process can play an important role only for weak anisotropy and a weak field H: $\epsilon_{20} \sim 10^{-2}T$, $\epsilon_{10} \sim 10^{-3}T$, or $\epsilon_{20} \sim 10^{-3}T$, $\epsilon_{10} \sim 10^{-4}T$, etc. For these cases, the contribution of the three-magnon coalescence to the relaxation rate has the form

$$\Delta\omega_{1k} \approx \frac{(\mu_B g)^2}{\pi\hbar(\Theta_c\Delta)^3}\frac{H_E}{M_0}\left(\frac{\mu_B gH}{\varepsilon_{10}}\right)^2\varepsilon_{20}^2 T, \tag{15}$$

whence, e.g., for $CsMnF_3$ we find the magnitude of $\Delta\omega_{1k} \lesssim 10^4\ \text{sec}^{-1}$. Apart from the first factor, the expression (15) coincides with the formula obtained in [21] for $k = 0$. In the intermediate case, when $2\epsilon_{10} \sim (0.3\text{–}0.7)\,\epsilon_{20}$, the process $11 \to 2$ gives a contribution to $\Delta\omega_{1k}$ that is also no greater than $10^4\ \text{sec}^{-1}$, provided that $\epsilon_{20} \lesssim 10^{-2}T$, $\alpha_k \ll \epsilon_{10}$. In the above cases, there is agreement with the conclusions of [21] concerning $\Delta\omega_{10}$. The three-magnon process $11 \to 2$ is found to be the most important, although we must take into account that the values mentioned for ϵ_{10} usually correspond to fields H in which the crystal is not saturated, i.e., a domain structure exists.

We also estimate the effect on the spin-wave relaxation of the coalescence of two magnons of the LF branch into one phonon. We calculate first of all the amplitude of this process in the simplest case of a body-centered cubic lattice. The spin, phonon and spin-phonon Hamiltonians have the following forms respectively:[22]

$$\mathcal{H}_s = -\sum_{\mathbf{R}_1\Delta} J(\Delta)S_{\mathbf{R}_1}S_{\mathbf{R}_1+\Delta} - \mu_B g\mathbf{H}\sum_{\mathbf{R}}\mathbf{S}_{\mathbf{R}},$$

$$\mathcal{H}_{ph} = \frac{1}{2}\sum_{\mathbf{R}}\left\{\lambda_1 u_{ii}^2(\mathbf{R}) + \lambda_2\left[u_{ik}(\mathbf{R}) - \frac{1}{3}\delta_{ik}u_{ll}(\mathbf{R})\right]^2 + \rho\dot{u}_i^2(\mathbf{R})\right\},$$

$$\mathcal{H}_{s-ph} = -\sum_{\mathbf{R}_1\Delta}\eta J(\Delta)\mathbf{S}_{\mathbf{R}_1}\mathbf{S}_{\mathbf{R}_1+\Delta}u_{ii} + \ldots,$$

$$\mathbf{u}(\mathbf{r}) = \left(\frac{\hbar}{2\rho V}\right)^{1/2}\sum_{\mathbf{q}f}\frac{\mathbf{e}_{\mathbf{q}f}}{\omega_{\mathbf{q}f}^{1/2}}\left(b_{\mathbf{q}f}e^{i\mathbf{q}\mathbf{r}} + b_{\mathbf{q}f}^+ e^{-i\mathbf{q}\mathbf{r}}\right).$$

Going over to second-quantized operators and diagonalizing $\mathcal{H}_S$ and $\mathcal{H}_{ph}$ separately, we find that the amplitude of the process under study can be estimated by the expression

$$\Psi \sim -\eta\left(\frac{\hbar}{2\rho\omega_{\mathbf{q}f}V}\right)^{1/2}\frac{\varepsilon_{1k}q}{2}, \tag{16}$$

where usually $\eta \sim 1$.

Performing the calculations, we find that, in the case $\epsilon_{1k} \gg T$, the process under study gives an exponentially small contribution to $\Delta\omega_{1k}$, while for the case $\epsilon_{1k} \ll T$, $\Theta_C \ll \Theta_D$, we obtain

$$\Delta\omega_{1k} \sim \frac{\eta^2\hbar}{2\pi\rho(\Theta_D\Delta)^5}T\varepsilon_{1k}^4, \tag{17}$$

which, in any case, is much less than 1 sec^{-1}. For $\Theta_C \to \Theta_D$, the contribution of the given process to the relaxation rate tends to zero. Other processes with participation of phonons also give a small contribution to the relaxation rate, if we exclude the direct interaction of the phonon and magnon branches, which is a local effect.

Thus, it can be stated that the relaxation of spin waves of the LF branch in antiferromagnets with "easy-plane" anisotropy, when the constant magnetic field lies in the basal plane, is determined by the four-magnon scattering of magnons of the LF branch. In the case of small anisotropy ($\epsilon_{20} \ll \epsilon_{10}$), the relaxation of spin waves of the LF branch is determined by four-magnon scattering processes involving two magnons of the LF branch and two magnons of the HF branch.

2. THE HIGH-FREQUENCY BRANCH

The technique developed above for calculating the relaxation rate of magnons of the LF branch can be applied in the study of the relaxation of magnons of the HF branch.

To simplify the notation, we introduce γ_k by means of the relation

$$\Delta\omega_{2k} = 10^{-3}\frac{(\mu_B g)^4}{\hbar(\Theta_c\Delta)^6}\left(\frac{H_E}{M_0}\right)^2\gamma_k.$$

Along with an indication of the ranges of applicability of the formulas, we indicate the processes determining the magnon relaxation in these ranges. The expressions for γ_k have the following forms:

A) $\alpha_k \gg T \gg \varepsilon_{10}, \varepsilon_{20}$; $21 \to 21 + 22 \to 11$, $\gamma_k \sim 6(\alpha_k T)^{5/2}$;
B) $T \gg \alpha_k \gg \varepsilon_{10}, \varepsilon_{20}$; $22 \to 11 + 21 \to 21$,

$$\gamma_k \sim 15(\alpha_k^2 T^3 + 0.4\alpha_k^{5/2}T^{5/2});$$

C) $T \gg \varepsilon_{10} \gg \varepsilon_{20} \gg \alpha_k$; $21 \to 21$, $\gamma_k \sim 0.16[\varepsilon_{10}^3 T^2 + 11(\varepsilon_{10}\varepsilon_{20})^{3/2}T^2 + 20(\varepsilon_{20}T)^{5/2}]$;
D) $T \gg \varepsilon_{20} \gg \varepsilon_{10} \gg \alpha_k$; $22 \to 11$, $\gamma_k \sim 3{,}6\,\varepsilon_{20}^2 T^3$;
E) $\varepsilon_{10} \gg T \gg \varepsilon_{2k}$; $22 \to 22$,

$$\gamma_k \sim 0{,}6\left[a(k)\varepsilon_{20}^3T^2 - a'\varepsilon_{20}^{3/2}\alpha_k^{5/2}\frac{\varepsilon_{2k}+3\alpha_k}{\varepsilon_{2k}^2}T^2 + a''\alpha_k^{7/2}\frac{\varepsilon_{2k}+12\alpha_k}{\varepsilon_{2k}^2}T^{5/2}\right];$$

F) $\varepsilon_{20} \gg T \gg \varepsilon_{10}$; $21 \to 21$, $\gamma_k \sim 0.2\varepsilon_{20}^{3/2}T^{7/2}$; (18)
G) $\varepsilon_{10} \gg \varepsilon_{20} \gg T \gg \alpha_k$; $22 \to 22$, $\gamma_k \sim 0.8\varepsilon_{20}^4 T\exp(-\varepsilon_{20}/T)$;
H) $\varepsilon_{20} \gg \varepsilon_{10} \gg T \gg \alpha_k$; $21 \to 21$, $\gamma_k \sim 0.15(\varepsilon_{20}\varepsilon_{10})^{3/2}T^2\exp(-\varepsilon_{10}/T)$.

In the case of weak anisotropy, small magnetic fields H (with $\epsilon_{20} > 2\epsilon_{10}$) and small k, it may turn out to be important to take account of the decay of a magnon of the HF branch into two magnons of the LF branch. For the cases (B) and (D) respectively, we find

$$\Delta\omega_{2k} \approx 2\frac{(\mu_B gH)^2}{\pi\hbar(\Theta_c\Delta)^3}\frac{H_E}{M_0}T\ln\frac{2\alpha_k}{\varepsilon_{20}}, \tag{19}$$

$$\Delta\omega_{2k} \approx 2\frac{(\mu_B gH)^2}{\pi\hbar(\Theta_c\Delta)^3}\frac{H_E}{M_0}T.$$

3. DISCUSSION OF THE RESULTS

It is not difficult to see that the results obtained are applicable in the investigation of spin-wave relaxation in antiferromagnets in all cases when the antiferromagnetic vector $L = M_1 - M_2$ is oriented perpendicular to

3)The other three-magnon processes are forbidden either because the amplitudes Ψ are equal to zero or by the requirements of energy and quasi-momentum conservation.

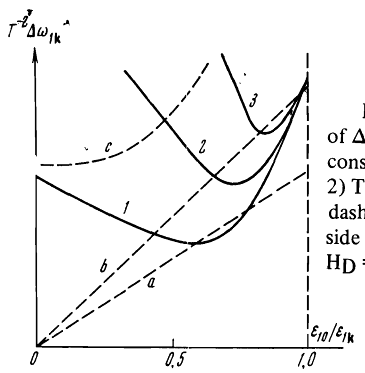

FIG. 3. Approximate dependence of $\Delta\omega_{1k}$ on ϵ_{10} for $\epsilon_{20} \gg T$, ϵ_{1k} = const, $T \ll \Theta_C$. Curve 1) $T = 10\,\epsilon_{1k}$; 2) $T = 50\,\epsilon_{1k}$; 3) $T = 250\,\epsilon_{1k}$. The dashed curves are the right-hand side of the inequality (20): a and b) $H_D = 0$, $h^{(b)} > h^{(a)}$; c) $H_D \neq 0$.

the field **H**, $H \ll H_E$. The shape of the spectrum remains unchanged, namely, $\epsilon_{jk} \approx (\epsilon_{j0}^2 + \alpha_k^2)^{1/2}$ (j = 1, 2). The form of ϵ_{j0} is given for various cases in [23]. In particular, the above remark can be extended to antiferromagnets with "easy-axis" anisotropy in fields greater than the field that inverts the sublattices, i.e., $(2H_AH_E)^{1/2} < H \ll H_E$, if **H** is parallel to the "easy axis." The latter inequality presupposes that the magnetic anisotropy in the crystal is not too strong.

Returning to the possibility of obtaining information on the relaxation of spin waves in the study of parametric phenomena, we note certain features of the behavior of the imaginary part of the susceptibility of a spin system beyond the instability threshold, based on the results of the calculation performed above. Intense energy absorption from an alternating magnetic field begins when the amplitude h of the field exceeds a certain threshold value h^{th}. In the general case of a superposition of "extra absorption" and "parallel pumping"[13] for $\epsilon_{20} \gg \hbar\omega$, where ω is the frequency of the alternating field, taking into account an expression for h^{th} [24,13] we can write the above condition in the following form:

$$\Delta\omega_{1k} < \gamma h[\varepsilon_{10}^2 + (\tfrac{1}{2}\mu_B g H_D)^2]^{1/2} / \hbar\omega, \qquad (20)$$

where, using the fact that ϵ_{1k} = const, we represent $\Delta\omega_{1k}$ as a function of ϵ_{10}. In Fig. 3, we give the dependence $\Delta\omega_{1k}(\epsilon_{10})$ for various ϵ_{1k}/T, with $\epsilon_{1k} \ll T$. The dashed curves correspond to the function in the right-hand side of the inequality, for different h and H_D. We may conclude from Fig. 3 that the absorption first arises only in a certain well-defined range of fields H. For sufficiently large H_D, the parametric excitation is extended down to H = 0.

With regard to the published papers on the theoretical study of spin-wave relaxation in antiferromagnets, we note that the results obtained by Harris, Kumar, Halperin and Hohenberg[7] for the case of single-ion anistropy, H = 0 and k = 0 refer only to easy-axis antiferromagnets. But it makes no sense to study the isotropic case in the absence of a magnetic field, since, in this case, the ground state of the spin system is not defined. For a correct calculation in the isotropic case, we must take a magnetic field into account, and this leads to an arrangement of the magnetic moments perpendicular to the field direction. In this case, the transformation to spin-deviation operators and the diagonalization of the Hamiltonian are performed by means other than those used in the paper.[7]

In the paper by Woolsey and White,[6] it is noted that the four-magnon processes give a contribution to the relaxation rate that is too small to explain the experimental values of $\Delta\omega_{1k}$. We now turn to experiment.

The value $\Delta\omega_{10} \sim 10^5\ \text{sec}^{-1}$ was obtained by Prozorova and Borovik-Romanov for $T \sim 1.5°$K and $\epsilon_{1k} \sim 2 \times 10^{-16}$ erg for $CsMnF_3$[11] and by Kotyuzhanskiĭ and Prozorova for $MnCO_3$.[12] The dependence of $\Delta\omega_{1k}$ on k is described qualitatively by the formula (9), if we allow for the fact that, in the experiment, $\epsilon_{1k} \ll T$ was insufficiently rigorously fulfilled. A numerical comparison is made difficult by the requirement of an exact determination of the quantity $\Theta_C\Delta$, which occurs to the sixth power in the formulas. By making use of the data of Seavey for $CsMnF_3$[8] and of Kotyuzhanskiĭ and Prozorova for $MnCO_3$,[12] we find, respectively, $(\Theta_C\Delta)_X \approx 1.63 \times 10^{-22}$ erg·cm and $(\Theta_C\Delta)_X \approx 1.56 \times 10^{-22}$ erg·cm, which, for $T \sim 2°$K and $\epsilon_{10} \sim 3 \times 10^{-17}$ erg ($\epsilon_{10} \ll T$) give $\Delta\omega_{10} \sim 10^2\ \text{sec}^{-1}$.

Comparison with the experiments of Seavey on $CsMnF_3$[8] is made difficult by the strong magnetoelastic interaction observed in them.

The dependence $\Delta\omega_{1k}(k)$ obtained by Hinderks and Richards from experiments on $RbMnF_3$ (weak anisotropy)[10] using the calculations of Richards[25] are qualitatively explained by formula (13). For the quantity $\Delta\omega_{10}$ with $T \sim 4°$K and $\epsilon_{10} \sim 7 \times 10^{-17}$ erg, assuming that the value of $\Theta_C\Delta$ for $RbMnF_3$ is the same as for $CsMnF_3$, we find $\Delta\omega_{10} \sim 10^6\ \text{sec}^{-1}$, which, in order of magnitude, coincides with the experimental value.

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APPENDIX

The function a(k) appearing in the expression (9) for $\Delta\omega_{1k}$ is determined as follows:

$$a(k) \approx [I_1(k, T) + I_2(k, T)] / I_2(0, T),$$

$$I_1(k, T) = \frac{2\pi T^2}{\varepsilon_{1k}\varepsilon_{10}} \int_0^1 \frac{(\sqrt{1 + e_1^2} - \sqrt{x^2 + e_2^2})^{1/2}(2 - x^2)x^2}{(\sqrt{1 + e_1^2} + \sqrt{x^2 + e_2^2})^{3/2}(x^2 + e_2^2)^{1/2}} dx$$

$$I_2(k, T) \approx 2 \iiint dx\, dy\, dz \left[1 - \left(\frac{\sqrt{y^2 + e_2^2} - \sqrt{1 + e_1^2}}{1 + xy}\right)^2\right]$$

$$\times \frac{y}{x[(1 - x^2)(y^2 + e_2^2)]^{1/2}} \frac{e^z}{(e^z - 1)(e^{g\sqrt{y^2 + e_2^2}} - 1)}$$

$$\times (\exp\{z + g(\sqrt{1 + e_1^2} - \sqrt{y^2 + e_2^2})\} - 1)^{-1},$$

with, for the limits of integration of the second integral,

$$[\sqrt{y^2 + e_2^2} - (1 + \sqrt{1 + e_1^2})] / y < x < 1,$$

$$[2(1 + \sqrt{1 + e_1^2})]^{1/2} < y < [(z/y)^2 - e_1^2]^{1/2},$$

$$e_1 g < z < \infty,$$

where $g = \alpha_k/T$ and $e_1 = e_2 = \epsilon_{10}/\alpha_k$. Approximate numerical calculations give the estimate cited in the text. To calculate b(k), one uses the same formulas, but with $e_1 = \epsilon_{10}/\alpha_k$ and $e_2 = 0$.

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