BREMSSTRAHLUNG SPECTRUM OF ULTRARELATIVISTIC ELECTRONS IN A DENSE MEDIUM

A. A. VARFOLOMEEV, V. A. BAZYLEV and N. K. ZHEVAGO

I. V. Kurchatov Atomic Energy Institute

Submitted April 7, 1972

Zh. Eksp. Teor. Fiz. **63**, 820–830 (September, 1972)

The bremsstrahlung of high energy electrons in a dense absorbing medium is investigated. It is shown that the change of the electron multiple-scattering constant as the result of energy loss leads to further suppression of radiation relative to the ordinary multiple-scattering effect. The question is considered of separating the total energy loss of an ultrarelativistic electron in absorbing media into bremsstrahlung and direct electron-positron pair production. Analytic expressions are obtained for the bremsstrahlung spectrum with inclusion of virtual-photon absorption, polarization of the medium, and multiple scattering with a varying constant.

INTRODUCTION

It is well known that the bremsstrahlung of a high energy electron in a dense medium has a collective nature, as a result of which the bremsstrahlung spectrum differs substantially from the usual Bethe-Heitler spectrum which corresponds to interaction with isolated atoms (a rarefied medium). The effect of the medium on bremsstrahlung appears in the form of the multiple-scattering effect (see Landau and Pomeranchuk[1]), the effect of the polarization of the medium (Ter-Mikaelyan[2]), and the virtual-photon absorption effect[1]. The shape of the electron-bremsstrahlung spectrum with inclusion of multiple scattering and polarization of the medium has been most rigorously obtained by Migdal[3,4]. The papers of Galitskiĭ and Gurevich[5] and Galitskiĭ and Yakimets[6] have been devoted to taking into account virtual-photon absorption in electron bremsstrahlung. A detailed review of the problem being discussed is contained in the monograph by Ter-Mikaelyan[7] (see also ref. 8).

In all of these studies mentioned the energy of the radiating electron and consequently also the multiple-scattering constant have been assumed constant. From the physical point of view this assumption is not always justified. The necessity of taking into account the energy loss of the electrons in path lengths which are substantial for radiation has been demonstrated in our earlier paper[9], the main result of which can be explained by using the classical description of the field and introducing the concept of a coherence radiation length[8], as was done by Galitskiĭ and Gurevich[5].

The coherence length l in a medium with inclusion of multiple scattering of the electron is found from the condition $2\omega l(1 - v\overline{\cos\theta_S}) = 1$, where v is the electron velocity, $\overline{\cos\theta_S}$ is the average cosine for the multiple scattering angle in a length l, ω is the frequency of the radiation ($\hbar = m = c = 1$). Expanding $\overline{\cos\theta_S}$ in a series and using for $\overline{\theta_S^2}$ an expression taking into account the change in electron energy in a length l, we obtain for the quantity $\lambda = l/L$ the following equation:

$$\lambda(e^{2\lambda} - 1) = 2\lambda_s^2, \tag{1}$$

where L is the radiation length; $\lambda_S = E/E_S\sqrt{L\omega}$ is the ratio of the coherence length with inclusion of scattering to L; E is the electron energy; $E_S = 21$ MeV.

For a small energy loss by the electron ($\lambda \ll 1$) we obtain

$$\lambda = \lambda_s(1 - \lambda_s/2), \qquad I = \frac{e^2}{3\pi}\frac{E_s}{E}\left(\frac{\omega}{L}\right)^{1/2}\left(1 - \frac{\lambda_s}{2}\right). \tag{2}$$

The expression for the bremsstrahlung intensity in a medium (2) contains in this case an additional factor $(1 - \lambda_S/2)$ in comparison with the Landau-Pomeranchuk formula[1].

In the case of large energy loss ($\lambda \gg 1$) a qualitative result can be obtained by using the approximate solution of Eq. (1): $\lambda = \ln\lambda_S[1 + O(1/\ln\lambda_S)]$. Thus, in the limiting case of a substantial change of the multiple-scattering constant the bremsstrahlung spectrum should change substantially in comparison with the spectrum taking into account the ordinary multiple-scattering effects[1,3].

If the electron energy loss in a coherence length cannot be neglected, as the result of variation of the scattering constant, then in this case, generally speaking, it is necessary to take into account also the effect of virtual-photon absorption. This statement is obvious for large λ_S. In particular, for the frequencies considered the coherence length cannot become greater than the radiation length, as the result of photon absorption, while inclusion of multiple scattering leads to a coherence length value $l = L\ln\lambda_S$ which increases with increasing electron energy. It can be shown[9] that inclusion of virtual-photon absorption is necessary also in the case of relatively low energy loss by the electron in the coherent length.

The effects of virtual-photon absorption, multiple scattering with a varying constant, and polarization of the medium can appear simultaneously. Therefore in determining the bremsstrahlung spectrum over a wide range of frequencies it is necessary to take into account all of these effects. In the present work we have used a classical approach to calculate the energy loss of ultrarelativistic electrons at a frequency ω as the result of bremsstrahlung and electron-positron pair production with inclusion of all the dense-medium effects mentioned.

FORMULATION OF THE PROBLEM AND METHOD OF CALCULATION

Let us consider an ultrarelativistic electron with energy E_0 which at the moment $t = 0$ is normally incident on a layer of condensed matter of thickness $T \gg L$. We will calculate the electron energy loss in passage through the layer, using the phenomenological approach and introducing the dielectric permittivity of the medium $\epsilon(\kappa)$ (compare refs. 3 and 6) ($\kappa = (\mathbf{k}, \omega)$). We will write the expression for the absorbed electromagnetic energy of waves with frequency $\omega \ll E_0$, produced as the result of the field in the layer of matter, in the form

$$E_\omega = \omega \int \varepsilon''(\kappa) \, |\vec{\mathscr{E}}_\kappa|^2 \, d\mathbf{k}. \tag{3}$$

The dielectric permittivity $\epsilon(\kappa) = \epsilon'(\kappa) + i\epsilon''(\kappa)$ is defined by the relation

$$\mathbf{D} = \int \varepsilon(\kappa) \, \vec{\mathscr{E}}_\kappa e^{-i\kappa x} d^4x, \tag{A}$$

where $\vec{\mathscr{E}}$ is the solution of Maxwell's equation for an electron moving in a medium along a trajectory $\mathbf{r}(t)$:

$$\vec{\mathscr{E}}_\kappa = \frac{i}{4\pi^3} \frac{e}{\mathbf{k}^2 - \varepsilon\omega^2} \int_0^T \left[\omega \mathbf{v} - \frac{\mathbf{k}(\mathbf{k}, \mathbf{v})}{\varepsilon\omega}\right] e^{i[\omega t - \mathbf{k}\mathbf{r}(t)]} dt. \tag{4}$$

For sufficiently high frequencies $\omega \gg 1$ the imaginary part of the dielectric permittivity is determined mainly by pair production: $\epsilon'' = n\sigma/\omega = 1/L_c\omega$, where σ is the cross section for production of electron-positron pairs and n is the nuclear density of the medium. For the frequencies considered ϵ' is close to unity: $\epsilon' = 1 - \omega_0^2/\omega^2$, where $\omega_0^2 = 4\pi N e^2$, and N is the electron density of the medium.

Substituting expression (4) into (3) and averaging E_ω over all possible electron trajectories in the medium, as has been done in ref. 6, we obtain

$$E_\omega = \frac{e^2\omega^3}{\pi^3} \operatorname{Re} \int \int_0^T \int_0^{T-t} \frac{\varepsilon''(\kappa) \xi \eta W^{(-k)}(\eta - \xi, t, \tau)}{(\mathbf{k}^2 - \varepsilon'\omega^2)^2 + \omega^2 L_c^{-2}} e^{-i\omega\tau} d\tau \, dt \, d\xi \, d\eta \, d\mathbf{k}; \tag{5}$$

$$\xi = \mathbf{v}_1 / v_1 - \mathbf{n}, \quad \eta = \mathbf{v}_2 / v_2 - \mathbf{n}, \quad \mathbf{n} = \mathbf{k} / k, \quad k = |\mathbf{k}|,$$

$\mathbf{v}_1$ is the velocity at the moment t; $\mathbf{v}_2$ is the velocity at the moment $t + \tau$. The function $W^{(-k)}$ satisfies the equation

$$\partial W^{(-k)} / \partial\tau - ik(1 - E_0^{-2} e^{2(t+\tau)/L} - \eta^2/2) W^{(-k)} = q_0 e^{2(t+\tau)/L} \Delta_\eta W^{(-k)} \tag{6}$$

with the initial condition

$$W^{(-k)} |_{\tau=0} = \delta(\xi - \eta),$$

where

$$q_0 = 4\pi n E_0^{-2} Z^2 e^4 L_R -$$

is the mean square multiple-scattering angle at the moment $t = 0$. The radiation logarithm L_R in the general case should be determined with inclusion of the effect of the medium on the maximum electron-scattering angle taken into account (see ref. 4). Equation (6) differs from the ordinary Fokker-Planck equation, which was used in ref. 3, in the existence of a dependence of the mean multiple scattering-angle and velocity on the time.

Making the transformation in Eq. (5) from the variable t to $d = t + \tau$, we determine the differential energy loss of the electron at frequency ω in the absorbing medium by the formula

$$I(d) = \frac{e^2\omega^3}{\pi^3} \iint_0^\infty \frac{\varepsilon''(\kappa) \xi \eta W^{(-k)}(\eta - \xi, d - \tau, \tau)}{(\mathbf{k}^2 - \varepsilon'\omega^2)^2 + \omega^2 L_c^{-2}} e^{-i\omega\tau} d\tau \, d\xi \, d\eta \, d\mathbf{k}. \tag{7}$$

In this definition the differential loss corresponds to the energy loss at frequency ω per unit path in matter if this portion of the path is sufficiently remote from the material boundary and its effect can be neglected. In other words, we are considering distances d substantially greater than the coherent radiation length. In this case the quantity I should depend on d only through the electron energy[1)] $E(d) = E_0 e^{-d/L}$.

The change in the magnitude of the electron velocity in the coherence radiation length is unimportant in comparison with the change of the multiple-scattering constant. When we take this into account Eq. (7) can be reduced by elementary transformations[9] to the form

$$I(d) = \frac{4e^2\omega^2}{\pi^2 L_c} \int_0^\infty \frac{F(k) \, dk}{(k^2 - \varepsilon'\omega^2)^2 + \omega^2 L_c^{-2}}, \tag{8}$$

where

$$F(k) = \frac{1}{L\lambda_0} \operatorname{Re} \int_0^\infty \int_0^\infty e^{-2is_0(k)\tau} \chi(z, y, \tau) |_{y=y(\tau)} \, d\tau dz,$$

$$y(\tau) = \frac{2d}{L\lambda_0} - \tau, \qquad s_0(k) = \frac{\omega - kv(d)}{4\sqrt{q_0 k}}, \qquad \lambda_0 = \frac{1}{L\sqrt{q_0 k}}.$$

The function $\chi(z, y, \tau)$ satisfies the equation

$$\frac{1}{2z} \frac{\partial\chi}{\partial\tau} + \frac{i}{8}\chi = e^{\lambda_0(y+\tau)} \frac{\partial^2\chi}{\partial z^2}, \qquad \chi = (z, y, 0) = z/8, \quad \chi(0, y, \tau) = 0 \tag{9}$$

which is an equation of the Fokker-Planck type with variable coefficients.

SOLUTION OF THE FOKKER-PLANCK EQUATION WITH VARYING SCATTERING CONSTANT

We will look for a solution of Eq. (9) in the form $\chi(z, y, \tau) = (z/8)\exp[\varphi_1(x, y) + z\varphi_2(x, y)]$, where $x = 2\exp[\lambda_0(y + \tau)]$. Substituting this expression into (9) and equating the coefficients of identical powers of z, we obtain the system of differential equations

$$\lambda_0 \frac{\partial\varphi_1}{\partial x} = 2\varphi_2, \qquad \lambda_0 \frac{\partial\varphi_2}{\partial x} - \varphi_2^2 + \frac{i}{4x} = 0, \qquad \varphi_1(x_0, y) = \varphi_2(x_0, y) = 0, \ x_0 \equiv 2e^{\lambda_0 y}. \tag{10}$$

We will look for the function $\varphi_2(x, y)$ in the form $\varphi_2 = u^{-1}\partial u/\partial x$, and in this case $\varphi_1 = 2\lambda_0^{-1}(\ln u + \text{const})$. The function

$$\left[-\lambda_0^{-1} \int \varphi_2 \, dx\right]$$

satisfies the equation

$$\frac{\partial^2 u}{\partial x^2} - \frac{i}{4\lambda_0 x} u = 0. \tag{11}$$

A particular solution of this equation is the function $u(x) = x^{1/2} Z_1(\nu)$, where $Z_1(\nu)$ is the cylindrical function of index 1, $\nu \equiv \lambda_0^{-1} i^{3/2} x^{1/2}$. As the general solution of Eq. (11) it is convenient to choose a linear combination of Hankel functions. Then we obtain the following expressions for the functions φ_1 and φ_2:

$$\varphi_1(x, y) = -2\ln\left\{\frac{\pi\nu}{4}[H_0^{(1)}(\rho) H_1^{(2)}(\nu) - H_0^{(2)}(\rho) H_1^{(1)}(\nu)]\right\},$$

$$\varphi_2(x, y) = \frac{i^{3/2} x^{1/2}}{2} \frac{H_0^{(2)}(\rho) H_0^{(1)}(\nu) - H_0^{(1)}(\rho) H_0^{(2)}(\nu)}{H_0^{(2)}(\rho) H_1^{(1)}(\nu) - H_0^{(1)}(\rho) H_1^{(2)}(\nu)},$$

$$\rho \equiv i^{3/2} x_0^{1/2} / \lambda_0.$$

[1)] Another possible definition of the differential energy loss is discussed in detail in ref. 9.

GENERAL EXPRESSION FOR THE DIFFERENTIAL ENERGY LOSS

Using the solution of Eq. (9), the function F(k) (see Eq. (8)) can be represented in the form

$$F(k)=\mathrm{Re}\frac{i}{2\lambda L}\int_0^\infty\int_0^\infty e^{-2is(k)\tau}\frac{d}{d\tau}\exp\left[\frac{i^{3/2}z}{2\sqrt{2}}\frac{\psi_2(\lambda,\tau)}{\psi_1(\lambda,\tau)}\right]d\tau\,dz, \qquad (12)$$

where

$$\psi_1(\lambda,\tau)=H_0^{(1)}(\beta)H_1^{(2)}(\delta)-H_0^{(2)}(\beta)H_1^{(1)}(\delta),$$

$$\psi_2(\lambda,\tau)=H_0^{(2)}(\beta)H_0^{(1)}(\delta)-H_0^{(1)}(\beta)H_0^{(2)}(\delta),$$

$$s(k)=\frac{\omega-kv(d)}{4\sqrt{qk}},\qquad q=q_0e^{2d/L},\qquad \beta=2^{1/2}i^{3/2}\lambda^{-1}e^{-\lambda\tau/2},$$

$$\delta=2^{1/2}i^{3/2}\lambda^{-1},\ \lambda=\lambda_0e^{-d/L}.$$

Integrating by parts in Eq. (12) and then changing the order of integration over the variables τ and z, we obtain

$$F(k)=\mathrm{Re}\frac{2\cdot2^{1/2}s(k)}{\lambda Li^{3/2}}\int_C\frac{\psi_1(\lambda,\tau)}{\psi_2(\lambda,\tau)}e^{-2is(k)\tau}d\tau.$$

The integration is carried out over a contour C consisting of an arc of a small circle of radius $\epsilon\to0$ and a radial line (ϵ,∞); see Fig. 1. Finally we will represent F(k) in a form convenient for the subsequent calculations:

$$F(k)=\frac{1}{\lambda L}\left\{\mathrm{Re}\frac{2\cdot2^{1/2}s(k)}{i^{3/2}}\times\int_0^\infty\left[\frac{\psi_1(\lambda,\tau)}{\psi_2(\lambda,\tau)}-\frac{2^{1/2}}{i^{3/2}\tau}\right]e^{-2is(k)\tau}d\tau-4\pi s(k)\theta[-s(k)]\right\},$$

where $\theta(x)$ is the Heaviside function.

In the integral (8) we will change from integration over the variable k to integration over s. Taking into account that $\omega L_c\gg1$, we obtain

$$I(d)=\frac{e^2}{\pi L_c}\left\{\mathrm{Re}\frac{1}{2^{1/2}\pi i^{3/2}}\int_{-\infty}^{\infty}\int_0^\infty\left[\frac{\psi_1(\lambda,\tau)}{\psi_2(\lambda,\tau)}-\frac{2^{1/2}}{i^{3/2}\tau}\right]\frac{se^{-2is\tau}d\tau\,ds}{(s-s_1)^2+(\lambda_s^{(c)}/8)^2}+\int_0^{s_\eta}\frac{sds}{(s-s_1)^2+(\lambda_s^{(c)}/8)^2}\right\}; \qquad (13)$$

$$\lambda_s\equiv\frac{1}{L\sqrt{q\omega}},\qquad \lambda_s^{(c)}\equiv\frac{1}{L_c\sqrt{q\omega}},\qquad s_1\equiv\frac{\omega[1-v(d)\sqrt{\varepsilon'}]}{8\sqrt{q\omega}}.$$

The choice of the upper limit

$$s_\eta=s_1-\eta^2/8\omega\sqrt{q\omega},$$

where η is a constant equal in order of magnitude to unity, is conditioned by the nature of the function $\sigma(\kappa)$ for large k. This choice, as has already been noted in ref. 6, corresponds to cutoff of the integrals in the pseudophoton method at $k_{\perp m}=\eta$. The quantities λ_s and $\lambda_s^{(c)}$ introduced by us are practically identical, but for comparison of the results with the formulas obtained previously it is desirable to retain the difference in the designations.[2)]

Interchanging the order of integration over the variables s and τ in Eq. (13), we obtain

$$I(d)=\frac{e^2}{\pi L_c}\left\{\mathrm{Re}\frac{4\cdot2^{1/2}}{\lambda_s^{(c)}i^{3/2}}\int_0^\infty\left[\frac{\psi_1(\lambda_s,\tau)}{\psi_2(\lambda_s,\tau)}-\frac{2^{1/2}}{i^{3/2}\tau}\right]\left(s_1-\frac{i\lambda_s^{(c)}}{8}\right)\exp\{-2i(s_1-i\lambda_s^{(c)}/8)\tau\}d\tau+\int_0^{s_\eta}\frac{sds}{(s-s_1)^2+(\lambda_s^{(c)}/8)^2}\right\}. \qquad (14)$$

For direct calculations it turns out to be more convenient to write this equation with expansion of the first term in a series:

$$(d)=\frac{e^2}{\pi L}\mathrm{Re}\left\{-2+\sum_{n=2}^{\infty}\frac{C_n\delta^n}{2^{n-2}}\frac{(n-1)!}{(\gamma+1)\ldots(\gamma+n-1)}+\gamma[\ln\gamma-\psi(\gamma)]\right\}+\frac{e^2}{\pi L_c}\int_0^{s_\eta}\frac{sds}{(s-s_1)^2+(\lambda_s^{(c)}/8)^2},$$

where

$$\gamma=2i\lambda_s^{-1}s_1+\lambda_s^{(c)}(4\lambda_s)^{-1},$$

and the coefficients C_n are determined by the following relations:

$$C_n=A_n-\sum_{m=0}^{n-1}B_{n-m}C_m;$$

$$A_0=1,\ \ A_1=0,\ \ B_0=1,\ \ B_1=\delta^{-1},$$

$$A_n=\frac{2(n-1)}{\delta n}A_{n-1}-\frac{A_{n-2}}{n(n-1)},\qquad B_n=\frac{2n}{\delta(n+1)}B_{n-1}-\frac{B_{n-2}}{n(n+1)};$$

ψ is the logarithmic derivative of the Γ function.

The result presented is the simplest form in which the general expression can be written for the differential energy loss of an electron in a medium as the result of bremsstrahlung and direct electron-positron pair production with inclusion of the polarization of the medium, virtual-photon absorption, and multiple scattering with a varying constant. Further simplifications of Eq. (14) are possible for specific portions of the radiated photon spectrum where one of the effects discussed dominates.

BREMSSTRAHLUNG IN A NONABSORBING MEDIUM

Equation (14) takes into account the effect on the bremsstrahlung of all the effects of the medium which we have discussed. Let us analyze in more detail some limiting cases which follow from the general solution (14) with the purpose of emphasizing the relation with previously obtained results and clarifying the region of appearance of the variable scattering-constant effect and photon-absorption effect. Let us begin with the case of bremsstrahlung in a nonabsorbing medium with inclusion of the multiple-scattering and polarization effects.

If we can neglect absorption of virtual photons and energy loss of the electron in the coherent length $(\lambda_s,\lambda_s^{(c)}\to0)$ then the second term in (14) does not contribute to the differential loss, and in the first term we can set $\psi_1(\lambda_s,\tau)/\psi_2(\lambda_s,\tau)\approx-i\,\mathrm{cth}[(1+i)\tau/2]$. As a result this term can be expressed in terms of the logarithmic derivative of the Γ function $\psi(x)=d\ln\Gamma(x)/dx$. Thus, we arrive at the result obtained by Migdal[3] with the difference that now q and s_1 depend in a trivial way on d in terms of the electron energy:

$$I(d)=\frac{2e^2}{\pi}\sqrt{q\omega}\,\mathrm{Im}\left\{4s_1\left[\psi(\varkappa)+\frac{1}{2\varkappa}-\frac{i\pi}{4}\right]\right\},\qquad \varkappa=(1+i)s_1. \qquad (15)$$

Analysis of Eq. (14) shows that the first term in curly brackets falls off with increasing energy and decreasing frequency, and the second term depends weakly on these parameters and is equal in order of magnitude to $10e^2L_c^{-1}$. Therefore in the region of frequencies and energies where the right-hand part of (15) is still large in comparison with $10e^2L_c^{-1}$, the differential energy loss by bremsstrahlung (15) is identical to the total loss. If this condition is not satisfied, then the direct production of electron-positron pairs plays an important role.

2)The radiation logarithm (see above), by analogy with ref. 4, can be represented in the form $L_R=\ln(183Z^{-1/3}f^{-1/2})$, where $f=s_1\max\{1,(\lambda_s^{(c)})^{-1}\}$.

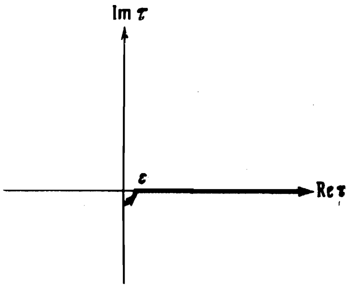

FIG. 1

In this case the concept of energy loss to bremsstrahlung requires some clarification (see below). In principle, a situation is possible in which there is no photon absorption but the effect of change in the scattering constant appears. The corresponding expressions are given in ref. 9.

BREMSSTRAHLUNG WITH INCLUSION OF THE ORDINARY MULTIPLE-SCATTERING EFFECT, POLARIZATION OF THE MEDIUM, AND PHOTON ABSORPTION

As has been noted above, in the region of frequencies considered, the photon-absorption effect and variable scattering-constant effect should appear simultaneously. In spite of this, we will carry out an analysis of the formal case corresponding to appearance of the photon-absorption effect together with the ordinary multiple-scattering effect (without variation of the scattering constant). This permits a comparison to be made with the results of refs. 5 and 6 and a discussion of the problem of separating the total energy loss into bremsstrahlung and pair production.

If we neglect the change in the scattering constant in the coherence length ($\lambda_s \ll 1$), then Eq. (14) takes the form

$$I(d) = I_1(d) + I_2(d),$$

$$I_1(d) = \frac{4e^2}{\pi}\sqrt{q\omega}\left\{\operatorname{Im}\left[\frac{\mu}{1+i}\left(\ln\frac{\mu}{2} - \psi\left(\frac{\mu}{2}\right) - \frac{1}{\mu}\right)\right] + \frac{1}{4\lambda_s^{(c)}}\int_0^{-b}\frac{s\,ds}{(s-s_1)^2 + (\lambda_s^{(c)}/8)^2}\right\},$$

$$I_2(d) = \frac{e^2}{\pi L_c}\int_{-b}^{s_\eta}\frac{s\,ds}{(s-s_1)^2 + (\lambda_s^{(c)}/8)^2}, \quad \mu \equiv 2(1+i)\left(i\frac{\lambda_s^{(c)}}{8} - s_1\right). \quad (16)$$

The last term in the curly brackets of Eq. (14) is represented here in the form of a sum of two integrals in which b is a constant satisfying the condition $0 < b < |s_\eta|$.

In the case of weak absorption of virtual photons and weak polarization for an unchanging scattering constant ($s_1 \ll \lambda_s^{(c)} \ll 1$) the approximate equality

$$\psi(\mu/2) \approx -C - 2/\alpha,$$

is valid, where C is Euler's constant,

$$\alpha \equiv (1-i)\lambda_s^c/4.$$

Here for the intensities I_1 and I_2 we obtain the following expressions:

$$I_1(d) = \frac{2e^2}{\pi}\sqrt{q\omega}\left(1 + \frac{C + \ln(b\sqrt{2})}{2}\,\frac{1}{L_c\sqrt{q\omega}}\right), \quad (17)$$

$$I_2(d) = \frac{e^2}{\pi L_c}\ln\frac{\eta^2}{8b\omega\sqrt{q\omega}}.$$

In the region of energies E and frequencies ω where polarization is absent and the virtual-photon absorption effect is dominant ($\lambda \gg \max[1, s_1]$), for an unchanging scattering constant we have:

$$\psi(\mu/2) \approx \ln(\alpha/2) - 1/\alpha,$$

$$I_1 = \frac{32e^2b^2}{\pi}q\omega L_c, \qquad I_2 = \frac{2e^2}{\pi L_c}\ln\eta\sqrt{\frac{L_c}{\omega}}. \quad (18)$$

The expression for I_1(18) completely coincides with the corresponding result obtained by Galitskiĭ and Yakimets[6] if we take the arbitrary constant b as unity. Thus, the expression (14) obtained by us for the total loss by radiation and pair production contains the results of ref. 6 as a formal limiting case for $\lambda_s \to 0$.

In the absence of scattering the total energy loss of the electron reduces to the energy loss in pair production. On the other hand, it is easy to see that for $q \to 0$ there remain in equations (16)—(18) only the terms I_2. Galitskiĭ and Yakimets[6] interpret the expressions analogous to I_1 and I_2 respectively as loss in bremsstrahlung and loss in pair production, and in the general case $q \neq 0$. With this breakdown the bremsstrahlung spectrum coincides in shape with the spectrum obtained previously in ref. 5 on the assumption that the bremsstrahlung intensity is proportional to the coherent radiation length. The corresponding derivation was carried out for a nonabsorbing medium in which the intensity of radiation is completely determined by the phase relations of the elementary radiating waves. The result was then automatically transferred to the case of a medium with absorption where it, generally speaking, is not valid. Thus, it is impossible to consider as correct the derivation of the bremsstrahlung spectrum[5] in a medium with absorption, whose shape was actually used in ref. 6 without additional justification to separate bremsstrahlung losses from the total energy loss.

If, by analogy with ref. 6, we assign the term I_1 (17) to bremsstrahlung, setting b = 1, we obtain in general a contradictory result for $\lambda_s^{(c)} \ll 1$. The radiation intensity $I_1(d)$ increases with increasing $\lambda_s^{(c)}$ (see Eq. (17)), while an increase of the virtual-photon absorption (decrease of the coherence length) should lead, according to ref. 5, to a decrease in the intensity of bremsstrahlung. Separation of the total energy loss of a relativistic electron into bremsstrahlung and pair production in accordance with Eq. (16) is generally artificial, since there is no experimental criterion for distinguishing these two forms of loss.

Thus, in the general case the separation of the electron energy loss in a medium into radiation of bremsstrahlung photons and virtual photons which produce pairs can be carried out only purely arbitrarily. It is possible, however, to separate more definitely that part of the energy loss in pair production I_0 which is not subjected to the effect of multiple scattering. At small scattering angles ($q \to 0$) this loss will be the only loss (bremsstrahlung will be absent), which permits it to be distinguished from the remaining loss I_s, which it is natural to call bremsstrahlung loss (compare ref. 10). On the other hand, if there is no photon absorption ($\epsilon'' \to 0$), then the loss to pair production I_0 disappears, and here I_s is identical with the ordinary definition of bremsstrahlung in a medium[3]. Separation of the energy loss on the basis of this principle leads to the following results (compare Eqs. (17), (18)):

$$I(d)=I_0(d)+I_s(d),\quad I_0(d)=\frac{e^2}{2\pi L_c}\ln\frac{\eta^4}{(\omega^2E^{-2}+\omega_0^2)^2+\omega^2L_c^{-1}},$$
$$I_s(d)=\frac{2e^2}{\pi}\sqrt{q\omega}\left[1+\frac{1}{2L_c\sqrt{q\omega}}\left(C+\ln\frac{\sqrt{2}}{8L_c\sqrt{q\omega}}\right)\right],\quad \lambda_s^{(c)}\ll 1;$$
$$I_s(d)=\frac{128e^2}{15\pi}q^2\omega^2L_c^3,\quad \lambda_s^{(c)}\gg 1. \tag{19}$$

BREMSSTRAHLUNG WITH INCLUSION OF PHOTON ABSORPTION AND ELECTRON ENERGY LOSS IN THE RADIATION PROCESS

Let us return now to the more correct expression for the differential energy loss, in which in addition to virtual-photon absorption we take into account the change in the scattering constant in the coherence length. Special interest is presented by the case of appearance of multiple-scattering and virtual-photon absorption effects in the region of E and ω where the effect of polarization is practically nonexistent ($\lambda_s \approx \lambda_s^{(c)} \gg 8s_1$). In this case the expression for bremsstrahlung loss $I_s(d)$ can be simplified:

$$I_s(d)=\frac{2e^2}{\pi L_c}\operatorname{Im}\left\{\frac{2^{1/2}}{4i^{3/2}}\int_0^\infty\left[\frac{\psi_1(\lambda_s,\tau)}{\psi_2(\lambda_s,\tau)}-\frac{2^{1/2}}{i^{1/2}\tau}\right]\exp(-\lambda_s^{(c)}\tau/4)\,d\tau\right\}. \tag{20}$$

For small energy loss in a coherence length, and consequently also a weak absorption of virtual photons ($\lambda_s \approx \lambda_s^{(c)} \ll 1$) we have with an accuracy to terms of order λ_s^2

$$\frac{\psi_1(\lambda_s,\tau)}{\psi_2(\lambda_s,\tau)}\approx -i\,\mathrm{cth}\left(\frac{1+i}{2}\tau\right)\left[1-i\lambda_s\,\mathrm{th}\left(\frac{1+i}{2}\tau\right)\Big/2(i-1)\right].$$

Integration in (20) with allowance for the smallness of $\lambda_s^{(c)}$ leads to the following result:

$$I_s(d)=\frac{2e^2}{\pi}\sqrt{q\omega}\left[1+\frac{1}{2L_c\sqrt{q\omega}}\left(C+\ln\frac{\sqrt{2}}{8L_c\sqrt{q\omega}}\right)-\frac{1}{2L\sqrt{q\omega}}\right]. \tag{21}$$

Thus, for $\lambda_s \ll 1$ the absorption of virtual photons and the change in the multiple-scattering constant have approximately an identical degree of influence on bremsstrahlung (on the part I_s of the total energy loss of the electron, which we have separated). We note that if we formally neglect absorption in Eq. (21) ($\lambda_s^{(c)} = 0$), and if we take into account only the effect of change in the constant, then I_s is identical to the result of ref. 9, which differs only in a numerical factor from Eq. (2), obtained from simple physical considerations.

In the inverse limiting case of strong photon absorption and change of scattering constant, it is necessary to set $\lambda_s = \lambda_s^{(s)} \gg 1$. With an accuracy to terms of order λ_s^{-6} we obtain

$$\frac{\psi_1}{\psi_2}=\frac{2^{1/2}\lambda_s}{i^{1/2}\tau'}\left\{1+\frac{i\lambda_s^{-2}}{2}\left[\tau'-2-2\frac{1-e^{-\tau'}}{\tau'}\right]\right. \tag{22}$$
$$\left.-\lambda_s^{-4}\left[\frac{5-\tau'}{8}-\frac{21-16e^{-\tau'}-5e^{-2\tau'}}{16\tau'}+\left(\frac{1-e^{-\tau'}}{\tau'}\right)^2\right]\right\},\quad \tau'\equiv\lambda_s\tau$$

In integration in Eq. (20) only the last term in the curly brackets of Eq. (22) contributes to the integral, and we arrive at the following result:

$$I_s(d)=\frac{ae^2}{\pi}q^2\omega^2L^3,\quad a=\frac{5}{8}+\frac{117}{32}\ln 3-\frac{45}{16}\ln 5\approx 0.12. \tag{23}$$

Thus, in the case discussed ($\lambda_s = \lambda_s^{(c)} \gg 1$) inclu-

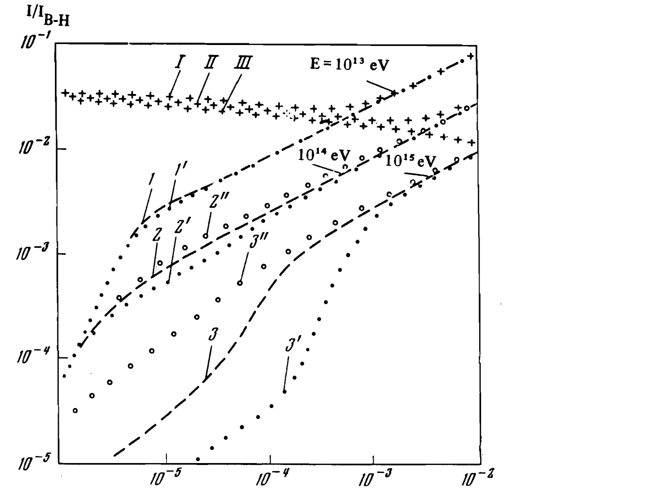

FIG. 2

sion of a change in the multiple-scattering constant does not change the shape of the bremsstrahlung spectrum in comparison with the last formula (19) but leads to a substantially differing numerical factor (~ 0.12 instead of 128/15 ~ 8.6). It is interesting to note also that the frequency dependence of the bremsstrahlung spectrum (23) is the same as in the case of manifestation of the polarization of the medium[2], but the dependence on the energy and charge of the electron and on the density of the medium are completely different.

In the region of frequencies and energies where λ_s is of the order of unity, for determination of I_s it is necessary to use the general expression (14) (without the second term in the curly brackets), and here the effect of change in the scattering constant can substantially affect the shape of the bremsstrahlung spectrum.

For illustration of the results obtained, we have shown in Fig. 2 curves for bremsstrahlung loss in lead by an electron with different energies. As the abscissa we have plotted the ratio of the radiated photon energy to the electron energy, and as the ordinate the ratio of the radiated intensity with inclusion of the effects of the medium to the intensity of radiation according to the Bethe-Heitler formula. Curves I, II, and III correspond to the total energy loss of an electron in production of electron-positron pairs and bremsstrahlung. The remaining curves describe the electron energy loss to bremsstrahlung. Curves 1—3 correspond to the ordinary multiple-scattering effect together with the effects of virtual-photon absorption and polarization of the medium. Curves 1′—3′ take into account the variation in energy, and consequently also of the multiple-scattering constant in the coherence length. For comparison with the results of ref. 6 we have shown curves 2″ and 3″, which were calculated from Eq. (16) with the constant b = 1.

It is evident that for electrons with energies not exceeding 10^{14} eV, in the soft-photon region $\omega \lesssim 10^{-3}E^{2/3}$ eV in lead, the effect of the medium polarization is dominant, and in the region $\omega \gtrsim 10^{-3}E^{2/3}$ eV the effects of virtual-photon absorption and change in the scattering constant appear noticeably. At these energies the polarization of the medium affects the radiation of photons with frequencies 10^8 eV $\gtrsim \omega$, and in the hard-photon region $\omega \gtrsim 10^{-20}E^2$ eV the ordinary multiple-scattering effect is dominant. Thus, for high electron energies the effect of change in the multiple-scattering constant in the radiation process leads to an appreciable suppression of bremsstrahlung in a photon-frequency region which widens rapidly with increasing electron energy.

[1] L. D. Landau and I. Ya. Pomeranchuk, Dokl. Akad. Nauk SSSR **92**, 535, 735 (1953).

[2] M. L. Ter-Mikaelyan, Dokl. Akad. Nauk SSSR **94**, 1033 (1954).

[3] A. B. Migdal, Dokl. Akad. Nauk SSSR **96**, 49 (1954).

[4] A. B. Migdal, Zh. Eksp. Teor. Fiz. **32**, 633 (1957) [Sov. Phys.-JETP **5**, 527 (1957)]; Phys. Rev. **103**, 1811 (1956).

[5] V. M. Galitsky and I. I. Gurevich, Nuovo Cimento **32**, 396 (1964).

[6] V. M. Galitskiĭ and V. V. Yakimets, Zh. Eksp. Teor. Fiz. **46**, 1066 (1964) [Sov. Phys.-JETP **19**, 723 (1964)].

[7] M. L. Ter-Mikaelyan, Vliyanie sredy na elektromagnitnye protsessy pri vysokikh energiyakh (Influence of the Medium on Electromagnetic Processes at High Energies), Academy of Sciences, Armenian SSR, Erevan, 1969.

[8] E. L. Feinberg, Usp. Fiz. Nauk **58**, 193 (1956). E. L. Feinberg and I. Ya. Pomerančuk, Nuovo Cimento, Series 10, Suppl. **3**, 652 (1956).

[9] V. A. Bazylev, A. A. Varfolomeev, and N. K. Zhevago, I. V. Kurchatov Atomic Energy Institute preprint IAÉ-2135, 1971.

[10] I. N. Toptygin, Zh. Eksp. Teor. Fiz. **46**, 851 (1964) [Sov. Phys.-JETP 19, 583 (1964)].

Translated by C. S. Robinson
86