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DESCRIPTION OF ELECTROPRODUCTION PROCESSES BY USING THE APPROXIMATION OF QUASI-REAL PHOTONS

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A general approach is formulated for the description of two-photon electroproduction processes ($e^{\pm}e^{-} \to e^{\pm}e^{-} + N$) under conditions when only the particles produced with a small total transverse momentum are observed. It is shown that under these conditions a number of combinations of the amplitudes for the photon-photon process $\gamma + \gamma \to N$ can in principle be determined, one of these combinations being the photon-photon cross section for unpolarized photons, and the remaining combinations enter into the photon-photon cross section with linearly polarized photons.

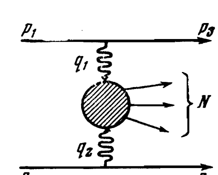
1. Electroproduction of particles in colliding-beam experiments ($e^{\pm}e^{-} \to e^{\pm}e^{-} + N$) has recently been widely discussed.[1-7] The electroproduction of electron-positron pairs ($N = e^{+}e^{-}$) at large angles was recently observed at Novosibirsk.[8] In what follows it is proposed to use this process in order to investigate hadron production by two photons (see the accompanying figure); however the contributions of the one-photon diagrams to the cross section for the electroproduction of hadrons are expressed in terms of functions which have been studied in the process of one-photon annihilation of $e^{+}e^{-}$ pairs, and these contributions are substantially smaller in magnitude than the contribution from the two-photon diagrams (see, for example,[5]).

It is necessary to distinguish between the region of "quasi-real" photons, where $\Delta_{1,2}^2 \ll \mu^2$ ($\Delta_{1,2}^2 = -q_{1,2}^2$ and μ denotes the characteristic mass of the created hadrons), and the "deep inelastic" region $\Delta_{1,2}^2 \gg \mu^2$. In the first region the electroproduction cross section is expressed in terms of the amplitude for the photoprocess $\gamma + \gamma \to N$, which to a certain extent can also be investigated in processes involving real photons, and which leads itself to a simpler analysis than is required in the second region. In addition, the electroproduction cross section in the first region is much larger than in the second region, so that only the region of "quasi-real" photons is actually accessible for the presently available experimental arrangements involving colliding beams, and for this reason it is of special interest.

If the final electrons with energies $\epsilon_{3,4}$ emitted at angles $\theta_{3,4}$ are detected, then the "quasi-real" region can be isolated by selecting events with small values of $\Delta_{1,2}^2$. In the case when only the created particles are detected, the "quasi-real" region can also be distinguished by fixing their kinematic characteristics, for example, the value of $\Delta_\perp$ ($\Delta = q_1 + q_2$, $\Delta_\perp \cdot p_1 = 0$). Such an analysis was recently carried out by Cheng and Wu[9] for the special case of the electroproduction of pairs of spinless bosons (for definiteness, $\pi^{+}\pi^{-}$ pairs); in their treatment the electron parts of the diagram were calculated exactly, and a standard parametrization of the amplitude[1)] was used to describe the block diagram $\gamma + \gamma \to \pi^{+} + \pi^{-}$. A general (tensor) formulation of the problem of electroproduction in the region of "quasi-real" photons is given in the present article. It is shown that here the problem is essentially a two-dimensional (transverse) problem, where the tensor of the electrons (the product of the current tensors), after integration over the final states, can be expanded in terms of five independent tensor combinations whose coefficients are explicitly calculated.

After contraction of this universal tensor with the square of the appropriately parametrized amplitude for any arbitrary process $\gamma + \gamma \to N$, we obtain the electroproduction cross section, differentiated with respect to the states of the created particles; the resultant expression is valid to power-law accuracy (to terms $\sim |\Delta_\perp|/\mu$). In such a form this cross section cannot in general be expressed in terms of a photon-photon collision cross section averaged over the polarizations of the photons, and for this reason it is more "informative" (for example, in the particular case $\gamma + \gamma \to \pi + \pi$ one can separately determine both amplitudes and their relative phase,[9] and in the case of the photon-photon process $\gamma + \gamma \to N + \overline{N}$ (when the polarization is not measured) four independent combinations of the amplitudes can be determined). The information that can be obtained through such a formulation of the problem is discussed in the general case, and also two examples are treated in detail, namely, the electroproduction of pairs of spin-½ particles and the electroproduction of the $\rho\pi$ system. We also compare our results with the results obtained by using the approximation of equivalent photons, an approximation which has been extensively used in the quasi-real region.

It should be kept in mind that previous analyses of the electroproduction cross section in terms of the tensor structure or helicity amplitudes were carried out either in situations where an integration was performed over all states of the created particles (see,



1) In the limit $\Delta_{1,2}^2 \to 0$ it is expressed in terms of two invariant amplitudes.

for example,[10,11]) or else the analysis was made for the total differential cross section (see, for example,[11,12]). In both cases the kinematic characteristics of the final electrons were fixed. However, in a present-day experiment it is actually the created particles which are detected, and the electrons are not detected. Even in the case of a magnetic analysis of the reaction products, in which optimum conditions exist for the detection of the final electrons, the measurement of the very small angles of flight of the electrons in the important region represents an extremely complicated problem. Therefore, in the present work, just as in[9], we integrate over the final states of the electrons at fixed kinematics of the created particles.[2)]

2. Let us represent the cross section for the electroproduction process, depicted by the diagram (shown in the accompanying figure) in the center-of-mass system, in the form

$$d\sigma=\frac{2\alpha^2}{\varepsilon^2\Delta_1^4\Delta_2^4}J_{\mu\nu}(p_1,p_3)J_{\rho\sigma}(p_2,p_4)T^{\mu\rho}(q_1,q_2,N)T^{\nu\sigma*}(q_1,q_2,N)\cdot \tag{1}$$
$$\times\delta(p_1+p_2-p_3-p_4-\Delta)\frac{d^3p_3}{2\varepsilon_3}\frac{d^3p_4}{2\varepsilon_4}\prod_{i=1}^{N}\frac{d^3k_i}{2\varepsilon_{k_i}}\frac{1}{(2\pi)^3},$$

where $\Delta_{1,2}^2=-q_{1,2}^2$, $\epsilon(m)$ denote the energy (mass) of the initial electrons, and $J_{\mu\nu}(p_1,p_3)$ is the current tensor:

$$J_{\mu\nu}(p_1,p_3)=-{}^1\!/_2\Delta_1^2g_{\mu\nu}+p_{1\mu}p_{3\nu}+p_{1\nu}p_{3\mu},\qquad s\Delta=\sum_{i=1}^{N}k_{i},\tag{2}$$

$T^{\mu\rho}(q_1,q_2,N)$ denotes the amplitude for the transition of two photons into the state N. For $q_{1,2}^2=0$ and for unpolarized photons we have

$$d\sigma_{\gamma\gamma\to N}=\frac{\pi^4}{(q_1q_2)}T^{\mu\rho}T_{\mu\rho}^{*}\delta(q_1+q_2-\Delta)\prod_{i=1}^{N}\left(\frac{d^3k_i}{2\varepsilon_{k_i}}\frac{1}{(2\pi)^3}\right).\tag{3}$$

The cross section (1) is large in the region of small $\Delta_{1,2}^2\ll\mu^2$, and in what follows we shall confine our attention to this region. Here, under the assumption that the final electrons are ultrarelativistic, the following relations hold:

$$\Delta_{1,2}^2=\frac{\varepsilon}{\varepsilon-\omega_{1,2}}\left(\mathbf{q}_{1,2\perp}+\frac{m^2}{\varepsilon^2}\omega_{1,2}^2\right),\qquad q_{1\parallel}=\omega_1,\qquad q_{2\parallel}=-\omega_2,\tag{4}$$

where $\omega_{1,2}=q_{1,2}^0$, and $q_{\parallel}=q_z$ ($q_{\perp}(q_x,q_y)$) are the components of the vector q parallel (perpendicular) to the vector p_1.

Since the amplitude $T^{\mu\rho}(q_1,q_2,N)$ is gauge invariant,

$$q_{1\mu}T^{\mu\rho}=q_{2\rho}T^{\mu\rho}=0,\tag{5}$$

the electroproduction cross section is not changed if we add terms containing $q_{1\mu}$, $q_{1\nu}$ ($q_{2\rho}$, $q_{2\sigma}$) to the current tensor $J_{\mu\nu}(p_1,p_3)$ ($J_{\rho\sigma}(p_2,p_4)$). Utilizing this fact, let us represent the current tensor $J_{\mu\nu}(p_1,p_3)$ in the form

$$J_{\mu\nu}(p_1,p_3)=-{}^1\!/_2\Delta_1^2g_{\mu\nu}+2R_{1\mu}R_{1\nu},\tag{6}$$

where $R_1=p_1-\epsilon q_1/\omega_1$ has the following components:

$$R_{10}=0,\quad \mathbf{R}_{1\perp}=-\frac{\varepsilon}{\omega_1}\mathbf{q}_{1\perp},\quad R_{1\parallel}=-\frac{m^2}{p}+\frac{\varepsilon q_1^2}{2p\omega_1}\tag{7}$$

(here $p^2=\epsilon^2-m^2$). The relations (7) are exact.

From formulas (4) and (7) it follows that one can neglect the longitudinal components in the second term of expression (6) for the current tensor, and in the first term the zero component and the longitudinal component mutually cancel upon contraction with $T^{\mu\rho}T^{\nu\sigma*}$ according to relation (5); hence only the perpendicular components contribute to the cross section. Taking what has been said into consideration, one can represent the current tensor in the form

$$J_{ij}(p_1,p_3)={}^1\!/_2\Delta_1^2\delta_{ij}+2\frac{\varepsilon^2}{\omega_1^2}q_{1i}q_{1j},\qquad i,j=1,2(x,y).\tag{8}$$

The remaining components of the current tensor are equal to zero. Similarly

$$J_{kl}(p_2,p_4)=\frac{\Delta_2^2}{2}\delta_{kl}+2\frac{\varepsilon^2}{\omega_2^2}q_{2k}q_{2l}.\tag{9}$$

Thus the problem turns out to be essentially two-dimensional.[3)]

Substituting expressions (8) and (9) into expression (1) for the cross section, we obtain the differential cross section for electroproduction in the region of small values of $\Delta_{1,2}^2$. However, if the final electrons are not detected, then it is necessary to integrate over their states. In this connection we shall everywhere consider the region where $|\Delta_{\perp}|\ll\mu$; then values $q_{1,2\perp}^2\ll\mu^2$ give the major contribution; under this condition one can neglect the dependence of $T^{\mu\rho}$ on $q_{1,2\perp}$ and then only the electron tensor is integrated.

Let us represent

$$d^3p_3d^3p_4=d\mathbf{q}_{1\perp}d\mathbf{q}_{2\perp}dq_{1\parallel}dq_{2\parallel}.\tag{10}$$

After integration with the δ-function dependence (4) on the longitudinal and zero components taken into account, we obtain

$$\omega_1={}^1\!/_2(\Delta_0+\Delta_{\parallel}),\qquad \omega_2={}^1\!/_2(\Delta_0-\Delta_{\parallel}).\tag{11}$$

After this we only have to evaluate the integral

$$F_{ijkl}=\frac{\omega_1^2\omega_2^2}{\pi\varepsilon^4}\int\delta(\mathbf{q}_{1\perp}+\mathbf{q}_{2\perp}-\Delta_{\perp})\frac{J_{ij}(p_1,p_3)J_{kl}(p_2,p_4)}{(\mathbf{q}_{1\perp}^2+m^2\zeta_1^2)^2(\mathbf{q}_{2\perp}^2+m^2\zeta_2^2)^2}d\mathbf{q}_{1\perp}d\mathbf{q}_{2\perp},\tag{12}$$

where $\zeta_{1,2}=\omega_{1,2}/\epsilon$.

The tensor F_{ijkl} only depends on the vector $\Delta_{\perp}$ and is symmetric with respect to the indices i, j, and k, l; therefore it can be expanded in terms of five independent tensors:

$$F_{ijkl}=\frac{1}{4}\sum_{n=1}^{5}g^{(n)}E_{ijkl}^{(n)},\tag{13}$$

$$E_{ijkl}^{(1)}=a_{ij}\tilde{a}_{kl}=\delta_{ij}\delta_{kl},$$
$$E_{ijkl}^{(2)}=2^{1/2}(e_{1i}^{-}e_{1j}^{+*}e_{2k}^{-}e_{2l}^{+*}+e_{1i}^{+}e_{1j}^{-*}e_{2k}^{+}e_{2l}^{-*})=2^{-1/2}(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}-\delta_{ij}\delta_{kl}),\tag{14}$$
$$E_{ijkl}^{(3)}=-a_{ij}\tilde{b}_{kl}=\delta_{ij}(2n_kn_l-\delta_{kl}),$$
$$E_{ijkl}^{(4)}=-b_{ij}\tilde{a}_{kl}=\delta_{kl}(2n_in_j-\delta_{ij}),$$
$$E_{ijkl}^{(5)}=2^{1/2}b_{ij}\tilde{b}_{kl}-E_{ijkl}^{(2)}=2^{1/2}[4n_in_jn_kn_l-2\delta_{ij}n_kn_l-2\delta_{kl}n_in_j-{}^1\!/_2(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})+{}^3\!/_2\delta_{ij}\delta_{kl}].$$

[2)]Integration over the final states of the electrons was also performed in a number of articles (see, for example, [6,7,12]); there, however in contrast to the work of Cheng and Wu [9] and the present work, the method of equivalent photons was used.

[3)]This fact was indicated by us in [13]; in the approximation assumed there, it was possible to omit the first term in expressions (8) and (9) for the currents.

Here
$$a_{ij}=e_{1i}^{+}e_{1j}^{+*}+e_{1i}^{-}e_{1j}^{-*},\quad b_{ij}=e_{1i}^{+}e_{1j}^{-*}e^{-2i\theta}+e_{1i}^{-}e_{1j}^{+*}e^{2i\theta},$$
$$\tilde{a}_{ij}=a_{ij}(1\to 2),\quad \tilde{b}_{ij}=b_{ij}(1\to 2,\ (\theta)\leftrightarrow(-\theta)),$$
$$\mathbf{n}=\Delta_{\perp}/|\Delta_{\perp}|,\quad e_1^{\pm}=\mp 2^{-1/2}i(\mathbf{e}_x\pm i\mathbf{e}_y),\tag{15}$$
$$\mathbf{e}_2^{\pm}=\mp 2^{-1/2}i(\mathbf{e}_x\mp i\mathbf{e}_y)$$
are helicity unit vectors, θ is the angle between $\mathbf{n}$ and the unit vector $\mathbf{e}_x$, and $\mathbf{n}=\mathbf{e}_x\cos\theta+\mathbf{e}_y\sin\theta$. The set of tensors $(1/2)E^{(n)}_{ijkl}$, in terms of which the tensor F_{ijkl} is being expanded, is orthonormal. Writing down $E^{(n)}_{ijkl}$ in terms of combinations of helicity unit vectors is convenient for an analysis of the final states in terms of helicity amplitudes.[4)]

Replacing the F_{ijkl} in expression (12) by the $E^{(n)}_{ijkl}$ given by (14), we obtain the coefficients $g^{(n)}$. The explicit form of the $g^{(n)}$ is given in the Appendix.

3. With (12) taken into account, one can write the cross section (1) for electroproduction, integrated over the final states of the electrons, in the form
$$d\sigma=\frac{\pi\alpha^2}{4\omega_1^2\omega_2^2}(1-\zeta_1)(1-\zeta_2)F^{ijkl}T_{ik}T_{jl}^{*}\prod_{i=1}^{N}\left(\frac{d^3k_i}{2\varepsilon_{k_i}}\frac{1}{(2\pi)^3}\right).\tag{16}$$
Taking (13) and (14) into consideration, one can rewrite this electroproduction cross section in terms of the helicity amplitudes:
$$d\sigma=\frac{\pi\alpha^2}{16\omega_1^2\omega_2^2}(1-\zeta_1)(1-\zeta_2)\{g^{(1)}[|T^{++}|^2+|T^{+-}|^2+|T^{-+}|^2+|T^{--}|^2]$$
$$+2^{3/2}g^{(2)}\operatorname{Re}(T^{--}T^{++*})-2g^{(3)}\operatorname{Re}[e^{2i\theta}(T^{++}T^{+-*}+T^{-+}T^{--*})]$$
$$-2g^{(4)}\operatorname{Re}[e^{2i\theta}(T^{-+}T^{++*}+T^{--}T^{+-*})]+$$
$$+2^{3/2}g^{(5)}\operatorname{Re}[e^{4i\theta}(T^{-+}T^{+-*})]\}\prod_{i=1}^{N}\left(\frac{d^3k_i}{2\varepsilon_{k_i}}\frac{1}{(2\pi)^3}\right),\tag{17}$$
where $T^{+-}=e^{+}_{1i}e^{-}_{2k}T^{ik}$, etc. This result is quite general and may be used to analyze any electroproduction processes.

Now let us consider several specific examples of the application of formula (17). In the case of the production of two spinless particles with momenta k_1 and k_2 (see, for example,[9]), we have
$$T_{\mu\rho}=\left(g_{\mu\rho}-\frac{q_{2\mu}q_{1\rho}}{(q_1q_2)}\right)A_1(q_1q_2,kq_1)$$
$$+\left(k_\mu-\frac{q_{2\mu}(kq_1)}{(q_1q_2)}\right)\left(k_\rho-\frac{q_{1\rho}(kq_2)}{(q_1q_2)}\right)A_2(q_1q_2,kq_1),\tag{18}$$
where $k=k_1-k_2$. In this case the helicity amplitudes (the x axis is directed along $\mathbf{k}_1$; then θ denotes the angle between $\mathbf{k}_\perp$ and $\mathbf{n}$) are given by
$$T^{++}=T^{--}=A_1-{}^1/_2A_2\mathbf{k}_\perp^2,\quad T^{+-}=T^{-+}={}^1/_2A_2\mathbf{k}_\perp^2.\tag{19}$$

Substituting (19) into (17), we arrive at the result obtained by Cheng and Wu.[9] In the general case of photoproduction it is convenient to parametrize the photoproduction amplitude in the following form (see, for example,[14]):
$$T_{\mu\rho}=G_0(e_\mu^{(1)}e_\rho^{(1)}+e_\mu^{(2)}e_\rho^{(2)})+G_1(e_\mu^{(1)}e_\rho^{(2)}+e_\mu^{(2)}e_\rho^{(1)})$$
$$+G_2(e_\mu^{(2)}e_\rho^{(1)}-e_\mu^{(1)}e_\rho^{(2)})+G_3(e_\mu^{(1)}e_\rho^{(1)}-e_\mu^{(2)}e_\rho^{(2)}),\tag{20}$$
where
$$e_\mu^{(1)}=P_\mu/(-P^2)^{1/2},\qquad e_\mu^{(2)}=N_\mu/(-N^2)^{1/2}.$$
Here
$$P^\mu=k^\mu-\frac{q_1^\mu(q_2k)+q_2^\mu(q_1k)}{q_1q_2},\qquad k=k_1-k_2,\qquad N^\mu=\varepsilon^{\mu\lambda\sigma\rho}P_\lambda q_{1\sigma}q_{2\rho},$$
the vectors $e^{(1)}$ and $e^{(2)}$ have only transverse components, $e^{(1)}$ is directed along the x axis, and $e^{(2)}$ is along the y axis.

In the case of the photoproduction of a pair of spin-1/2 particles, the functions G_m have the following form (see, for example,[14]):
$$G_m=\bar{u}(k_2)O_mv(k_1),$$
$$O_0=f_1+f_2\hat{Q},\quad O_1=f_3\gamma_5\hat{Q},\quad O_2=f_4\gamma_5,\tag{21}$$
$$O_3=f_5+f_6\hat{Q};$$
here $Q=q_1-q_2$. Thus in this case the photoproduction amplitude can be expressed in terms of six invariant amplitudes $f_m=f_m(q_1q_2,kq_1)$ $(m=1\ldots6)$. Using the parametrization described by (20), the helicity amplitudes are represented in the form
$$T^{++}=-G_0-iG_2,\quad T^{--}=-G_0+iG_2,$$
$$T^{-+}=-iG_1+G_3,\quad T^{+-}=iG_1+G_3.\tag{22}$$

Since the polarization of the created particles is not measured, we carry out a summation over their spins. Then the electroproduction cross section can be expressed in terms of the amplitudes G_m in the following way:
$$d\sigma=\frac{\pi\alpha^2(1-\zeta_1)(1-\zeta_2)}{(2\pi)^6 16\omega_1^2\omega_2^2}\sum_{n=1}^{5}(g^{(n)}B^{(n)})\frac{d^3k_1}{2\varepsilon_{k_1}}\frac{d^3k_2}{2\varepsilon_{k_2}},\tag{23}$$
$$B^{(1)}=2\langle|G_0|^2+|G_1|^2+|G_2|^2+|G_3|^2\rangle,$$
$$B^{(2)}=2^{3/2}\langle|G_0|^2-|G_2|^2\rangle,$$
$$B^{(3)}=4\langle\cos 2\theta\operatorname{Re}(G_0G_3^*+G_1G_2^*)+\sin 2\theta\operatorname{Re}(G_0G_1^*-G_2G_3^*)\rangle,$$
$$B^{(4)}=4\langle\cos 2\theta\operatorname{Re}(G_0G_3^*-G_1G_2^*)+\sin 2\theta\operatorname{Re}(G_0G_1^*+G_2G_3^*)\rangle,$$
$$B^{(5)}=2^{3/2}\langle\cos 4\theta(|G_3|^2-|G_1|^2]+2\sin 4\theta\operatorname{Re}(G_1G_3^*)\rangle,\tag{24}$$
where $\langle\ldots\rangle$ denotes summation over the polarizations.[5)]

In the case of the production of a pair of spin-1/2 particles (the spinors are normalized according to $\bar{u}u=2\mu$) we obtain
$$\langle|G_0|^2\rangle=4\{|f_1|^2(q_1q_2-2\mu^2)+2|f_2|^2\varkappa+4\operatorname{Re}(f_1f_2^*)\mu(q_1k)\},$$
$$\langle|G_1|^2\rangle=4|f_3|^2\{2(q_1q_2)(q_1q_2-2\mu^2)-2(q_1k)^2\},$$
$$\langle|G_2|^2\rangle=4|f_4|^2(q_1q_2),\quad \langle|G_3|^2\rangle=\langle|G_0|^2\rangle(f_{1,2}\to f_{5,6}),$$
$$\langle G_0G_3^*\rangle=4\{(f_1f_5^*)(q_1q_2-2\mu^2)+2(f_2f_6^*)\varkappa$$
$$+2(f_1f_6^*+f_2f_5^*)\mu(q_1k)\},\tag{25}$$
$$\langle G_1G_2^*\rangle=\langle G_0G_1^*\rangle=\langle G_2G_3^*\rangle=\langle G_1G_3^*\rangle=0,$$
where μ denotes the particle's mass, and $\kappa=(q_1q_2)^2-(q_1k)^2$. In the case of muon production (the muons are point particles), the invariant amplitudes have the form
$$f_1=f_5=2e^2\mu(q_1q_2)/\varkappa,\quad f_2=0,\quad f_6=-e^2(q_1k)/\varkappa,$$
$$f_3=-ie^2(q_1q_2)/\varkappa,\quad f_4=2ie^2\mu(q_1q_2)/\varkappa,\tag{26}$$
and the functions $B^{(n)}$ are given by
$$B^{(1)}=\frac{16e^4}{\varkappa}\left[(q_1q_2)^2+(q_1k)^2+4\mu^2(q_1q_2)-\frac{8\mu^4(q_1q_2)^2}{\varkappa}\right],$$
$$B^{(2)}=-\frac{2^{1/2}\cdot 64e^4\mu^4(q_1q_2)^2}{\varkappa^2},$$
$$B^{(4)}=B^{(3)}=\frac{64e^4\cos 2\theta\mu^2(q_1q_2)}{\varkappa}\left[1-\frac{2\mu^2(q_1q_2)}{\varkappa}\right],$$
$$B^{(5)}=2^{1/2}\cdot 16e^4\cos 4\theta\left[-1+\frac{4\mu^2(q_1q_2)}{\varkappa}-\frac{4\mu^4(q_1q_2)^2}{\varkappa^2}\right].\tag{27}$$

Let us also consider the case of electroproduction of $\rho+\pi$ $(k_1^2=m_\pi^2,\ k_2^2=m_\rho^2)$, which can be regarded as

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4) It follows from expression (13) that "quasi-real" photons are, in general, linearly polarized.

5) Formulas (23) and (24) can be used to describe the electroproduction of any arbitrary final state. In the case of the production of two particles, it follows from P-invariance that $\langle G_0G_1^*\rangle=\langle G_2G_3^*\rangle=\langle G_1G_3^*\rangle=0$.

a model for the production of 3π. We shall again use the parametrization (20) of the photoproduction amplitudes, but now the G_m will have the form

$$G_0 = F_1 \varepsilon_{\mu\nu\rho\sigma} e^\mu q_1^\nu q_2^\rho k^\sigma, \quad G_1 = F_2(eq_1) + F_3(eq_2), \quad G_2 = F_4(eq_1) + F_5(eq_2), \quad G_3 = F_6 \varepsilon_{\mu\nu\rho\sigma} e^\mu q_1^\nu q_2^\rho k^\sigma. \tag{28}$$

where e^μ denotes the polarization vector of the vector meson. Thus, in this case the photoproduction amplitude can also be expressed in terms of six invariant amplitudes. After summing over the spins of the vector meson, the electroproduction cross section is again given by formulas (23) and (24), only now we find

$$\begin{aligned}
\langle G_0 G_3^* \rangle &= F_1 F_6^* (q_1 q_2)[2(q_1 k)(q_2 k) - (q_1 q_2) k^2], \\
\langle G_1 G_2^* \rangle &= F_2 F_4^* (q_1 k_2)^2 / m_\rho^2 + (F_2 F_5^* + F_3 F_4^*) \cdot \\
&\cdot [-(q_1 q_2) + (q_1 k_2)(q_2 k_2)/m_\rho^2] + F_3 F_5^* (q_2 k_2)^2 / m_\rho^2, \\
\langle |G_0|^2 \rangle &= \langle G_0 G_3^* \rangle (F_6 \to F_1), \quad \langle |G_3|^2 \rangle = \langle G_0 G_3^* \rangle (F_1 \to F_6), \\
\langle |G_1|^2 \rangle &= \langle G_1 G_2^* \rangle (F_{4,5} \to F_{2,3}), \quad \langle |G_2|^2 \rangle = \langle G_1 G_2^* \rangle (F_{2,3} \to F_{4,5}), \\
\langle G_0 G_1^* \rangle &= \langle G_2 G_3^* \rangle = \langle G_1 G_3^* \rangle = 0.
\end{aligned} \tag{29}$$

Here $\langle \ldots \rangle$ denotes, just as previously, summation over polarizations.

4. Let us proceed to a discussion of the obtained results. The coefficient associated with $g^{(1)}$ in the cross sections (17) and (23) is proportional to the photon-photon cross section,[6)] which can thereby be investigated in the electroproduction process. However, the cross sections (17) and (23) contain four more combinations of the amplitudes which do not reduce to the photon-photon cross section, and this enables us to obtain additional important information about the photoproduction process by studying electroproduction.[7)]

In the case when averaging over the directions of the vector $\Delta_\perp$ is not carried out, one can in principle determine the four combinations of the invariant amplitudes (see Eqs. (23)–(25)) for the production of a pair of spin-½ particles if the polarization of the produced particles is not measured, and correspondingly five combinations of the invariant amplitudes (see Eqs. (23), (24), and (29)) for the case of production of the $(\rho + \pi)$ system. In order to establish the feasibility of measuring the above indicated combinations of the invariant amplitudes, it is necessary to estimate the magnitude of the functions $g^{(n)}$ in the region $\Delta_\perp^2 \gg m^2\zeta$ (see the formulas given in the Appendix)

$$g^{(1,3,4)} \sim \frac{1}{\Delta_\perp^2} \ln \frac{\Delta_\perp^2}{m^2 \zeta_1 \zeta_2}, \qquad g^{(2)} \sim \frac{m^2(\zeta_1^2 + \zeta_2^2)}{\Delta_\perp^2}, \qquad g^{(5)} \sim \frac{1}{\Delta_\perp^2}. \tag{30}$$

Thus, under the given kinematic conditions $|\Delta_\perp| \ll \mu$ the coefficients $g^{(3)}$ and $g^{(4)}$ are of the same order of magnitude as $g^{(1)}$ (the coefficient associated with the photon-photon cross section), the contribution from the combination associated with $g^{(5)}$ does not contain a "large" logarithm, and the contribution of the combination associated with $g^{(2)}$ is important only in the very narrow region in which $m^2\zeta_{1,2}^2/\Delta_\perp^2 \lesssim 1$.

However, if averaging is carried out over the directions of $\Delta_\perp$, then, as is clear from expression (17) for the cross section, one can in principle determine only two combinations of the invariant amplitudes (the combinations associated with $g^{(1)}$ and $g^{(2)}$), where the photon-photon cross section is associated with $g^{(1)}$, but it is only necessary to take account of the function $g^{(2)}$ itself in the region $m^2\zeta_{1,2}^2/\Delta_\perp^2 \lesssim 1$, which does not give a logarithmically large contribution to the electroproduction cross section. Confining our attention to only taking account of the terms associated with $g^{(1)}$, we arrive at an electroproduction cross section which can be expressed in terms of the photon-photon cross section. Precisely such a situation occurs in the approximation of equivalent photons. However, if we also integrate over $|\Delta_\perp|$ (up to $|\Delta_\perp| \sim \mu$) and keep the leading logarithmic terms, then we arrive at the standard expression for the electroproduction cross section in the approximation of equivalent photons. Only in this situation does it make sense to compare the obtained result with the approximation of equivalent photons, in which an integration over $\Delta_\perp$ is carried out in the initial formulas.

From what has been said above, it follows that it is only in specially selected circumstances that an analysis of the electroproduction cross section (17) enables us to determine the combinations of the photoproduction amplitudes which are contained in it. Namely, this can be done when the angular and energy characteristics of the particles created in each event are determined with a high degree of accuracy, and when one is able to reconstruct $\Delta_\perp$ from this data ($\Delta_\perp$ is essentially the difference between large numbers). In this connection the electroproduction cross section is large, and the establishment of the appropriate conditions is an important experimental problem. At the same time we wish to emphasize that in the case when such conditions are not realized, for example, when the angles of flight of the created particles are measured but their energies are not determined (as was done in the experiment described in article[8]), then an effective averaging with respect to the vector $\Delta_\perp$ takes place, and then the leading logarithmic term in the electroproduction cross section can be obtained by using the standard approximation of equivalent photons.

APPENDIX

We present the explicit form of the coefficients $g^{(n)}$ appearing in the expansion (13):

$$\begin{aligned}
g^{(1)} = \frac{4}{m^2Q^2} &\{2L[\tau^2\lambda^6 - 2\zeta_1^2\zeta_2^2(\tau^2(\zeta_1^2+\zeta_2^2) + (\zeta_1^2-\zeta_2^2)^2)] - 2\tau^6 \\
&- 4\tau^2\zeta_1^2\zeta_2^2 + (\zeta_1^2+\zeta_1^2)(-3\tau^4 + (\zeta_1^2-\zeta_2^2)^2)\} + \frac{2}{m^2}\beta_1\beta_2 L \\
&+ \left\{\frac{2\beta_2}{m^2Q}[2L((\tau^2+\zeta_2^2)^2 + \zeta_1^2(\tau^2-\zeta_2^2)) - \tau^2 - \zeta_2^2 + \zeta_1^2] + (1 \leftrightarrow 2)\right\}.
\end{aligned}$$

$$g^{(2)} = \frac{2^{3/2}}{m^2Q^2}\{4\zeta_1^2\zeta_2^2[3\tau^2\lambda^2 - Q]L + (\zeta_1^2+\zeta_2^2)Q - 12\tau^2\zeta_1^2\zeta_2^2\},$$

$$\begin{aligned}
g^{(3)} = \frac{4}{m^2Q^2} &\left\{\left[(\zeta_1^2-\zeta_2^2)\left(\frac{Q^2}{\tau^2} + Q\lambda^2\right) + \tau^2\lambda^2(\lambda^4 + 8\zeta_1^2\zeta_2^2)\right]L\right. \\
&\left. - 2\tau^2(\lambda^4 + 2\zeta_1^2\zeta_2^2) - Q(\zeta_1^2-\zeta_2^2)\right\} - \frac{2}{m^2\tau^2}\ln\frac{\zeta_1^2}{\zeta_2^2} + \frac{2\beta_1}{m^2}\left\{\frac{1}{2\tau^2}\ln\frac{\zeta_2^2}{\zeta_1^2}\right. \\
&\left. - \frac{(\tau^2-\zeta_2^2+\zeta_1^2)}{Q} + \left[1 + \frac{\zeta_1^2-\zeta_2^2}{\tau^2} - \frac{2\zeta_2^2}{Q}(\tau^2+\zeta_2^2-\zeta_1^2)\right]L\right\},
\end{aligned}$$

$$g^{(4)} = g^{(3)} \quad (1 \leftrightarrow 2),$$

$$g^{(5)} = \frac{2^{3/2}}{m^2Q^2}\left\{\left[4\zeta_1^2\zeta_2^2(6Q - 3\tau^2(\zeta_1^2+\zeta_2^2) - 3(\zeta_1^2-\zeta_2^2)^2)\right.\right.$$

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6) Here and below we understand the photon-photon cross section to mean the cross section for the reaction $\gamma + \gamma \to N$ with unpolarized photons.

7) In the case of the electroproduction of two spinless particles, this property has already been pointed out by Cheng and Wu. [9]

$$-\frac{6Q^2}{\tau^4}(\tau^2(\zeta_1^2+\zeta_2^2)+(\zeta_1^2-\zeta_2^2)^2)\Big]L-3(\zeta_1^2+\zeta_2^2)Q-$$
$$-4\tau^2(\lambda^4-\zeta_1^2\zeta_2^2)\Big\}+\frac{2^{1/2}6}{m^2\tau^2}\left(\frac{\zeta_1^2-\zeta_2^2}{\tau^2}\ln\frac{\zeta_1^2}{\zeta_2^2}+2\right). \qquad \text{(A.1)}$$

Here

$$L=\frac{1}{Q^{1/2}}\ln\left(\frac{\lambda^2+Q^{1/2}}{2\zeta_1\zeta_2}\right), \qquad \beta_{1,2}=\frac{\zeta_{1,2}}{1-\zeta_{1,2}},$$
$$Q=\lambda^4-4\zeta_1^2\zeta_2^2, \qquad \lambda^2=\tau^2+\zeta_1^2+\zeta_2^2, \qquad \tau^2=\Delta_\perp^2/m^2.$$

We note that the coefficients $g^{(1)}$, $g^{(2)}$, $g^{(5)}$, and $g^{(3)} + g^{(4)}$ can be obtained from the results of the work by Cheng and Wu;[9] in order to do this it is necessary to compare expressions (17) and (19) with the corresponding formulas from[9].

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