INFLUENCE OF DISLOCATIONS ON THE DE HAAS–VAN ALPHEN EFFECT

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The influence of extended crystal defects (dislocations) on the de Haas–van Alphen effect is considered. It is shown that the influence reduces to the appearance of the Dingle factor, i.e., to the reduction of the amplitude of oscillation of the magnetic moment by a factor $e^{-1/\Omega\tau}$ (Ω is the cyclotron frequency). For dislocations which do not possess a short-range potential, the relaxation time τ does not depend on the magnetic field and the value of this time is of the same order as for impurities. In the case of long-range dislocations (deformation potential) τ is proportional to the field strength and is smaller than that for impurities by an order of magnitude.

It is known that the defects of a crystal lattice have a significant influence on the de Haas-van Alphen effect which essentially reduces to a decrease in the amplitude of oscillations of the magnetic moment. This decrease has been well studied both theoretically and experimentally[1,2] in the case of point defects such as impurities, vacancies, interstitial atoms and so on, and is described by the Dingle factor $e^{-1/\Omega\tau}$ (Ω is the cyclotron frequency, τ the time of free flight of the electron). The influence of extended defects of the crystal (dislocations) on the de Haas-van Alphen effect has not been studied, so far as we know. At the same time, the dislocations represent a very widespread type of crystal lattice defect. Furthermore, their extension leads, as will be seen below, to significant specification of their influence on the magnetic properties of the metal or semiconductor. In this connection, the problem of the calculation of the magnetic moment and the susceptibility of a sample containing dislocations is considered in the present work for the simplest of models.

Let us assume the dispersion law for the electrons to be isotropic and quadratic: $E = p^2/2m$. Since the radius of curvature of the dislocation is usually much greater than the magnetic length $r_H = (c_0\hbar/eH)^{1/2}$ (where c_0 is the speed of light), the dislocations are naturally assumed to be linear and passing through the entire crystal. The interaction of an electron with the dislocations will be described by a potential which is independent of the coordinates along the axis of the dislocation. The form of the potential in a plane perpendicular to the dislocation is determined by the specific type of crystal. Therefore, following the work of Kaner and one of the authors,[3] we consider separately two limiting cases: long-range deformation potentials and short-range delta-type potentials.

1. In many cases, we can assume that the interaction of electrons with dislocations is due to deformations of the lattice, i.e., the potential has the form $U = \Lambda_{ik}u_{ik}$, where Λ_{ik} is the deformation potential tensor, and u_{ik} the tensor of dislocation deformations, known from elasticity theory. The scattering of electrons by dislocations leads to a change in the single-particle density of states, in terms of which, as is known, all the thermodynamic quantities that depend on the properties of the electrons are expressed. The altered density of states can be found by starting from the following considerations.

An electron moving in a constant, homogeneous magnetic field has discrete Landau energy levels. Randomly distributed dislocations create a perturbing random potential, which leads to a broadening of these levels. Inasmuch as deformations created by an individual dislocation diminish in inverse proportion to the distance from its axis, the characteristic length of the considered random field will be the mean distance $r_c \equiv c^{-1/2}$ between dislocations (c is the concentration of dislocations, i.e., their total length per unit volume). If the dimensions of the classical orbit of the electron (Larmor radius R) is much less than the distance r_c over which the perturbation potential changes materially, then one can assume this potential to be constant, equal to its value U in the region of localization of the electron. It then follows that in the considered case each level is shifted by an amount U. Since this shift is a random quantity with some distribution function p(U), it is evident that each discrete level is "smeared out" in a band in which, the density of states is determined by the form of the function p(U). This broadening, following the terminology of I. Lifshitz, is naturally called the classical broadening of the quantum level. A simple generalization of this fact leads to the following formula for the perturbed density of states:

$$D(E)=\int_{-\infty}^{\infty}D_0(E-U)p(U)\,dU, \qquad (1)$$

where $D_0(E)$ is the known (see, for example,[4]) unperturbed density of electron states in the magnetic field.

It has been shown previously[3] that p(U) is itself a Gaussian distribution with rms fluctuations $\epsilon_* = \Lambda bc^{1/2}$, where Λ is a quantity of the same order as the arbitrary component of the tensor Λ_{ik} and b is the Burgers vector (the interatomic distance).

To find the oscillating increment to the density of states, which determines the de Haas-van Alphen effect, an expansion of $D_0(E)$ is usually carried out over the sum of the harmonic components. If we then integrate this series term by term in accord with (1), we obtain as a result in front of each harmonic (the number of

which we denote by the letter l) the factor

$$\exp[-2\pi^2 l^2 \epsilon_*^2/(\hbar\Omega)^2].$$

The exponent thus found describes the decrease in the amplitude of the oscillations due to dislocations, i.e., it is the Dingle dislocation factor. This factor is apparently contained also in the expressions for the magnetic susceptibility and the magnetic moment.

If the broadening ϵ_* of the individual level exceeds the distance $\hbar\Omega$ between the Landau levels, then the individual peaks in the density of states are so "smeared out" that there it is meaningless to speak of any oscillations. The same result is obtained by considering the Dingle factor, inasmuch as in this case the argument of the exponential has a modulus much greater than unity. The ratio R/r_C which enters into the criterion for applicability of Eq. (1) is, as is not difficult to establish, the same as in the case of a dislocation perturbation with the ratio $\epsilon_0/\hbar\Omega$. Therefore Eq. (1), and also the corresponding Dingle factor, describe qualitatively the amplitude of the de Haas–van Alphen oscillations when the oscillations can be observed.

Let us consider in more detail the Dingle dislocation factor described above. The specific character of dislocations, in contrast with impurities, is first of all that the magnetic field enters into the argument of the exponential in second and not in first power. This is connected with the long-range action of the dislocations. The dependence of the oscillations on the number of the harmonic is also very strong; therefore, one can neglect the higher harmonics (beginning with $l = 2$). The oscillations can be observed distinctly, as has already been noted above, if

$$\epsilon_*/\hbar\Omega \equiv \Lambda b c^{1/2}/\hbar\Omega \ll 1. \qquad (2)$$

The inequality (2) is the essential limit on the dislocation concentration.

We note especially that the angular distribution of the long-range dislocations is not taken into account by Eq. (1). Consequently, in the approximation described by the inequality (2), the angle between the direction of the dislocations and the magnetic field does not enter into the expression for the magnetic moment.

2. In a number of cases (for example, for certain types of semiconductors), the principal role in the interaction of electrons with dislocations is played by the strong distortion of the crystalline lattice near the nucleus, or by the screened charge of the current carriers, which is deposited on the dislocations. Then the potential of the individual dislocation is short-range, and we shall consider it to have the shape of a delta function, to simplify the calculations. One can show that the change in the density of electron states under the action of randomly distributed dislocations does not depend on their orientation relative to the magnetic field. For this reason, the dislocations can be regarded as parallel.

We direct the z axis along the axis of the dislocations. Then the system of dislocations defines a random two-dimensional, statistically homogeneous field:

$$U(\rho) = -\beta \sum_s u(\rho - \rho_s),$$

where ρ is a two-dimensional vector in the xy plane, ρ_s are independent uniformly distributed random vectors, $u(\rho)$ is the "smeared" delta function, β is the "intensity" of the dislocation perturbation, and summation is carried out over all dislocations.

Let the magnetic field be located in the xz plane at an angle θ to the dislocation axis. Then it is convenient to choose the following gauge of the vector potential $\mathbf{A}$:

$$A_x = -yH\cos\theta, \quad A_y = 0, \quad A_z = yH\sin\theta.$$

This gauge was proposed in a paper by Kaner and one of the authors,[5] where the problem of the effect of a single dislocation on the spectrum of an electron in a magnetic field was studied. For such a gauge, the two-dimensionality of our problem is obvious, and the momentum p_z appears everywhere as a parameter over which integration is to be carried out in the final analysis. The eigenfunctions and eigenvalues of the unperturbed two-dimensional Schrödinger equation are:

$$\psi_\alpha(\rho) = (2\pi^{1/2}\hbar 2^n r_H n!)^{-1/2} \exp\left[\frac{i}{\hbar}xp_x - \frac{(y-y_0)^2}{2r_H^2}\right] \mathcal{H}_n\left(\frac{y-y_0}{r_H}\right),$$
$$E_\alpha = \hbar\Omega\left(n+\frac{1}{2}\right) + \frac{1}{2m}(p_x\sin\theta + p_z\cos\theta)^2 \qquad (3)$$

where $u_0 = c_0(p_z\sin\theta - p_x\cos\theta)(eH)^{-1}$; α denotes the set of quantum numbers n, p_x; $\mathcal{H}_n(\xi)$ is a Hermite polynomial.

It is convenient to perform the calculations[2] in a representation determined by the unperturbed wave functions. The two-dimensional density of states $D_2(E)$ is (with accuracy to a numerical factor) the imaginary part of the trace of the perturbed Green's function $\mathcal{G}_{\alpha\beta}$:

$$D_2(E) = -\frac{1}{\pi}\operatorname{Im}\sum_\alpha \mathcal{G}_{\alpha\alpha}. \qquad (4)$$

The Green's function satisfies the Lippman-Schwinger equation

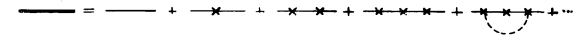
$$\mathcal{G}_{\alpha\beta} = g_\alpha\delta_{\alpha\beta} + g_\alpha\sum_\gamma U_{\alpha\gamma}\mathcal{G}_{\gamma\beta}. \qquad (5)$$

Here $g_\alpha = (E - E_\alpha)^{-1}$ is the unperturbed Green's function, $\delta_{\alpha\beta}$ the Kronecker delta,

$$U_{\alpha\beta} = \sum_s u_{\alpha\beta}^s, \quad u_{\alpha\beta}^s = -\beta\int d\rho\,\psi_\alpha^*(\rho)u(\rho-\rho_s)\psi_\beta(\rho),$$

and the index s enumerates the dislocations.

Using notation of the diagram technique, we can write the solution of Eq. (5) in the form of the following series:



The heavy line corresponds to $\mathcal{G}_{\alpha\beta}$, the thin one to g_α, the cross to the total scattering amplitude on a single dislocation, the dotted line joins identical dislocations, Summation is carried out over all crosses and the internal thin lines. If we use a short-range potential, then the scattering amplitude on the s-th dislocation can be written in the form

$$a_{\alpha\gamma}^s = -\frac{\beta\psi_\alpha^*(\rho_s)\psi_\gamma(\rho_s)}{1+\beta G_N(\rho_s,\rho_s;E)}$$

$$G_N(\rho,\rho;E) = \int_{-\infty}^{\infty} dp_x \sum_{n=0}^{N} \frac{\psi_\alpha^*(\rho)\psi_\alpha(\rho)}{E-E_\alpha},$$

where N is a large parameter of the order of $(r_H/b)^2$.

It is known from the literature (see, for example,[2]) that the principal contribution to the density of states over a wide range of energy values is made by the diagrams without dashed lines. This approximation corresponds to allowance for only the field correlation function and neglect of higher moments. To find the mean Green's function $\langle \mathcal{G}_{\alpha\beta} \rangle = \langle \mathcal{G}_{\alpha\alpha} \rangle \delta_{\alpha\beta}$, it is necessary to average the separated graphs over the distribution of the dislocations. The each cross corresponds to

$$\langle a_{\alpha\gamma} \rangle = -c \int \frac{\beta \psi_\alpha^*(\rho_s)\psi_\gamma(\rho_s)\,d\rho_s}{1+\beta G_N(\rho_s, \rho_s; E)}.$$

The result of summation of such graphs is

$$\langle \mathcal{G}_{\alpha\beta} \rangle = \frac{\delta_{\alpha\beta}}{E - E_\alpha - \langle a_{\alpha\beta} \rangle}. \quad (6)$$

The study of the expression (6) for an arbitrary angle is rather tedious. We therefore limit ourselves to writing down the final result. If the concentration of dislocations is small, so that $c\bar{\beta}^2 r_H^2 \ll 1$ ($\bar{\beta} = m\beta/\pi\hbar^2$), then the oscillating part of the magnetic moment does not depend on the angle between the magnetic field and the dislocations, and takes the form

$$M' = \frac{mTE_F}{\pi r_H H} \sum_{l=1}^{\infty} \frac{(-1)^l}{l^{1/2}} \frac{\exp(-\pi^3 l \bar{\beta}^3 r_H^2 c)}{\operatorname{sh}(2\pi^2 lT/\hbar\Omega)} \sin\left(\frac{2\pi l E_F}{\hbar\Omega} - \frac{\pi}{4}\right).$$

The reason for the independence of the magnetic moment of the angle is different here than for long-range dislocations. Namely, the effect of the magnetic field is not felt in the scattering of an electron by a dislocation, since the magnetic length is much greater than the radius of action of the potential of the dislocations. Therefore, the scattering length, and hence the time of flight, does not depend on the characteristics of the magnetic field and, in particular on its orientation.

The effect of the short-range dislocations on the magnetic moment reduces to the appearance of the Dingle factor $e^{-1/\Omega\tau}$, where $\tau = 2m/\pi\bar{\beta}^2\hbar c$. The corresponding Dingle temperature $\hbar/\tau$, just as in the case of impurities, does not depend on the magnetic field and is proportional to the concentration. The ratio of the "dislocation" Dingle temperature to the "impurity" temperature is proportional to $c_d/c_i b$, where c_d is the concentration of the dislocations and c_i the concentration of the impurities. Usually, this ratio is of the order of unity; therefore, we can assert that the short-range dislocations decrease the amplitude of the oscillations of the magnetic moment by approximately as much as the point defects. The effect of the long-range dislocations on the de Haas–van Alphen effect is much greater. Actually, for typical magnetic fields and concentrations, the "dislocation" Dingle temperature is of the order of 10°K in this case, while for the short-range dislocations or impurities, it is of the order of 0.1–1°K. Moreover (which is most interesting), the Dingle temperature in the case of short-range dislocations depends strongly on the field (in inverse proportion to it). In the case of short-range dislocations, such a dependence is entirely lacking.

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