GRAVITATIONAL RADIATION IN THE SCALAR-TENSOR GRAVITATION THEORY

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The emission of scalar waves is considered in the scalar-tensor theory of gravitation. It is shown
that under certain conditions it can exceed by several orders the emission of tensorial gravitational
waves. In particular, the currently assumed turbulence in the initial stages of the expanding Universe
could lead to scalar radiation exceeding the tensorial one within a wide range of dimensions. Further-
more, for a spherically-symmetric collapse (which seems to occur in supernova flares), when tensor-
ial radiation is absent, the scalar emission is related to an energy loss comparable to the mechanical
energy of the exploding star envelope.

1. The problem of Dirac's large parameter[1] and
Mach's problem, which were acutely felt but not solved
by Einstein[2], were the stimulus for proposed changes
in the general theory of relativity (GTR), consisting in
the introduction of a scalar field $\varphi(\mathbf{r}, t)$ related to the
gravitational "constant" k, which thus becomes depen-
dent on the coordinates and time. Such a scalar-tensor
theory of gravitation (STTG) was proposed by Jordan[3]
(cf. also[4]) and later by Brans and Dicke[5]. The
Lagrangian density in this theory is[5]

$$\Lambda = \varphi R - \frac{\delta \varphi_{,i} \varphi^{,i}}{\varphi} - \frac{16\pi}{c^4} \Lambda_m.$$

Here Λ_m is the Lagrangian density of material sources,
R is the curvature scalar, $\delta \approx 6$[5] is the coupling con-
stant of the scalar and tensor fields; the comma denotes
partial differentiation and the semicolon denotes covar-
iant differentiation.

The scalar $\varphi^{-1}(\mathbf{r}, t) \sim k(\mathbf{r}, t)$ was denoted in[3]
and[5] by φ^ν with $\nu = +1$ and $\nu = -1$, respectively. How-
ever, this distinction does not affect the field equations
which follow from the above Lagrangian and neither does
it manifest itself in the equations of motion. Finkel'-
stein[6] has proposed a modification of this theory,
which we will not discuss, however.

In all its proposed versions the STTG does not solve
in fact the problem of Mach's principle, since its field
equations admit "anti-Mach" solutions, for which the
field $\varphi(\mathbf{r}, t)$ is not fully determined by the distribution
of the source tensor $T_{ik}(\mathbf{r}, t)$. More complete solutions
of this problem have been proposed in another direction
by Al'tshuler[7], Linden-Bell[8], and Sciama[9]. How-
ever, the STTG presents interest independently of the

Mach problem, namely as a theory of gravitational
interaction with a variable parameter k(**r**, t).

The above Lagrangian density leads to the following
field equations:

$$\varphi G_{ik} + g_{ik} \, \Box \, \varphi - \varphi_{,i;k} + \delta \varphi^{-1}(g_{ik}\varphi_{,l}\varphi^{,l} - \varphi_{,i}\varphi_{,k}) = 8\pi c^{-4} T_{ik},$$
$$\Box \varphi = [8\pi / (3+2\delta)c^4]T; \quad G_{ik} = R_{ik} - \tfrac{1}{2}g_{ik}R, \quad \Box \varphi \equiv \varphi_{,i}^{;i}.$$

Here T_{ik} is the energy-momentum tensor of the field
sources and T is its trace. At the same time the follow-
ing equation of GTR remains valid:

$$T_{i;k}^{k} = 0.$$

2. The purpose of the present paper is to show that
the STTG leads to gravitational radiation, which under
certain circumstances may surpass by many orders the
gravitational radiation in the GTR. Scalar radiation can
occur even for the case of a spherically symmetric sys-
tem. Therefore, if the scalar field $\varphi(\mathbf{r}, t)$ exists, the
metagalaxy may contain many more scalar gravitons
than tensor gravitons, and these scalar gravitons could
play an essential role in the process of formation of
the basic elements of structure of the metagalaxy, the
galactic clusters. We shall consider the radiation as a
small perturbation of the pseudoeuclidian metric. We
set

$$g_{ik} = g_{ik}^{(0)} + h_{ik}, \quad \varphi = \varphi_0(1 + \psi), \qquad (1)$$

where[5]

$$\varphi_0^{-1} = \frac{3+2\delta}{4+2\delta} k = \gamma k, \quad \gamma \approx 1.$$

We shall consider T_{ik} to be small of first order.

We select the coordinate system in such a way, that
the following conditions be satisfied[5] (these differ

somewhat from the Landau-Lifshitz relations[10]):

$$[h_i{}^k - 1/2\delta_i{}^k(h + 2\psi)]_{,k} = 0. \tag{2}$$

As was shown by Brans and Dicke[5], the following wave equations hold

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\left[h_{ik} - \frac{1}{2}g_{ik}^{(0)}(h + 2\psi)\right] = \frac{16\pi}{c^i}\gamma k T_{ik}, \tag{3}$$

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi = -\frac{8\pi\gamma k}{(3 + 2\delta)c^i}T.$$

Let us consider far from the source system a plane wave propagating into the x^1 direction. According to Landau-Lifshitz[10], in addition to the conditions of type (2), the choice of the coordinate system allows one to impose four more conditions, e.g.,

$$h_{00} = h_{01} = h_{02} = h_{03} = 0. \tag{4}$$

Since both the quantities h_{ik} and ψ depend only on the variable $x^1 - ct$, it follows from (2) and (4) that all components of h_{ik}, except h_{22}, h_{23} and h_{33} vanish, and also that the following relation holds:

$$h_{22} + h_{33} = 2\psi. \tag{5}$$

One can now calculate the components of the pseudo-tensor. The pseudotensor

$$t^{ik} = k\varphi t_0{}^{ik} + t_\varphi{}^{ik},$$

where t_0^{ik} is the pseudotensor from[10] and t_φ^{ik} is an additional pseudotensor, related to the presence of the scalar field $\varphi(\mathbf{r}, t)$, and has the form[1]

$$\begin{aligned}t_\varphi{}^{ik} = {} & (c^4/16\pi)\varphi^{-1}[2(\delta - 1)\varphi^{,i}\varphi^{,k} + (2 - \delta)g^{ik}\varphi_{,n}\varphi^{,n}] \\ & + (c^4/8\pi)[g^{ik}(2\varphi^{,m}\Gamma_{pm}{}^p - g^{lm}\Gamma_{lm}{}^p\varphi_{,p}) \\ & \quad g^{im}g^{kn}\varphi_{,\nu}\Gamma_{mn}{}^p + (\varphi^{,i}\Gamma_{ln}{}^k + \varphi^{,k}\Gamma_{ln}{}^i)g^{in} \\ & \quad - (\varphi^{,i}g^{km} + \varphi^{,k}g^{im})\Gamma_{mn}{}^n - (g^{im}\Gamma_{mn}{}^k + g^{km}\Gamma_{mn}{}^i)\varphi^{,n}]. \end{aligned} \tag{6}$$

Substituting the above-mentioned expressions for h_{ik} from (5), we obtain

$$t^{01} = \frac{c^2}{16\pi\gamma k}\left[\dot{h}_{23}{}^2 + \frac{1}{4}(\dot{h}_{22} - \dot{h}_{33})^2 + (3 + 2\delta)\dot{\psi}^2\right],$$

Here and in the sequel the dot above a letter denotes differentiation with respect to time.

3. We compute the quantities h_{22}, h_{33}, h_{23} and ψ from (3). According to[10], one may neglect the retardation in the first equation of (3), and thus

$$h_{ik} - g_{ik}^{(0)}\left(\frac{1}{2}h + \psi\right) = -\frac{4\gamma k}{c^4 r_0}\int T_{ik}\left(t - \frac{r_0}{c}\right)dV, \tag{7}$$

where the dimensions of the radiating system have been assumed small compared to the distance r_0 from the selected origin to the point of observation. In the sequel the variable $t - r_0/c$ will be omitted. From (2) and the first equation of the system (3) it is easy to derive an equation like in the GTR

$$T_{i,k}^{k} = 0.$$

Then, by analogy with Landau-Lifshitz[10], we obtain from (7)

$$h_{23} = -\frac{2\gamma k}{c^4 r_0}\frac{\partial^2}{\partial t^2}\int \rho x_2 x_3 dV = -\frac{2\gamma k}{3c^4 r_0}\ddot{D}_{23},$$

$$h_{22} - h_{33} = -\frac{2\gamma k}{3c^4 r_0}(\ddot{D}_{22} - \ddot{D}_{33}). \tag{8}$$

The solution of the second equation of the system (3) is

$$\psi(\mathbf{r}_0, t) = [2\gamma k/c^4(3 + 2\delta)]\int \frac{T(t - |\mathbf{r} - \mathbf{r}_0|/c)}{|\mathbf{r} - \mathbf{r}_0|}dV. \tag{9}$$

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[1]We are grateful to A. M. Finkel'shteĭn for indicating to us the form of the symmetric pseudotensor t_φ^{ik}.

The energy losses due to the usual (tensorial) gravitational radiation coincide with the equations of Landau-Lifshitz[10] and the energy loss per unit time due to scalar radiation is:

$$\left(-\frac{dE}{dt}\right)_\varphi = \frac{k\gamma}{4\pi c^5(3 + 2\delta)}\cdot\int\left[\frac{\partial}{\partial t}\int T(t')dV\right]^2 do. \tag{10}$$

Here $t' = t - |\mathbf{r}_0 - \mathbf{r}|/c$, do is the element of solid angle on a sphere of radius r_0. One cannot neglect the retardation in the argument of T. Effecting in T an expansion in powers of the variable r which enters into the retardation, and neglecting r compared to r_0 in the denominator, we obtain

$$\begin{aligned}\dot{\psi}(\mathbf{r}_0, t) = {} & \frac{2\gamma k}{c^4 r_0(3 + 2\delta)}\frac{\partial}{\partial t}\int\left[\dot{T}\left(\mathbf{r}, t - \frac{r_0}{c}\right)\frac{\mathbf{rn}}{c}\right. \\ & \left. + \frac{1}{2}\ddot{T}\left(\mathbf{r}, t - \frac{r_0}{c}\right)\left(\frac{\mathbf{rn}}{c}\right)^2\right]dV\end{aligned} \tag{11}$$

where $\mathbf{n} = \mathbf{r}_0/r_0$.

4. The energy lost by the material sources during the emission of the scalar field surpasses in some cases by several orders of magnitude the energy lost through tensorial radiation. One of these cases is the turbulence of a hot plasma interacting with electromagnetic radiation, an effect, which according to Ozernoĭ and Chechin[11], occurred in the pregalactic stage of expansion of the metagalaxy and was the mechanism responsible for the formation of its present structure. Considering the scalar radiation of such turbulent vortices one may retain the first term under the integral in (11). Then we obtain

$$\psi(\mathbf{r}_0, t) = \frac{2\gamma k}{(3 + 2\delta)c^3 r_0}\left(\frac{\partial^2}{\partial t^2}\int \rho \mathbf{r}\, dV, \mathbf{n}\right).$$

The integral here is proportional to the radius-vector of the center of inertia of the system, and its second derivative is the rate of change of the total momentum as a function of time, $d\mathbf{P}/dt$. If the system is closed and its total momentum does not change with time, the derivative vanishes. However, when the system interacts with electromagnetic radiation, equation (11) will contain only the energy-momentum tensor of the plasma, since for the electromagnetic field the trace of the tensor vanishes. If this is so, $d\mathbf{P}/dt \neq 0$ and the emission of scalar radiation is possible already in this approximation. According to the estimates of Ozernoĭ and Chechin[11] the velocity of vortex motions is $v \approx 10^8 - 3 \times 10^9$ cm/sec during the epoch $t_* \approx 3 \times 10^{13}$ sec, adjacent to the epoch of hydrogen recombination, when the interaction of the plasma with radiation is very strongly diminished. In the period preceding the recombination the plasma density is $\rho \approx 10^{-22}$ g/cm³. If one considers the turbulent vortices enveloping a mass exceeding by three orders the mass of the Galaxy M_G: $M \approx 10^{14}$ $M_\odot \approx 10^{17}$ g, the dimensions of the vortex are $r \approx (M/\rho)^{1/3} \approx 10^{23}$ cm, and the characteristic time is $t = r/v = 10^{23}/10^9 \approx 10^{14}$ sec, then the power of scalar radiation of this vortex in the epoch t_* is

$$\left(-\frac{dE}{dt}\right)_* = 10^{-2}\frac{kM^2c}{r^2}\left(\frac{v}{c}\right)^4,$$

$3(4 + 2\delta) \approx 10^2$. For $v \approx 10^9$ cm/sec this yields $(-dE/dt)_* \approx 3 \times 10^{43}$ erg/sec.

In the epoch preceding the instant t_*, v = const, $r \sim t^{1/2}$. Setting $\ln(t_*/t_{init}) \approx 30$, we obtain for the en-

ergy during the epoch of coupling of nonrelativistic plasma to the radiation

$$\Delta E \approx 10^{58} \quad \text{erg},$$

which is by ten orders smaller than the energy of electromagnetic radiation in the volume of the vortex under consideration at time t_*. In the following epoch, i.e., after recombination, the turbulent motions are damped out and an estimate of the scalar radiation during that epoch does not essentially modify the value obtained above. If the mass of the gas comprised by the vortex is $M \approx 10^{17} M_{\odot}$ and the velocity of the vortex motion is $v \approx 3 \times 10^9$ cm/sec, then $r \approx 10^{24}$ cm and $\Delta E \approx 3 \times 10^{64}$ erg. We note that during that epoch of nonrelativistic plasma interacting with electromagnetic radiation, the ratio of the power of the scalar field to that of the tensor radiation in GTR is $2 \times 10^{-2} rc^2/kM$. Setting $M = (4/3)\pi\rho r^3$, we arrive at the conclusion that the tensor radiation is comparable to the scalar one or exceeds it for vortices of the order of

$$r \geqslant c / 15(k\rho)^{1/2}.$$

The mass of such regions in the epoch discussed above is $M \approx 10^{17} M_{\odot}$, if one takes into account the small decrease of the gravitational constant compared to its present value.

5. A spherically symmetric system is well known to be incapable of losing energy on account of tensorial radiation, but it can emit scalar radiation. This effect is particularly significant in the case of spherically-symmetric collapse of a star. The calculation which follows has only an estimation character, and therefore we use a simplified model of the star. We shall assume that the star remains homogeneous throughout its collapse and that its outer layer falls freely towards the center. This leads to the following order-of-magnitude relation:

$$\rho(t) = \rho_0(1 - t/t_0)^{-2}, \qquad R = R_0(1 - t/t_0)^{2/3}, \qquad (12)$$

where ρ is the density and R is the radius of the star and ρ_0 and R_0 are their initial values; at the same time

$$t_0 \approx (k\rho_0)^{-1/2}.$$

Setting $T = \rho c^2$ we obtain from (10) and (11)

$$-\frac{dE}{dt} = \frac{1}{4 + 2\delta} \frac{kM^2 R_0^4}{t_0^3 c^5} \left(1 - \frac{t}{t_0}\right)^{-10/3}$$

and the total energy losses due to scalar radiation for the whole collapse time from radius R_0 to $R_1 \ll R_0$ is

$$\Delta E = \frac{1}{4 + 2\delta} \left(\frac{r_g}{R_1}\right)^{5/2} \frac{kM^2}{R_1} \qquad r_g = \frac{kM}{c^2}$$

This energy does not depend in our approximation on the initial radius R_0. In the case of a supernova eruption we set $R_1 = 2 \times 10^6$ cm, $M = 2 \times 10^{33}$ g, which yields $\Delta E \approx 10^{49}$ erg. In some cases this energy is comparable to the mechanical energy of the expanding shell in a supernova flare.

6. We now consider the gravitational radiation of a spherically symmetric pulsating star. We shall see that for usual stars the damping time of the oscillations due to this radiation is much larger than the cosmological time. However, for pulsars and quasars the radiation

may turn out to be substantially stronger. Unfortunately we do not have data for the appropriate estimates, and therefore consider an ordinary star, using two simplified models.

a) The total volume is subject to small harmonic oscillations, remaining homogeneous, so that the radius of the star is

$$R = R_0 + r_1\cos \omega t, \quad r_1 \ll R_0.$$

Then the energy of oscillations of the star is

$$E = {}^3/_{10}r_1^2\omega^2 M,$$

and the energy loss per unit time averaged over a period, due to gravitational radiation is

$$-\frac{dE}{dt} = \frac{1}{20} \frac{k}{c^5(4 + 2\delta)} M^2\omega^6 r_1^2 R_0^2.$$

Therefore the characteristic damping time of the oscillations is

$$\tau = 6c^5(4 + 2\delta)(kM\omega^4 R_0^2)^{-1}.$$

setting $M = 10^{22}$ g, $\omega = 10^{-4}$ sec, $R_0 = 10^{11}$ cm, we have $\tau \approx 10^{22}$ sec.

b) The star consists of a stationary interior part of radius R_a and mass m_a and a harmonically oscillating exterior part of radius:

$$R = R_0 + r_1\cos \omega t$$

and mass m_b, with

$$R_a = (1 - \beta)R_0, \quad \beta \ll 1, \quad r_1 \ll \beta R_0.$$

The energy of the oscillations of the star is

$$E = {}^1/_6 m_b r_1^2 \omega^2.$$

Then the energy loss per unit time averaged over one period is

$$-\frac{dE}{dt} = \frac{1}{70} \frac{k}{\beta^2 c^5(4 + 2\delta)} m_b^2\omega^6 r_1^2 R_0^2.$$

Therefore the characteristic damping time of the oscillations is

$$\tau = 12c^5(4 + 2\delta)\beta^2(km_b\omega^4 R_0^2)^{-1}.$$

Setting $\beta = 0.1$, $m_b = 10^{22}$ g, $R_0 = 10^{11}$ cm, $\omega = 10^{-4}$ sec^{-1}, we get $\tau \approx 10^{21}$ sec.

In both models the characteristic times are very large.

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