NONLINEAR RESPONSE OF A FERROMAGNET TO THE SWITCHING ON OF A CONSTANT FIELD

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The behavior of the spin system of a ferromagnet is investigated when a constant longitudinal magnetic field of arbitrary magnitude is switched on. Formulas are obtained for the nonequilibrium magnetic moment after switching on of the field. Kinetic equations are also derived; they describe the relaxation of the system to the new equilibrium state, corresponding to the switched-on field.

THE interaction of a ferrodielectric with a uniform alternating magnetic field, directed along the preferred axis, is described by the Zeeman energy

$$\mathcal{H}_t = -\mu h_t \sum_n S_n^z.$$

Here h_t is the field intensity, μ is the magneton, and S_n^z is the operator of the projection of the spin on the n-th site of the lattice along the preferred axis. In the spin-wave approximation, which is valid at low temperatures,

$$\mathcal{H}_t = -\mu NSh_t + \mu h_t \sum_{\mathbf{k}} a_{\mathbf{k}}^+ a_{\mathbf{k}}, \tag{1}$$

where $a_{\mathbf{k}}^+$ and $a_{\mathbf{k}}$ are the Bose operators of creation and annihilation of a spin wave with wave vector $\mathbf{k}$, S is the magnitude of the spin on the site, and N is the number of atoms in the lattice. If the amplitude of the field h_t is sufficiently small, the response of the system to such a field can be described by a complex susceptibility, which links the contribution to the magnetization with the alternating field in the linear approximation.[1] At fields exceeding a certain threshold value, so-called parametric excitation of spin waves is possible,[2-4] and the linear approximation ceases to be valid. Usually, in the case of a monochromatic field $h_t = h_0 \cos \omega t$, after transformation to "true" spin waves with the Hamiltonian (1) there remain only resonance terms, corresponding to the conservation law $\hbar\omega = 2\epsilon_{\mathbf{k}}$ ($\epsilon_{\mathbf{k}}$ is the energy of a spin wave, including the dipole interaction).[4] At sufficiently low frequencies ($\hbar\omega < 2\epsilon_{\min}$), the neglected terms must make an appreciable contribution. Therefore an exact treatment of the interaction (1) becomes necessary. The present paper considers the interaction of the spin system with a constant field, of arbitrary magnitude h, which is switched on at the instant t = 0.

1. The Hamiltonian of noninteracting spin waves, which includes both exchange and relativistic interaction in the spin system, has the form

$$\mathcal{H}_0 = \sum_{\mathbf{k}} \{A_{\mathbf{k}} a_{\mathbf{k}}^+ a_{\mathbf{k}} + {}^1/_2 (B_{\mathbf{k}} a_{\mathbf{k}} a_{-\mathbf{k}} + B_{\mathbf{k}}^* a_{\mathbf{k}}^+ a_{-\mathbf{k}}^+)\} + E_0, \tag{2}$$

where E_0 is the energy of the classical ground state, and the coefficients $A_{\mathbf{k}}$ and $B_{\mathbf{k}}$ are given, in the long-wave approximation, by the expressions

$$\begin{aligned} A_{\mathbf{k}} &= \Theta_C (ak)^2 + \mu (H_0 + \beta M_0) + \mu^2 a^{-3} \cdot 2\pi \sin^2 \vartheta_{\mathbf{k}}, \\ B_{\mathbf{k}} &= \mu^2 a^{-3} \cdot 2\pi \sin^2 \vartheta_{\mathbf{k}} \exp (2i\varphi_{\mathbf{k}}). \end{aligned} \tag{3}$$

Here Θ_C is the exchange energy, of the order of the Curie temperature; a is the lattice constant; H_0 is the constant external field, applied along the preferred axis; β is the anisotropy constant; M_0 is the saturation magnetization; and $\vartheta_{\mathbf{k}}$ and $\varphi_{\mathbf{k}}$ are the polar angles of the wave vector $\mathbf{k}$. The Hamiltonian (2) is transformed, by means of the linear uv-transformation

$$\begin{aligned} a_{\mathbf{k}} &= u_{\mathbf{k}} b_{\mathbf{k}} + v_{\mathbf{k}}^* b_{-\mathbf{k}}^+, \quad a_{-\mathbf{k}} = u_{\mathbf{k}} b_{-\mathbf{k}} + v_{\mathbf{k}}^* b_{\mathbf{k}}^+; \\ &|u_{\mathbf{k}}|^2 - |v_{\mathbf{k}}|^2 = 1 \end{aligned} \tag{4}$$

to the diagonal form

$$\mathcal{H}_0 = \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} b_{\mathbf{k}}^+ b_{\mathbf{k}}, \quad \varepsilon_{\mathbf{k}} = (A_{\mathbf{k}}^2 - |B_{\mathbf{k}}|^2)^{1/2}. \tag{5}$$

Let the system, at the initial instant t = 0, be in thermodynamic equilibrium, which is described by a Gibbs distribution ρ_0 with Hamiltonian $\mathcal{H}_0$. For $t > 0$ the system is described by a density matrix $\rho(t)$ that satisfies the equation

$$i\hbar \frac{\partial \rho}{\partial t} = [\mathcal{H}_0 + \mathcal{H}_t, \rho] \tag{6}$$

with the initial condition ρ (t = 0) = ρ_0. Since in our case there is no explicit dependence of the Hamiltonian $\mathcal{H}_t$ on time, the solution of equation (6) can be written in the form

$$\rho(t) = \exp [-i(\mathcal{H}_0 + \mathcal{H}_t)t/\hbar] \rho_0 \exp [i(\mathcal{H}_0 + \mathcal{H}_t)t/\hbar]. \tag{7}$$

The density matrix (7) will have a simpler form if the Hamiltonian $\mathcal{H}_0 + \mathcal{H}_t$ is reduced to diagonal form by means of the uv-transformation

$$\begin{aligned} a_{\mathbf{k}} &= \tilde{u}_{\mathbf{k}} c_{\mathbf{k}} + \tilde{v}_{\mathbf{k}}^* c_{-\mathbf{k}}^+, \quad a_{-\mathbf{k}} = \tilde{u}_{\mathbf{k}} c_{-\mathbf{k}} + \tilde{v}_{\mathbf{k}}^* c_{\mathbf{k}}^+, \\ &|\tilde{u}_{\mathbf{k}}|^2 - |\tilde{v}_{\mathbf{k}}|^2 = 1. \end{aligned} \tag{8}$$

As a result we have (to within a constant term)

$$\mathcal{H}_0 + \mathcal{H}_t = \sum_{\mathbf{k}} \tilde{\varepsilon}_{\mathbf{k}} c_{\mathbf{k}}^+ c_{\mathbf{k}}, \quad \tilde{\varepsilon}_{\mathbf{k}} = [(A_{\mathbf{k}} + \mu h)^2 - |B_{\mathbf{k}}|^2]^{1/2}. \tag{9}$$

The magnetization of the system is expressed in terms of the correlators

$$\langle a_{\mathbf{k}}^+ a_{\mathbf{k}} \rangle = \mathrm{Sp} [\rho(t) a_{\mathbf{k}}^+ a_{\mathbf{k}}].$$

On transforming from the operators $a_{\mathbf{k}}$ to the operators

c_k, we have with the aid of (8) and (9),

$$\langle a_k^+ a_k\rangle = |\tilde u_k|^2\langle c_k^+ c_k\rangle_0 + |\tilde v_k|^2\langle c_{-k} c_{-k}^+\rangle_0 + \tilde u_k^* \tilde v_k \langle c_k c_{-k}\rangle_0 \times \exp\left(-\frac{2i}{\hbar}\tilde\varepsilon_k t\right) + \tilde u_k \tilde v_k^* \langle c_k^+ c_{-k}^+\rangle_0 \exp(2i\tilde\varepsilon_k t/\hbar),$$

where $\langle\ldots\rangle_0 = \mathrm{Sp}(\rho_0\ldots)$. On further expressing the operators c_k in terms of the operators b_k and performing the averaging, we find the magnetic moment of the system:

$$M \equiv \langle M_z\rangle = M_0 - \mu\sum_k\left(\frac{A_k}{\varepsilon_k}n_k + \frac{A_k-\varepsilon_k}{2\varepsilon_k}\right) + \mu\sum_k \frac{\mu h|B_k|^2}{\varepsilon_k\tilde\varepsilon_k^2}\left(n_k+\frac{1}{2}\right)\left(1-\cos\frac{2\tilde\varepsilon_k}{\hbar}t\right), \qquad (10)$$

where $M_0 = \mu NS$ is the nominal magnetic moment. The first two terms in (10) correspond to the static magnetization in field H_0; the third term is the contribution due to the switching on of the field h. As is seen from (10), it is positive; this is quite natural. But its dependence on the value of the field h turns out to be nonmonotonic: this contribution vanishes for h = 0 and for h = ∞. An estimate at zero temperature gives for the field corresponding to a maximum contribution a value of the order of the magnitude of the gap in the spin-wave spectrum. At small fields, one obtains from (10) a result agreeing with the linear response.[1]

By means of the distribution (7), we find for the energy of the system after switching on of the field h

$$E \equiv \mathrm{Sp}\{\rho(t)(\mathcal{H}_0+\mathcal{H}_t)\} = \mathrm{Sp}\{\rho_0(\mathcal{H}_0+\mathcal{H}_t)\} = \langle\mathcal{H}_0\rangle_0 - \mu h\langle M_z\rangle_0. \qquad (11)$$

As is seen from (11), the energy of the ferromagnet changes discontinuously by the amount $\mu h\langle M_z\rangle_0$ when the field is switched on, whereas the magnetization changes smoothly. In particular, in the absence of magnetic dipole interaction the magnetization does not change at all when the field is switched on.

2. The behavior of the magnetic system considered above occurs only at times small in comparison with the smallest relaxation time. Relaxation processes lead to establishment of the equilibrium state. Thus after switching on of the field h the system, which was originally in the equilibrium state corresponding to field H_0, changes to a new equilibrium state corresponding to field H_0+h. Here we shall present the derivation of a kinetic equation that describes this process. We shall take account only of exchange interaction between the magnons, the Hamiltonian of which, to terms of the fourth order in the Bose operators, has the form

$$\mathcal{H}_{int} = \sum_{1,2,3,4}\psi_{1,2;3,4}a_1^+ a_2^+ a_3 a_4,$$

explicit expressions for the real amplitudes ψ are given, for example, in the book of Akhiezer and others.[5] We consider the case $|B_k| \ll A_k$ (see [3]).[1)] In this case, in the transformation from the operators a_k to the operators c_k it may be supposed that $\tilde u_k \approx 1$ and $\tilde v_k \approx 0$. Consequently the Hamiltonian $\mathcal{H}_{int}$ preserves its form:

$$\mathcal{H}_{int} = \sum_{1,2,3,4}\psi_{1,2;3,4}c_1^+ c_2^+ c_3 c_4. \qquad (12)$$

In the derivation of the kinetic equation, we shall use Bogolyubov's splitting method in the form given in [6].

[1)] This inequality is always satisfied when the gap in the spin-wave spectrum is sufficiently large or, if this is not the case, when the temperature is not too low.

In our case, in contrast to those usually treated, it is necessary to take into account not only the one-particle distribution function $\langle c_k^+ c_k\rangle$ but also the anomalous correlators $\langle c_k c_{-k}\rangle$ and $\langle c_k^+ c_{-k}^+\rangle$. These correlators differ from zero because of the initial condition, according to which the system is in an equilibrium state until switching on of the field h. For the time derivatives of the correlators in question we have

$$-i\hbar\frac{d}{dt}\langle c_k^+ c_k\rangle = 2\sum_{1,2,3}\psi_{1,2;3,k}(\langle c_1^+ c_2^+ c_3 c_k\rangle - \text{c. c.}),$$
$$i\hbar\frac{d}{dt}\langle c_k c_{-k}\rangle = 2\tilde\varepsilon_k\langle c_k c_{-k}\rangle + 2\sum_{1,2,3}\psi_{-k,1;2,3}\times\langle c_1^+ c_2 c_3 c_k\rangle + 2\sum_{1,2,3}\psi_{k,1;2,3}\langle c_1^+ c_2 c_3 c_{-k}\rangle. \qquad (13)$$

As usual, in order to obtain a closed system of equations, a splitting is performed in the right sides of the equations for the correlators that appear in equation (13). In the splitting, it is necessary to take into account not only the normal but also the anomalous pair correlators. As a result we get the system of equations

$$\frac{dn_k}{dt} = -\frac{32\pi}{\hbar}\sum_{1,2,3}\{\psi^2_{1,2;3,k}[(n_1+1)(n_2+1)n_3 n_k - n_1 n_2(n_3+1)(n_k+1)] \times\delta(\varepsilon_{1,2;3,k}) + \psi_{1,2;3,k}[\psi_{-2,3;-k,1}(n_3-n_1)(\sigma_2^*\sigma_k+\sigma_2\sigma_k^*)\delta(\varepsilon_{1,k;2,3}) + \psi_{-2,k;1,-3}(n_k-n_1)(\sigma_2^*\sigma_3+\sigma_2\sigma_3^*)\delta(\varepsilon_{1,3;2,k})]\},$$
$$\frac{d\sigma_k}{dt} = -\frac{2i\tilde\varepsilon_k}{\hbar}\sigma_k + \frac{16\pi}{\hbar}\sum_{1,2,3}\{(\psi^2_{-k,1;2,3}+\psi^2_{k,1;2,3})[(n_1+1)n_2 n_3 \qquad (14)$$
$$-n_1(n_2+1)(n_3+1)]\sigma_k\delta(\varepsilon_{1,k;2,3}) + 2(\psi_{-k,1;2,3}\psi_{2,k;-3,1}+\psi_{k,1;2,3}\psi_{2,-k;-3,1}) \times[(n_1+1)n_2 n_k - n_1(n_2+1)(n_k+1)]\sigma_3\delta(\varepsilon_{1,3;2,k}) - 2(\psi_{-k,1;2,3}\psi_{-1,2;-3,-k} + \psi_{k,1;2,3}\psi_{-1,2;-3,k})\sigma_1^*\sigma_3\sigma_k\delta(\varepsilon_{1,2;3,k}) - (\psi_{-k,1;2,3}\psi_{-1,k;-2,-3} + \psi_{k,1;2,3}\psi_{-1,-k;-2,-3})\sigma_1^*\sigma_2\sigma_3\delta(\varepsilon_{1,k;2,3}); \qquad (15)$$

$\tilde\varepsilon_{1,2;3,k} = \tilde\varepsilon_1 + \tilde\varepsilon_2 - \tilde\varepsilon_3 - \tilde\varepsilon_k$, with an analogous equation for σ_k^*. In the derivation of equations (14) and (15), use has been made of the fact that the correlator σ contains an exponential factor that attenuates rapidly with time and which, according to (13), is equal to $\exp(-2i\tilde\epsilon_k t/\hbar)$. The presence of this factor manifests itself, in particular, in the laws of conservation of energy in (14) and (15).

Equations (14) and (15) describe completely the relaxation to the new equilibrium state; they constitute a system of integrodifferential equations not only for the distribution functions n_k but also for the correlators σ_k and σ_k^*. For times large in comparison with the characteristic periods $\hbar/\tilde\epsilon_k$, these equations simplify considerably, because for such times those terms drop out that contain rapidly oscillating factors of the type $\exp[2i(\tilde\epsilon_k - \epsilon_1)t/\hbar]$. As a result, the equations are significantly simplified:

$$\frac{dn_k}{dt} = -\frac{32\pi}{\hbar}\sum_{1,2,3}\psi^2_{1,2;3,k}[(n_1+1)(n_2+1)n_3 n_k - n_1 n_2(n_3+1) \times(n_k+1)]\delta(\varepsilon_{1,2;3,k}),$$
$$\frac{d\tilde\sigma_k}{dt} = -\frac{16\pi}{\hbar}\tilde\sigma_k\sum_{1,2,3}(\psi^2_{1,2;3,k}+\psi^2_{1,2;3,-k}) \times[(n_1+1)(n_2+1)n_3 - n_1 n_2(n_3+1)]\delta(\varepsilon_{1,2;3,k}), \qquad (16)$$

where $\tilde\sigma_k = \sigma_k\exp(2i\tilde\epsilon_k t/\hbar)$ is a slowly changing function of time. The first equation (16) is the usual kinetic equation for the distribution function n_k. In the linear approximation with respect to the nonequilibrium contribution, it contains both departure and arrival terms in the col-

lision integral. In the same approximation, the equation for $\tilde{\sigma}_{\mathbf{k}}$ contains only a "departure" term, and this enables us to introduce a relaxation time:

$$\frac{1}{\tau_k}=\frac{16\pi}{\hbar}\sum_{1,2,3}(\psi^2_{1,2;3,k}+\psi^2_{1,2;3,-k})[(n_1+1)(n_2+1)n_3-n_1n_2(n_3+1)]\delta(\varepsilon_{1,2;3,k}). \quad (17)$$

Thus the spin system of a ferromagnet after switching on of a constant field of arbitrary value is initially described by the nonequilibrium moment (10). At times large in comparison with the characteristic periods of the system, the anomalous correlators attenuate exponentially. The relaxation time for them is described by equation (17). At times coinciding in order of magnitude with (17), the distribution function becomes an equilibrium one, and the system is described by the Gibbs distribution corresponding to the total magnetic field.

[1] R. Kubo, Lectures in Theoretical Physics, Vol. 1 (New York, 1959).

[2] H. Suhl, J. Phys. Chem. Solids **1**, 209 (1957).

[3] V. Zakharov, V. S. L'vov, and S. S. Starobinets, Zh. Eksp. Teor. Fiz. **59**, 1200 (1970) [Sov. Phys.-JETP **32**, 656 (1971)].

[4] N. Zagury and S. M. Rezende, Phys. Rev. **B4**, 201 (1971).

[5] A. Akhiezer, V. Bar'yakhtar, and S. Peletminskiĭ, Spinovye volny (Spin Waves), "Nauka," 1967 (translation, North Holland, 1968).

[6] R. N. Gurzhi and V. M. Tsukernik, Zh. Eksp. Teor. Fiz. **43**, 977 (1962) [Sov. Phys.-JETP **16**, 692 (1963)].

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