GALVANOMAGNETIC PROPERTIES OF ANTIMONY AT LOW TEMPERATURES. SIZE EFFECT, ROLE OF SURFACE, AND SHAPE EFFECTS

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Submitted April 3, 1972

Zh. Eksp. Teor. Fiz. 63, 1036–1047 (September, 1972)

The effect of size, shape, and state of the surface on the electrical conductivity and magnetic resistance of antimony single crystals of various purity is studied at low temperatures. The experimental data are explained by invoking the static skin effect with allowance for intervalley scattering in the bulk and on the surface[15]. The intervalley scattering relaxation time in the bulk at 4.2°K is of the order of 10^{-8} sec and the intervalley scattering coefficient on the surface is 10^{-1}–10^{-2}.

INTRODUCTION

THE present work is a continuation of a study, started by Verkin and the authors[1,2] of the scattering mechanisms connected with imperfections in the crystal lattice of antimony. We have reported earlier[1,2] that the static conductivity of antimony is influenced by the transverse dimensions of the samples. It was noted there that, other conditions being equal, the kinetic properties depend strongly on the method of preparing the sample; it was proposed that the decisive factors are, first, carrier scattering by twin boundaries, and second, differences in the state of the crystal surfaces[2].

The role played by twin boundaries in static conductivity was discussed in[3]. The main task of the present study, besides further study of the size effect, is to determine the degree of influence of the state of the boundary surface of the sample and of their shape on the kinetic properties of antimony.

THE SAMPLES

The idea of the experiments is, first, to compare the results obtained with crystals of equal purity, cut by the electric spark method, before and after chemical polishing of the surface, and second, to compare data obtained with samples of equal dimensions but different purity. Accordingly, we present below the measurement results for samples 5, 6, 16, 17, and 18, which are of one degree of purity, and 19, 20, and 21, which are of another. With the exception of samples 5 and 6, the single crystals were cut in batches of three each from two rectangular antimony bars of different grades. The electric-spark cutting was with a copper filament 0.1 mm thick, actuated by an RD-09 motor that reversed at the instant when the filament made electric contact with the sample, so that the mechanical action on the crystals (which lay freely on the substrate during the course of cutting) reduced in fact to a spark impact. As a result, the crystal surface assumed the form of a sponge with pits not deeper than 0.01 mm. The chemical polishing in a CP-4 solution[4] removed a surface layer ~ 0.05 mm thick. The matrix of the series of samples with smaller amount of impurities (19–21) contained a single twin layer of thickness $<10^{-2}$ mm, the trace of which on the (111) planes that bounded the crystals turned out to be perpendicular to their longitudinal axis. The processing of the crystals introduced no additional twin layers. The characteristics of the samples are listed in Table I.

RESULTS OF EXPERIMENTS

1. Influence of Surface Finish on the Static Conductivity

The characteristic changes in the transport properties of the crystals at helium temperatures, as a result of polishing their surface, are as follows (see Fig. 1 and Table I):

1. The electric conductivity of the samples in a zero magnetic field increases, but not more than 20%.

2. The exponent n of the function $R_H = \gamma H^n$ increases (see Table II). In the initial (unpolished) samples, n increases with changing temperature from 4 to 14°K, and when $\mathbf{H} \parallel C_2$ the derivative $\partial n/\partial T$ is larger the thicker the sample. Conversely, an increase of the temperature is accompanied by a lowering of the values of n, provided $n \approx 2$ at 4.2°K. The error in the measurement of the exponent does not exceed ±0.01. This is seen, for

Table I

Sample number	Transverse dimensions, mm	r_0	α	
			4,2 °K	20 °K
5	3,9(C_2)×3,9(C_3)	1650	1.51	3.95
5A		1760	2.2	4.26
6	1,3(C_2)×1,3(C_3)	1070	0.97	3.14
6A		1080	1.3	3.56
16	6,0(C_2) × 0,6(C_3)	934	1.23	3.25
16A		970	1.12	3.6
17	0,6(C_2)×4,0(C_3)	770	1.08	2.95
17A		850	1.88	3.58
18	5,0(C_2)×3,8(C_3)	1420	1.64	3.81
18A		1340	2.24	4.3
19	5,8(C_2)×0,6(C_3)	1980	1.05	3.28
19A		2280	0.8	3.78
20	0,45(C_2)×5,5(C_3)	994	0.96	2.84
20A		1160	1.79	4.07
21	6,0(C_2)×4,8(C_3)	3400	0.89	3.79
21A		3400	0.98	4.26
23A	∅ 4,0	2460	1.35	4.35
10	see [3]	2100	3.7	4.45

Note. $r_0 = R_0(300°K)/R_0(4.2°K)$, $a = R(\mathbf{H} \parallel C_2)/R(\mathbf{H} \parallel C_3)$, H = 20 kOe. The symbols in the parentheses are the corresponding directions of the binary (C_2) and trigonal (C_3) crystallographic axes.

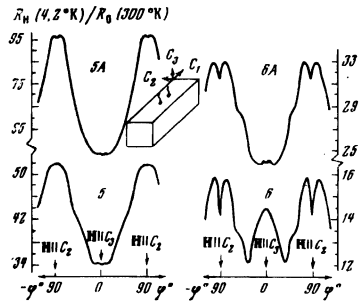

FIG. 1. Rotation diagrams of samples 5 and 6 before and after (5A, 6A) polishing, H = 20 kOe, $R_H(4.2°K)$ is the resistance in a magnetic at 4.2°K, $R_0(300°K)$ is the resistance in zero field at 300°K.

Table II

Sample number	T = 4.2 °K		T = 14 °K		Sample number	T = 4.2 °K		T = 14 °K	
	H ∥ C_2	H ∥ C_3	H ∥ C_2	H ∥ C_3		H ∥ C_2	H ∥ C_3	H ∥ C_2	H ∥ C_3
6	1.77	1.86	1.83	1.92	6A	1.94	1.99	1.95	1.95
5	1.82	1.95	1.94	2.0	5A	1.97	2.0	1.99	1.95
20	1.69	1.81	1.75	1.86	20A	1.99	1.99	1.97	1.94
21	1.79	1.87	1.9	1.94	21A	1.99	2.0	1.97	1.94

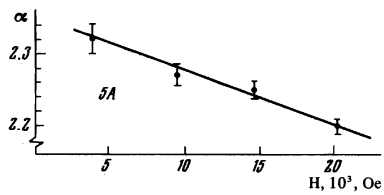

FIG. 2. Anisotropy of the magnetoresistance vs. the field, T = 4.2°K.

example, in Fig. 2, where $\alpha(H)$ satisfies the condition

$$\alpha_{H_1}/\alpha_{H_2} = (H_2/H_1)^{\nu}, \quad H_2 > H_1,$$

where $\nu = n(\mathbf{H} \parallel C_3) - n(\mathbf{H} \parallel C_2)$, which yields for sample 5A and for magnetic fields 5 and 20 kOe a value $\Delta n = 0.03 \pm 0.01$, in accordance with the result of Table II.

3. The magnetoresistance increases appreciably at $\mathbf{H} \parallel C_2$, and is practically the same for samples 5 and 6. In other words, polishing does not change the ratio of the resistances of the samples having a quadratic cross section, a given purity, and given dimensions at a given field direction[1)].

4. Considerable changes are observed in the rotation diagram, which is characterized by the relative values of the resistances at different directions of the vector H in a plane perpendicular to the current. In particular, polishing increases the anisotropy α of the magnetoresistance, which serves as an indicator of the size effect in samples with quadratic cross sections[1,2]. Using this property of α, we attempted to influence the kinetic characteristics of the samples, by successively varying the state of the boundary surface with etchants of different compositions. As a result, we observed that the chemical treatment of the surface exerts an influence (albeit quite weak) on the magnetoresistance of

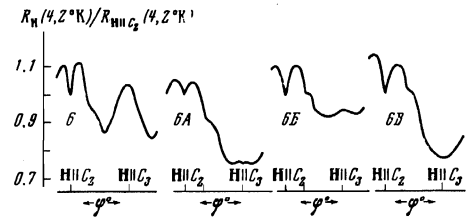

FIG. 3. Change of rotation diagram of sample 6 following chemical polishing of the surface.

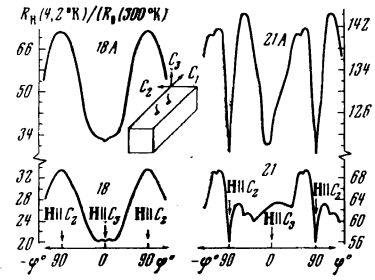

FIG. 4. Rotation diagram of samples of different purity before and after polishing, H = 20 kOe.

antimony. Figure 3 shows rotation diagrams of sample 6 before and after (6A) polishing in CP-4, etching in a solution of HF : HCl : HNO_3 = 2 : 2 : 1 (6B) and second polishing (6C). As a result of all the procedures, the transverse dimensions were changed by 30%.) It is clearly seen that etching of a polished surface bring the diagram closer to the initial form.

Comparison of the results obtained with samples of different purity, but with identical quadratic cross section dimensions, leads to the following conclusions.

1. In strong effective fields, at a given state of the boundary surface, the anisotropy of the magnetoresistance increases with increasing impurity concentration[2)] (see Fig. 4 and Table I).

2. The increase of the magnetoresistive effect upon polishing of crystal 18 and 21 for the case $\mathbf{H} \parallel C_2$ is practically the same and amounts to ~ 100%, whereas for $\mathbf{H} \parallel C_3$ the change of the magnetoresistance of sample 21 is much larger (Fig. 4).

3. At T = 20°K, the anisotropy of the magnetoresistance still depends on the surface finish: its value for samples 5, 18, and 21 amounts to 3.8 and increases to 4.26 after polishing.

In concluding this section we indicate that, just as in the case of Bi[5], polishing of the surface of antimony is accompanied by a growth of the amplitude of the Shubnikov-de Haas oscillations.

2. Shape Effects

We present below results from which it follows that at a fixed current direction the orientation of the plates, specified by the vector h normal to their broad plane,

[1)] The cross section of sample 5A was reduced to the dimensions of sample 6A by chemical polishing. In this case their electric conductivities and magnetoresistances coincided within 30%.

[2)] As already mentioned, one twin layer passes between the potential contact of sample 21, and scattering by the boundaries of this layer, generally speaking, decreases the anisotropy of the magnetoresistance [3]. In this case, however, the change of α does not amount to more than 30% (the influence of the boundaries of single twin layers on the static conductivity will be discussed in a separate article).

determines to a considerable degree the singularities of the kinetic properties of the antimony samples.

1. In the absence of a magnetic field, the electric conductivity of plates with $\mathbf{h} \parallel C_3$ is larger than with $\mathbf{h} \parallel C_2$, i.e., a unique shape effect is observed, the degree of which depends on the purity of the sample (see Fig. 5 and Table I).

2. Polishing of the plates doubles the magnetoresistance at $\mathbf{H} \parallel C_2 \perp \mathbf{h}$ ($T = 4.2°K$), whereas at $H \parallel C_3 \perp h$ a twofold increase of the magnetoresistive effect is observed only in the pure sample 20. This result is perfectly analogous to those described above for samples with quadratic cross sections (see, for example, Figs. 1 and 4).

3. At a given orientation of the magnetic field relative to the crystallographic axes, other conditions being equal, the growth of the magnetoresistive effect as a result of surface polishing is determined by the orientation of H relative to the broad plane of the plates. When $\mathbf{H} \parallel \mathbf{h}$, the growth of the magnetoresistance is much larger than in the case $\mathbf{H} \parallel \mathbf{h}$ (Fig. 5).

4. The dependence of the magnetoresistance of polished plates on the thickness at $\mathbf{H} \parallel \mathbf{h}$ is stronger than in the case $\mathbf{H} \parallel \mathbf{h}$, which leads to one more shape effect, first observed by Borovik and Lazarev in thin plates of Bi[6], namely the anisotropy of the magnetoresistance becomes a function of the direction of the vector normal to the broad plane of the plate (see Fig. 5 and Table I).

At a given purity, the anisotropy of the magnetoresistance of samples with quadratic cross section is smaller than for samples with round section of nearly equal dimensions (see Table I[3)], samples 5A, 10, 21A, 23A).

Thus, in the temperature interval 1.6–20°K (see also Fig. 6), the shape, dimensions, and surface state exert a very strong influence on the static conductivity of relatively bulky (up to 5 mm thick and possibly more) antimony single crystals equivalent in their physical properties to those investigated in the present study.

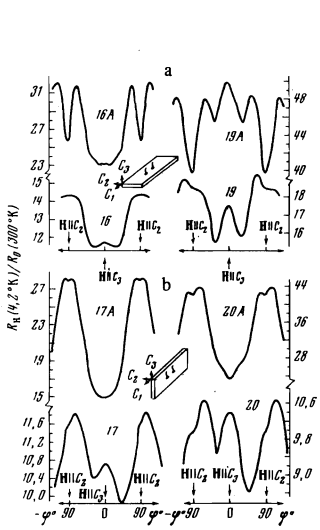

FIG. 5

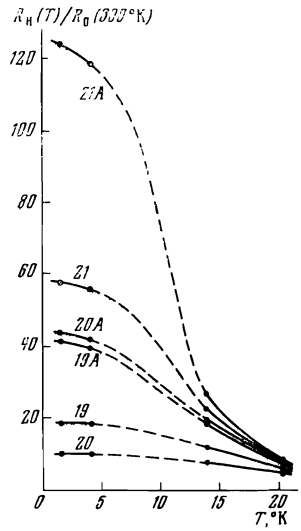

FIG. 6

FIG. 5. Rotation diagrams of plates: a–$\mathbf{h} \parallel C_3$, b–$\mathbf{h} \parallel C_3$. Samples 16, 17 (and 18, Fig. 4) were cut from a common matrix; the series 19, 20 (and 21) was prepared in the same manner.

FIG. 6. Temperature dependence of the magnetoresistance of samples 19–21, $\mathbf{H} \parallel C_2$, H = 20 kOe.

3. Temperature Dependence of Electric Resistivity

Since the error in the measurement of the electric resistivity at temperatures below 10°K is relatively large, the optimal curve describing the temperature dependence of the resistivity of antimony can be obtained by averaging a set of corresponding values for a large number of samples. The result of such a procedure for a set of 6 different single crystals is shown by the solid curve of Fig. 7. In the interval 1.6–4°K, the temperature-dependent part of the resistivity $\rho_T = [R_0(T) - R_0]/R_0(300°K)$ is proportional to $T^{1.4}$, $\rho_T \sim T^3$ in the region 10–40°K, and $\rho_T \sim T^{1.4}$ above the liquid-nitrogen temperature.

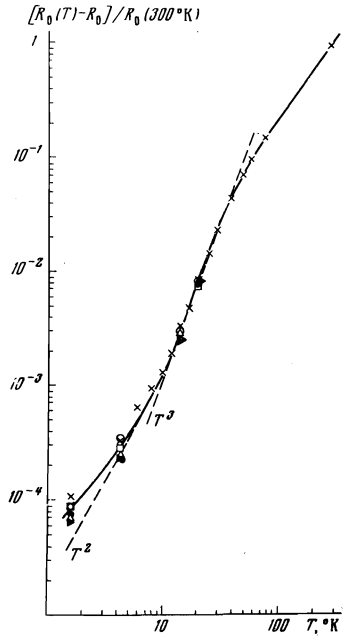

FIG. 7. Temperature dependence of the electroresistance of samples 6 (+), 16A (○), 17A (△), 18A (□), 19A (●), 20A (▲). R_0 is the resistance of T = 0°K and is determined by extrapolation.

DISCUSSION OF THE RESULTS

1. Electric Conductivity

According to the data of[2,7], the intravalley mean free path l of the carriers in antimony at helium temperatures is ~ 0.1 mm. Using the relation

$$\sigma_0 = \sigma_0^{\infty}(1 - 3lP/16A), \qquad (1)$$

which is a generalization of the Dingle formula[8] for the conductivity of a thick wire in a zero magnetic field to the case of a cross section of arbitrary geometry[9] (P is the perimeter, A is the cross section area, and σ_0^{∞} is the conductivity of a wire of infinite thickness), it is easy to estimate the change of the conductivity on going from sample 5A to 6A. According to the estimates, the sought value (relative to 5A) turns out to be not more

[3)] The data for sample 10 were taken from [3]; its cross section had a near-elliptic shape with maximum dimensions 3.9 × 2.9 mm; sample 23, with 4 mm diameter, was cut by the electric-spark method with a cutter of cylindrical shape from the same ingot as 21.

than 4%, whereas the corresponding value determined from experiment is not less than 40%. This is evidence in favor of the diffusion nature of the size effect[10], due to the multivalley structure of the electron energy spectrum of antimony.

The most favorable situation for observing the diffusion size effect in plates occurs at a definite combination of the plate orientation and the geometry of the multivalley Fermi surface, such that one valley goes over into the other by mirror reflection from a plane parallel to the plane of the plate, and the angle of inclination of the valleys to this plane is close to 45°. It is clear from symmetry considerations that in this case no transverse electric field is produced to compensate for the carrier concentration gradients, and the transverse particle fluxes are maximal. The geometry of the Fermi surface of antimony is such[11] that conditions analogous to those formulated above are satisfied when $\mathbf{h} \parallel C_3$ and are not satisfied when $\mathbf{h} \perp C_3$. On the other hand, the greatest change in the conductivity is observed also when $\mathbf{h} \perp C_3$ (cf. Figs. 4 and 5 and Table I).

Thus, we are faced with a qualitative correlation between the experimental results and the theoretical premises.

Further, assuming that the conductivity of sample 21 is close to that of a sample of infinite thickness, and comparing it with the conductivity of plate 20, we estimate the diffusion mean free path L. The analytic expression for the effective conductivity Σ of an antimony plate with $\mathbf{h} \perp C_3$ should have the same structure as formula (10) and (14) of[10], which have the following form, assuming a small ratio of the rate of surface recombination (determined from the phenomenological boundary condition that the current through the surface be equal to zero) to the diffusion rate $v_d = D/L$,

$$\Sigma \approx \sigma\left(1 - \frac{\mathrm{th}\,(d/L)}{d/L}\right), \tag{2}$$

where $L = (D\tau_{iv})^{1/2}$, D is the diffusion coefficient, τ_{iv} is the intervalley relaxation time, and d is the plate thickness.

Substituting the obtained value[4)] of L in the relation $L = l(\tau_{iv}/\tau)^{1/2}$ between the diffusion mean free path with the number of Brownian moves, and recognizing that at 4.2° K the intravalley relaxation time is $\tau \approx (2-5) \times 10^{-10}$ sec[2], we determine the value of τ_{iv}. According to the estimates, $\tau_{iv} \approx (2-5) \times 10^{-8}$ sec, which agrees with the data of Dolgopolov[12], who investigated electromagnetic excitation of sound in antimony.

2. Magnetoresistance

The Dingle formula for the conductivity, generalized to the case of strong magnetic fields perpendicular to the wire axis, takes the following form[9]

$$\sigma_H = \sigma_0^{\infty} \frac{1 + 3l/4\pi R}{(\omega_c\tau)^2}. \tag{3}$$

Here σ_0^{∞} is the conductivity of an infinite sample in a zero magnetic field ($l/R \to 0$, R is the radius of the wire cross section) and ω_c is the cyclotron frequency ($\omega_c\tau \gg 1$). Since $\sigma_H = 1/\rho_H$ for compensated metals, we obtain from (3), recognizing that $l \ll R$,

$$\rho_H = \rho_0^{\infty}(\omega_c\tau)^2[1 - 3l/4\pi R]. \tag{4}$$

Estimating the change of the resistance in the magnetic field on going from sample 5 to sample 6 with the aid of expression (4) and comparing the obtained value (~ 3%) with the experimental one (~ 60%), we conclude that the results of experiments in a magnetic field cannot be explained by means of the usual size effect.

Size can have a strong influence on the magnetoresistance of conductors having mean free paths that are small compared with the thickness under the conditions of the static skin effect, first considered by Azbel'[13] and by Azbel' and Peschanskiĭ[14] for the case of equal probabilities of intra- and intervalley scattering (i.e., for a single carrier-momentum relaxation time); in this case the current density is concentrated at a distance equal to the Larmor radius r from the surface of the semimetal. A recent analysis of the magnetoresistance of semimetals with allowance for multivalley effects[15] leads to a much larger depth of penetration of the current in the sample, of the order of the intervalley diffusion length $L_1 \approx r(\tau_{iv}/\tau)^{1/2}$.

In either case, the structure of the expressions for the average conductivity Σ_H of plates in a magnetic field is the same[5)]:

$$\Sigma_H \approx \sigma_H^{\infty} + k\sigma_0^{\infty} b/d \tag{5}$$

(b is the depth of the skin layer and k is a coefficient that depends on the magnetic field and on the state of the surface). Therefore, making use of the static skin effect to explain the experimental results, we shall not specify concretely the nature of this effect for the time being. We note only the very lucid physical meaning of formula (5): it can be obtained by regarding the skin layer at the center as two parallel-connected conductors with different thicknesses (with $b \ll d$) and with resistivities σ_H^{∞} and $k\sigma_0^{\infty}$. From (5) we can draw the following conclusions:

1. The anisotropy of the magnetoresistance

$$\alpha = R(\mathbf{H} \parallel C_2)/R(\mathbf{H} \parallel C_3) = \Sigma(\mathbf{H} \parallel C_3)/\Sigma(\mathbf{H} \parallel C_2)$$

of sufficiently thin samples with quadratic cross sections tends to unity, if it is assumed that[6)]

$$(kb)_{\mathbf{H} \parallel C_2} \approx (kb)_{\mathbf{H} \parallel C_3}$$

(see Fig. 1, sample 6).

2. Since in antimony at a given current orientation ($\mathbf{j} \parallel C_1$) in a strong magnetic field we have

$$\sigma^{\infty}(\mathbf{H} \parallel C_3) > \sigma^{\infty}(\mathbf{H} \parallel C_2)$$

(see Fig. 4, sample 18; Table I, samples 5 and 10), the condition $\Sigma_H \approx k\sigma_0^{\infty} b/d$ is satisfied for $\mathbf{H} \parallel C_2$ sooner (i.e., starting with larger thicknesses) than for $\mathbf{H} \parallel C_3$. It is clear that when this condition is satisfied, identical surface treatment does not change the ratio of the re-

4) Relation (2) yields L ≈ 0.5 mm. Actually this value is probably too low, since sample 21 is not bulky enough, as is evident by the influence of the surface treatment on the electric conductivity. The subsequent estimates are therefore made for L ≈ 1 mm.

5) The authors of the cited papers confined themselves to the case of spherical Fermi surfaces and magnetic fields parallel to the surface of the plate ($\mathbf{H} \perp \mathbf{h}$ in our notation).

6) Each premise is accompanied by an experimental result obtained in the present study and referred to in the parentheses.

sistances of samples having different thicknesses (Fig. 1, samples 5 and 6, 5A, and 6A, $\mathbf{H} \parallel C_2$). Performing the inverse operation, i.e., specifying the magnetoresistances of single crystals 5A and 6A, we obtain with the aid of relation (5) at $T = 4.2°K$ $\sigma^\infty(\mathbf{H} \parallel C_3)/\sigma^\infty(\mathbf{H} \parallel C_2) \approx 4.6$, i.e. $\rho_0(300°K)\sigma^\infty(\mathbf{H} \parallel C_2) \approx 2.7 \times 10^{-3}$, $\rho_0(300°K)\sigma^\infty(\mathbf{H} \parallel C_3) \approx 12 \times 10^{-3}$.

3. If the purity of the crystal is such that

$$\Sigma(\mathbf{H} \parallel C_2) \approx \Sigma(\mathbf{H} \parallel C_3) \approx k\sigma_0^\infty b/d$$

(i.e., $\sigma_H^\infty \sim 1/(\omega_c\tau)^2 \to 0$), then surface finish produces practically no change in the near-unity anisotropy of the magnetoresistance (Fig. 4, samples 21 and 21A). In addition, when $\sigma_H^\infty < k\sigma_0^\infty b/d$, the relative change of the magnetoresistance at a given surface finish does not depend on the purity (Fig. 4, samples 18 and 18A at $\mathbf{H} \parallel C_2$ and 21, 21A).

4. As soon as $\sigma_H^\infty \gg k\sigma_0^\infty b/d$, the anisotropy of the magnetoresistance takes on a value determined by the ratios of the cyclotron masses and the intravalley relaxation times. Such a result should follow, in particular, from an increase in the temperature (see Table I), since the temperature dependence of σ_H^∞ is much stronger than that of the second term in (5) (see Fig. 6, samples 20 and 21A).

5. Since a non-quadratic dependence of the resistance on the magnetic field can be due only to the second term in (5), the rapid temperature growth of σ_H^∞ relative to $k\sigma_0^\infty b/d$ should increase the exponent n, the change of which at $(d\sigma_H^\infty/k\sigma_0^\infty b)_{T=4.2°K} \ll 1$ is faster the thicker the sample (see Table II, $\mathbf{H} \parallel C_2$). When $n \approx 2$, its decrease with increasing T is due to violation of the condition $\omega_c\tau \gg 1$ at $T \geq 15°K$[2].

It was already mentioned that expression (5) was obtained for the case of spherical Fermi surfaces and magnetic fields parallel to the plane of the plate. The anisotropy of the Fermi surface and the presence of additional boundary planes in real samples should lead to the formation of a skin layer that surrounds the sample concentrically. For samples with a rectangular cross section, one can attempt to describe the concentricity of the skin layer by adding to relation (5) a term of the type $\sigma_0^\infty k_1 b_1/d_1$. If the cross section of the crystal is a square, then, as can be readily seen, the third term has no effect whatever on the anisotropy and on the ratio of the magnetoresistances of samples of equal thickness under the conditions $\sigma_H^\infty \ll \sigma_0^\infty k_i b_i/d$, $b_i < d$ (see Fig. 1, where the ratio of the resistances of samples 5 and 6, 5A and 6A at $\mathbf{H} \parallel C_2$ is equal to the thickness ratio). However, when comparing crystals with unlike shape of the transverse cross section and with different surface states, the same quantities will be determined by the ratio of the coefficients k_i, and also by the relative thicknesses b_i/d_i of the skin layer (see Figs. 4 and 5).

Let us examine now the results of concrete assumptions concerning the nature of the static skin effect. For the Azbel'-Peschanskiĭ skin effect[13-14]

$$k = \gamma/(q_1 + \gamma), \quad b = r$$

(q_1 is the diffuseness coefficient, $\gamma = r/l$), and the condition $\sigma_H^\infty \ll \sigma_0^\infty kb/d$ (where $\sigma_H^\infty \approx \sigma_0^\infty \gamma^{-2}$) is equivalent to the inequalities $q_1, \gamma \ll l/d$. The Larmor radius of antimony at $H \sim 10^4$ Oe is $r \sim 10^{-4}$ cm. Therefore at thicknesses $d \sim 10^{-1}$ cm and free paths $l \sim 10^{-2}$ cm the inequality $\gamma \ll l/d$ is satisfied and the coefficient of diffuseness of the samples, after electric-spark treatment of the surface, determined by the method proposed earlier[16] (by the deviation of the increase of the resistance in the magnetic field from the quadratic law), turns out to be $\sim 3 \times 10^{-2}$ (see also[17]). Within the framework of the considered model, the increase of the magnetoresistance and of the exponent n after polishing of the single crystals is a consequence of the increase of the diffuseness of the surface.

Allowance for multivalley effects, besides resulting in a larger depth of the skin layer, leads to a significant dependence of the average conductivity in the magnetic field on the fraction $\tilde{d}$, of the intervalley scattering by the surface, which can be obtained from the condition[15]

$$q + q_1 + \tilde{d} = 1 \qquad (6)$$

where q and q_1 are the specularity and diffuseness coefficients in intravalley scattering from the surface.

If there is no intervalley scattering from the surface, compensation of the bipolar carrier drift in the direction perpendicular to the electric and magnetic fields by the diffusion current leads to a strong increase of the conductivity of the surface layer. Inclusion of the intervalley scattering upsets the compensation and decreases the average conductivity. Since specular scattering calls for the conservation of the energy and of the tangential components of the momentum, it is customary to assume[18] that the probability of the intervalley transitions in collisions with the surface is maximal in the case of fully diffuse intravalley scattering (and the average magnetoconductivity is accordingly minimal).

When $\tilde{d} \to 0$, according to[15],

$$\Sigma_H = \sigma_H^\infty + \sigma_0^\infty \frac{L_1}{d} \operatorname{th} \frac{d}{L_1}. \qquad (7)$$

Assuming for samples of type 5 at $T = 4.2°K$

$$\rho_0(300°K)\sigma_0^\infty \approx 2000\ [2], \quad \rho_0(300°K)\sigma^\infty(\mathbf{H} \parallel C_2) \approx 2.7 \cdot 10^{-3}\ \text{cm}$$

and using the magnetoresistance data obtained for plate 16, we obtain in the case of $L_1 \leq d$ the value $L_1 \approx 2 \times 10^{-6}$ cm (as against $L_1 \sim r(\tau_{iv}/\tau)^{1/2} \sim 10^{-3}$ cm from the electric-conductivity data), which contradicts the condition $\tau_{iv} \gg \tau$, on which the solution method used in[15] is based. If $L_1 > d$, then, as follows from (7), the conductivity $\Sigma_H \sim \sigma_0^\infty$ should not depend on the magnetic field and on the dimensions of the object, and this contradicts the experimental data. In addition, in our concrete case $L_1 > d$ means $L_1 \gtrsim 1$ cm or $\tau_{iv} > 10^{-2}$ sec, which is in general not realistic, since it exceeds the τ_{iv} of bismuth at helium temperature[19] by at least six orders of magnitude[7)].

Values of L_1 satisfying the initial premises (see[15]) can be obtained by introducing in the second term of (7) a factor $k \ll 1$. Introduction of a small numerical factor in this case means a decrease of the conductivity of the surface layer as a result of intervalley scattering. According to[15], for the particular case $q = 0$, $k = (1 - 2\tilde{d})/(1 + \tilde{d})$ we have

[7)] The method of determining the intervalley diffusion length from the temperature dependence of the magnetoresistance, used by us in [2], is not convenient under the conditions of the static skin effect.

$$Q=\frac{3}{4}\tilde{d}\left(1+\frac{\gamma_c}{\gamma_v}\right)\frac{L_1}{\gamma_c r}\operatorname{th}\frac{d}{L_1}. \quad (8)$$

If $k \ll 1$ then $Q \gg 1$, and we obtain for the average conductivity

$$\Sigma_H \approx \sigma_H^{\infty} + \sigma_0^{\infty}\frac{L_1}{d}\frac{1-2\tilde{d}}{Q}\operatorname{th}\frac{d}{L_1}. \quad (9)$$

Assuming the intervalley scattering in antimony to be appreciable, we use the foregoing formulas to estimate the coefficient $\tilde{d}$.

Substituting (8) in (9), using at T = 4.2° K the values $\gamma_c \approx 10^{-2}$ and $\gamma_v \approx 10^{-3}$ from [2], and knowing the magnetoresistance of the sample 16, we obtain $\tilde{d} \approx 5 \times 10^{-2}$. In accordance with the statements made above, the growth of the magnetoresistance of the samples as a result of polishing of the surface means an increase of fraction of intervalley scattering. Thus, the experimental results on magnetoresistance can be explained with the aid of the diffusion size effect only if account is taken of intervalley scattering by the surface. Since the influence of the dimensions on the electric conductivity and on the magnetoresistance is most likely to have a common nature, the static skin-effect mechanism proposed by Babkin and Kravchenko [15] turns out in this case to be preferable. It should be noted, however, that to explain the dependence of the exponent n on the surface finish under the condition

$$\sigma_H^{\infty} \ll \sigma_0^{\infty}\frac{L_1}{d}\frac{1-2\tilde{d}}{Q}$$

the quantity Q should have in general a somewhat different structure than in (8), for example $Q \sim \alpha + \beta H$, where the ratio of the coefficients α and β is determined by the state of the surface.

3. Influence of the Surface

If it is assumed that in the case of specular (diffuse) intravalley scattering the intervalley scattering from the surface should also be specular (diffuse), then the growth of the fraction of intervalley transitions following polishing of the surface (which results also from data on the electric conductivity) means an increase in the diffuseness. This situation, which is paradoxical at first glance, can occur in two cases.

1. The electric-spark processing, producing defects on the surface, generates additional states, the population of which leads to a near-surface bending of the bands. Such a unique field effect can suppress the intervalley transitions, the role of which again increases as a result of elimination of the defects upon polishing the surface.

The influence of the bending of the bands will be noticeable if its magnitude is comparable with the Fermi energy of the carriers, i.e., $\epsilon_F \sim eER_D$, (where $E = 4\pi N_S e/\epsilon_0$ is the intensity of the field induced by the surface charge with density N_S, and ϵ_0 is the relative dielectric constant). Assuming $\epsilon_F \sim 10^{-13}$ erg, $R_D \sim 10^{-7}$ cm, and $\epsilon_0 \sim 10$, we find that the necessary density of the surface states is

$$N_s = \varepsilon_0 \varepsilon_F / 4\pi e^2 R_D \sim 5\cdot 10^{12}\,\mathrm{cm}^{-2}.$$

2. If the dimensions of the surface roughnesses resulting from the spark cutting are much larger than the de Broglie wavelength of the carriers, the scattering can be specular also in the case of an optically diffuse surface. Polishing decreases the dimensions of the roughnesses, which can become commensurate with the de Broglie wavelength λ_B ($\lambda_B \sim 10^{-6}$ cm in antimony). As a result, the surface becomes optically specular, but diffuse for the carriers.

Unambiguous information concerning the interaction of electrons and holes with the surface can be obtained in principle from experiments on the field effect by varying the near-surface bending of the bands. Such experiments are being planned for the nearest future.

In conclusion we indicate that since the influence of the dimensions on the kinetic properties of antimony is noticeable even at hydrogen temperature (see Fig. 6), intervalley scattering processes in bulk begin to play a decisive role at T > 20° K.

The authors are grateful to B. I. Verkin for interest in the work, V. Ya. Kravchenko and V. T. Dolgopolov for a discussion of the results, and V. G. Gerasimenko for technical help.

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Translated by J. G. Adashko
110