THEORY OF THE KAPITZA TEMPERATURE DISCONTINUITY AT A SOLID BODY-LIQUID HELIUM BOUNDARY

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The effect is considered of sound attenuation in a narrow surface layer of a solid on the heat flow through the interface between liquid helium and the solid. The additional heat flow depends materially on the state of the solid surface. It is less sensitive to external pressure and may have a different temperature dependence than the flow previously calculated. The process of heat exchange due to interaction between fermions and the solid surface is also discussed.

As is well known, a temperature discontinuity, called the Kapitza discontinuity, develops at the interface between a solid body and liquid helium when heat flows between them.

According to the theory of Khalatnikov,[1] the temperature discontinuity is explained by the difficulty of transition of the phonons from liquid helium to the solid and conversely. In this case, the heat flow W_0 from the liquid helium to the solid can be written in the form

$$W_0=\frac{\hbar}{(2\pi c)^2}\int\limits_0^\infty n\left(\frac{\hbar\omega}{T}\right)\omega^3\,d\omega\int\limits_0^1 w(\omega,\theta)\cos\theta\,d\cos\theta. \tag{1}$$

Here n is the Planck function, T the temperature, c the speed of sound in helium, and $w(\omega, \theta)$ the coefficient of transmission of a phonon with energy $\hbar\omega$, incident at an angle θ to the interface into the solid.

The flow of heat (1), in which the transmission coefficient w obtained in[1] is substituted, is small in fact for two reasons. First, as follows from the boundary conditions, the amplitude of the sound wave entering the solid (and, correspondingly, w) is proportional to the ratio of the helium density ρ to the solid density D ($\rho/D \ll 1$). Second, at incidence angles larger than critical, $\sin\theta_c = c/c_t \ll 1$ (c_t is the speed of transverse sound in the solid), total internal reflection of the incident sound takes place, i.e., $w(\theta > \theta_c) = 0$. Therefore, only a narrow cone of angles, amounting only to a small part $(c/\sqrt{2}\,c_t)^2$ of the solid angle of the entire hemisphere, makes a contribution to the integral over θ in (1).

Furthermore, as mentioned by Andreev,[2] the amplitude of the transmitted sound wave has a sharp maximum at incidence angles close to $\sin\theta_1 = c/\xi c_t$. The numerical values of ξ is of the order of unity and depends only on the ratio of the speed of transverse sound c_t to the longitudinal c_e (the graph of ξ is shown in[3] on p. 143). Inasmuch as $\xi < 1$, the θ_1 lies in the plane of total internal reflection. A Rayleigh surface wave, which does not carry energy to the interior of the solid, is then excited in the solid. However, if we take into account the ever present sound attenuation, then the Rayleigh wave makes the same contribution to the heat flow as the region of subcritical angles $\theta \le \theta_c$.

The heat flow due to the Rayleigh waves was fully studied by Khalatnikov,[1] although specific mechanisms of sound attenuation were not discussed. Somewhat later, Andreev[2] considered the interaction of conduction electrons with phonons as a specific mechanism which leads to heat flow due to Rayleigh waves. It is evident that the heat flow obtained in[2] is already included in the final result of[1]:

$$W=\frac{\rho}{D}c\frac{4\pi^5}{15}\frac{T^4F}{(2\pi\hbar c_t)^3}. \tag{2}$$

Here $F \approx 1$ is some function of the elastic constants of the solid.

As is known, the theoretical value of the heat transfer coefficient of the boundary, $Q = \partial W/\partial T$, is at least an order of magnitude smaller than the experimental values, which, in turn, differ somewhat from one other. So far as the temperature dependence is concerned, the exponent in the different experiments deviates within the limits of 0.5 on both sides of the theoretical (see the reviews of Pollack[4] and Snyder[5], and also the later works of Zinov'eva[6] and Jonson and Anderson[7]). Furthermore, according to all known experiments (see, for example, the works of Kuang Wey-yen[8] and Challis et al.[9]), the heat transfer coefficient depends more weakly on the external pressure than follows from Eq. (2).

The reasons why the experimental values disagree with one another and with Khalatnikov's theory have been considered in many researches. So far as we know, they contain no principally new theory that explains the Kapitza discontinuity. Everything reduces essentially to allowance for the non-ideality of the interface.

Thus, for example, the weak dependence of the temperature discontinuity on the pressure and the excessive values of Q are explained in[9] by the existence on the surface of the solid of a translucent denser layer of helium which is formed under the action of Van der Waals forces. Incidentally, the acoustically induced transparency connected with the transition layer was proposed in[10] to be used to improve the heat transfer between the helium and the solid.

The effect of the always present roughnesses of the helium-solid interface was considered in[11,12]. As shown in[12], when the wavelength of the phonon is of the order of the characteristic dimensions of the roughnesses, an unusual spatial resonance takes place, leading to an increase in the heat flow. True, the calculations in[12] are strictly satisfied only for sufficiently flat roughnesses, when the contribution to the heat flow is small in accord with the smallness of the angles of inclination.

In one of the recent theoretical papers,[12] heat transfer was considered which is due to the interaction of the helium atoms with the solid surface. It was assumed in this case that the interaction between the atoms of helium is absent. If $T \to 0$, then the coefficient of heat transfer is the same as in the radiation of phonons into a gas, i.e., the result of Khalatnikov is actually valid. In the region of higher temperatures, Q deviates in the direction of higher values. However, as the authors of the mentioned paper themselves note, a model in which the interaction of the atoms among themselves is neglected is scarcely adequate for the helium-solid system.

Finally, as follows from the results obtained in a number of researches[7,8,14], it is necessary to take into account the fact that the narrow surface layer of the solid, of thickness of the order 10^{-5}–10^{-6} cm, is as a rule greatly deformed and differs materially from the bulk solid.

In the present work, the effect of sound attenuation in the narrow surface layer of the solid on the heat flow across the interface is explained. It is shown here that the phonon can enter the solid at any angle of incidence at the interface and not only in the subcritical region and close to the angle corresponding to the excitation of Rayleigh waves. This leads to the result that the range of integration over θ in (1) increases by the factor $2(c_t/c)^2$.

Thus, let a plane monochromatic wave be incident from the liquid, which occupies the half-space $z > 0$. The potential of this wave is

$$\varphi = A_0 e^{-ik_z z} e^{i(k_x x - \omega t)}, \tag{3}$$

where $k^2 = k_x^2 + k_z^2 = \omega^2/c^2$. In addition to the incident wave, there is the reflected wave

$$\varphi_r = A e^{ik_z z} e^{i(k_x x - \omega t)} \tag{4}$$

and the transmitted longitudinal and transverse waves. The velocity field produced in the solid by the transmitted waves can be written in the form

$$\mathrm{v} = \nabla\varphi_2 + \mathrm{rot}\,\psi. \tag{5}$$

Here $\varphi_2 = A_l e^{-ik_{lz}z} e^{i(k_x x - \omega t)}$,

$$\psi_y = A_t e^{-ik_{tz}z} e^{i(k_x x - \omega t)}, \quad \psi_x = \psi_z = 0. \tag{6}$$

where k_l and k_t are the wave numbers corresponding to the longitudinal and transverse waves,

$$k_{lz} = \sqrt{k_l^2 - k_x^2}, \quad k_{tz} = \sqrt{k_t^2 - k_x^2}. \tag{7}$$

It follows from (6) and (7) that for real $k_{l(t)} = \omega/c_{l(t)}$ and for $k_x > k_t$, total internal reflection of the incident sound wave takes place, inasmuch as k_{tz} and k_{lz} are purely imaginary. Here, in the narrow skin layer of the solid, of thickness $\delta \sim 1/|k_{lz}|$ the so-called inhomogeneous wave[15] is propagated which, as in the case of Rayleigh waves, does not carry energy into the interior of the solid. If k_l and k_t have imaginary parts, due to attenuation of the sound wave in the solid, then a leaking of the energy into the interior of the solid takes place. In this case, the transmission coefficient $w(\theta > \theta_c) \neq 0$.

It is convenient to use the following definition of the transmission coefficient:

$$w = E_d / E_{\mathrm{inc}}. \tag{8}$$

Here

$$E_d = \frac{1}{2}\int_{-\infty}^{0} \eta_{iklm} \frac{\partial v_i}{\partial x_k} \frac{\partial v_l^*}{\partial x_m} dz \tag{9}$$

is the average energy dissipated in the solid per unit time, η_{iklm} is the viscosity tensor,

$$E_{\mathrm{inc}} = {}^1/_2 |A_0|^2 \rho c k k_z \tag{10}$$

is the energy incident per unit time per unit area of the interface.

The velocity field in the solid, in accord with (5) and (6), can be written in the form

$$v_x = ik_x\varphi_2 \left(1 + \frac{k_{tz}}{k_x}\frac{A_t}{A_l}\exp[iz(k_{lz} - k_{tz})]\right), \tag{11}$$

$$v_z = -ik_{lz}\varphi_2 \left(1 - \frac{k_x}{k_{lz}}\frac{A_t}{A_l}\exp[iz(k_{lz} - k_{tz})]\right).$$

The coefficients A_l and A_t in (11) can be found from the system of boundary conditions, which stipulate continuity of the normal displacements and pressures at $z = 0$. If we neglect the damping of the sound wave, which corresponds to account only of terms of first order of smallness in the absorption, the solutions of this system can be written as[15] 1)

$$A_t / A_l = 2k_x k_{lz} / (2k_x^2 - k_t^2), \tag{12}$$

$$\frac{A_l}{A_0} = \frac{2\varepsilon_1^2 \varepsilon_2 k^2 k_z (k_t^2 - 2k_x^2)}{k_z (k_t^2 - 2k_x^2)^2 + 4k_x^2 k_z k_{lz} k_{tz} + \varepsilon_1^2 \varepsilon_2 k^2 k_t^2 k_{lz}}, \tag{13}$$

where $\epsilon_1 = c/c_t$, $\epsilon_2 = \rho/D$.

We now consider the region of transcritical angles, where $k_x \sim k \gg k_t$. It then follows from (11) and (12) that the longitudinal and transverse inhomogeneous waves vibrate in counterphase, so that the total velocity field differs from zero only in the next approximation in ϵ_1^2, i.e.,

$$v \approx k_x \varphi_2 \varepsilon_1^2. \tag{14}$$

As a result, the factor ϵ_1^4 appears in the transmission coefficient (8).

It is appropriate to note the special role of the transverse wave in the solid. The absence of the latter in the narrow skin layer of the more dense medium would lead to a sharp increase in the transmission coefficient in the transcritical range of angles and, correspondingly, to a decrease in the thermal resistance of the boundary.

The derivatives of the velocity components with respect to the coordinates are conveniently written in the following compact matrix form, with accuracy to the first non-vanishing terms in ϵ:

$$\begin{pmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial z} \\ \frac{\partial v_z}{\partial x} & \frac{\partial v_z}{\partial z} \end{pmatrix} = \frac{A_l}{2} e^{k_x z} k_t^2 \left\{ (1 - \beta^2) k_x z \begin{pmatrix} -1 & i \\ i & 1 \end{pmatrix} - \begin{pmatrix} \beta^2 & -i \\ i & \beta^2 \end{pmatrix} \right\} e^{i(k_x x - \omega t)}, \tag{15}$$

1) Account of the dissipative components in the wave numbers and the boundary conditions would be necessary if the problem were solved by another, more complicated but, of course, equivalent method, in which the transmission coefficient is defined as $w = 1 - |A/A_0|^2$.

where $\beta = c_t/c_l$. Substituting (15) in (9), and taking into account the relations (10), (13) and (8), it is not difficult to obtain the transmission coefficient of the phonons in the transcritical range of angles:

$$w_3 = \frac{2\varepsilon_2^2 k_t^4}{(1-\beta^2)^2 \rho c k k_x k_z}\left(\frac{4}{3}\eta + \zeta + \eta\frac{1-2\beta^2}{\beta^4}\right). \quad (16)$$

In an isotropic body, the tensor η_{iklm} has only two independent components, the combinations of which are denoted in (16) by η and ζ.

It is convenient to write the result in terms of the relative damping of the longitudinal and transverse sound waves:

$$\gamma_l = \frac{\operatorname{Im} k_l}{k_l} = \frac{k_l}{2Dc_l}\left(\frac{4}{3}\eta + \zeta\right), \quad \gamma_t = \frac{\operatorname{Im} k_t}{k_t} = \frac{k_t \eta}{2Dc_t}, \quad (17)$$

after which we have finally for the transmission coefficient

$$w_3 = \frac{4}{(1-\beta^2)^2}\frac{k_t^2 \varepsilon_2}{k_x k_z}[\gamma_l \beta^2 + \gamma_t(1-2\beta^2)]. \quad (18)$$

We recall that $\beta^2 < 1/2$ for all known bodies.[3]

Further calculations are determined to a significant extent by the frequency dependence of the damping coefficients $\gamma_l(t)$. If it is assumed that the relative absorption does not depend on the frequency, as takes place in the damping of phonons on dislocations,[16,17] then substitution of (18) in (1) gives

$$W_3 = \frac{4\pi^6}{15(1-\beta^2)^2}\frac{T^4 \varepsilon_2}{(2\pi\hbar)^3 c_t^2}[\gamma_l \beta^2 + \gamma_t(1-2\beta^2)]. \quad (19)$$

It is interesting to compare the obtained heat flow with the flow (2)

$$\frac{W_3}{W} = \frac{\pi[\gamma_l \beta^2 + \gamma_t(1-2\beta^2)]}{(1-\beta^2)^2 F}\frac{c_t}{c}. \quad (20)$$

We note that although the region of transcritical angles is $2(c_t/c)^2$ times greater than the region of subcritical, the large parameter $1/\epsilon_1$ enters in the final result only in the first degree. This is connected with the fact that, as is seen from (15), the inhomogeneous wave moves in the narrow skin layer, the depth of which is of the order of $1/k$. To sum up, the transmission coefficient in the transcritical region of angles is proportional to $\operatorname{Im} k_t/k = \epsilon_i \gamma_i$.

A formula of type (20) will also occur when the damping coefficient γ is not small. Unfortunately, not only are the formulas obtained for the problem under consideration very cumbersome, but there are obscurities in the exact formulation of the problem. However, the order of magnitude of the effect can easily be obtained from a consideration of a model problem in which the solid is replaced by a liquid characterized by a density D and sound speed c_t. We easily find a simple expression for the transmission coefficient w in the transcritical region of angles, assuming only smallness of the ratio c^2/c_t^2 (but not γ):

$$w = \frac{\rho c^2}{Dc_t^2}\frac{1}{\cos\theta\sin\theta}4\gamma \qquad (\theta \gg \theta_c)$$

The resultant transmission coefficient is smaller than the corresponding coefficient for the subcritical region of angles in the ratio $(c/c_t)\gamma$. However, inasmuch as the region of transcritical angles is $2(c_t/c)^2$ times larger than the region of subcritical, the gain in the ratio of heat flows will be $\gamma(c/c_t)(c_t/c)^2 \sim \gamma(c_t/c)$ and consequently a formula of the type (20) will hold for damping that is not small ($\gamma \sim 1$).

It was noted above that the surface layer of the solid is as a rule strongly deformed. The thickness of the deformed amorphous layer is $\sim 10^{-6}$ cm. The total thickness of the transition zone of the deformed metal is $\sim 10^{-5}$ cm. This circumstance ought to lead (as follows, for example, from the work of Zusman and Rokhlin,[18]) to strong damping of the inhomogeneous sound wave, which moves in a skin layer of depth $\delta \approx 1/k \approx 4 \times 10^{-7}\, T^{-1}$ cm.

According to (20), at relative damping γ on the order of unity, the heat flow is increased and the Kapitza resistance decreased) by half an order of magnitude. This circumstance could explain the significant difference of the experimentally observed Kapitza resistance from the ideal, which is given by Eq. (2).[2)] In this case, the resulting heat transfer coefficient will be sensitive to the state of the surface of the solid, which is in qualitative agreement with the experimental data.[7,8]

According to (19), W_3 generally does not depend on the sound speed in the less dense medium in the considered approximation in ϵ_1. This leads to a weaker dependence of W_3 on the external pressure in comparison with (2), which agrees qualitatively with the results of the work of Kuang[8] and Challis et al.[9] and, finally, the temperature dependence of W_3 can differ from that given in (19) if the relative damping depends on the frequency.

The results are applicable for He^3 in the range of temperatures $T \ll \hbar\tau$ (here τ is the time of free flight of the quasi-particle), when the theory of a Fermi liquid is valid. The surface of the solid, executing small oscillations with a characteristic frequency $\omega \sim T/\hbar$, cannot radiate (absorb) a phonon, inasmuch as $\omega\tau \gg 1$.

The heat transfer process in this case, as shown by Bekarevich and Khalatnikov,[19] is determined by the interaction of the vibrating boundary of the solid with the quasiparticles.

Special interest was paid in[19] to the possibility of radiation of the vibrating boundaries in a Fermi liquid of the collision-free collective mode—zero sound. The constant a in the final results was represented as the sum of two components. They depended on the energy of interaction of the quasiparticles f(p, p′) or, more accurately, on the first two coefficients F_0 and F_1 of the expansion of f(p, p′) in spherical harmonics. Upon substitution of numerical values for F_0 and F_1, the constant a is equal to 0.3 and is determined essentially by the first component, which is equal to the residue at the pole corresponding to zero sound.

In one of the latest theoretical researches, Rice[20] considered the heat exchange process due to "inelastic" collisions of noninteracting (f(p, p′) = 0) quasiparti-

[2)]The explanation of the sources of absorption and phonon damping in the surface layer of the solid is very important for understanding the reasons for the decrease in the Kapitza resistance. The fact is that in the attempt at a description of the dissipative properties of the solid, we encounter a difficulty, which is that all the known viscosity mechanisms (absorption of phonons by electrons or scattering of phonons by impurities and dislocations) lead to the appearance of a small dimensionless parameter in the damping coefficient γ.

cles with the solid. Here it is appropriate to note that the "inelastic" mechanism of heat transfer is automatically contained in the final result of Bekarevich and Khalatnikov.[19] This can easily be established by letting F_0 and F_1 approach zero in the expression for the heat transfer coefficient given in[19]. Then the constant a is determined only by the second component, which is equal to 0.25 in this case.

By virtue of what was pointed out above, it is not surprising that the heat transfer process, obtained by Rice[20] actually coincides with the result of[19]. The circumstance has been discussed in detail in the work of Sheard, Toombs and Challis[21].

In conclusion, we shall show that the heat transfer process due to "inelastic" collisions of noninteracting fermions with the vibrating wall, can be described by another method, essentially equivalent to that described in[20], but much simpler from our point of view. For this, it suffices to note that such a process is completely analogous to exchange of energy between the solid and He II in "inelastic" collisions of rotons and phonons with the solid wall. According to[1], the heat flow for He II connected with this mechanism is small in comparison with the flow due to radiation (absorption) of sound in He II.

A fermion reflected from a solid can acquire (lose) some energy, a fact accompanied by absorption (radiation) of a phonon in the solid. Inasmuch as all subsequent discussions and calculations are absolutely analogous to the calculations of[1], we shall write out here only the final result for the heat transfer coefficient:

$$Q = 6\pi \frac{\rho}{m} \frac{p_0}{D} \frac{T^3 F I}{(2\pi\hbar c_t)^3}, \tag{21}$$

where $\rho m^{-1} = N$ is the number of quasiparticles per unit volume, $p_0 = \hbar(3\pi^2 N)^{1/3}$ is the boundary momentum and

$$I = \int_0 dx \int_{-\infty} \frac{(x-x_1)^3 e^{x+x_1}}{(e^x - e^{x_1})(e^x+1)(e^{x_1}+1)} dx_1 \approx 8.4.$$

This expression is naturally identical (in the sense noted above) with the result of[19]. If we now make use of the definition of the Debye temperature and the boundary momentum, then the relation (21) can be rewritten in a form practically identical with the results of the work of Rice.[20]

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