THEORY OF DIFFUSION AFTEREFFECT

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A mechanism of the diffusion after-effect is proposed. Basic equations describing the process are derived. A solution is obtained in a special case and compared with the experimental results. The anomalous aftereffect which occurs during a finite time interval after switching an external load on and off is discussed qualitatively.

1. A theory of diffusion-dislocation flow of crystals was developed in earlier papers[1-3]. The mechanism responsible for such a flow contains also a diffusion aftereffect, i.e., a directed change in shape of the sample after the removal of the external load. The physical cause of the diffusion aftereffect in this flow mechanism is quite obvious. Indeed, since each dislocation-loop source emits an entire train of such loops during the steady-state diffusion-dislocation flow, removal of the load leaves the crystal in a non-equilibrium state with a certain dislocation-loop distribution that depends on the previously-applied load. This nonequilibrium state of the crystal relaxes by diffusion and produces the corresponding diffusion aftereffect.

The system of equations describing this process is obtained by setting the external load equal to zero in the previously obtained equations[1] describing the diffusion-dislocation flow. For a sample previously under tension we have

$$\frac{d\Delta^v}{dt}=-4\pi^2\ln^{-1}\frac{8R_0}{a}\left\{\int_0^{3-1/2}du\int D^v\left(\Delta^v-\frac{\alpha^v}{R^v}\right)f^v R\,dR+\right.$$
$$\left.+\int_{3-1/2}^{1}du\int D^v\left(|\Delta^v+\frac{\alpha^v}{R^{in}}\right)f^{in}R\,dR\right\},\qquad(1)$$

$$\frac{d\Delta^{in}}{dt}=-4\pi^2\ln^{-1}\frac{8R_0}{a}\left\{\int_0^{3-1/2}du\int D^{in}\left(\Delta^{in}+\frac{\alpha^{in}}{R^v}\right)f^v R\,dR+\right.$$
$$\left.+\int_{3-1/2}^{1}du\int D^{in}\left(\Delta^{in}-\frac{\alpha^{in}}{R^{in}}\right)f^{in}R\,dR\right\}.$$

Here u = cos φ, φ is the Burgers vector of the loop relative to the axis of the previously applied load, $\Delta^v = \bar{c}^v - c_0^v$ and $\Delta^{in} = \bar{c}^{in} - c_0^{in}$ are the excess vacancies and interstitial atoms, the equilibrium concentrations of which are c_0^v and c_0^{in}; D^v and D^{in} are the diffusion coefficients of the vacancies and interstitial atoms;

$$\alpha^i = c_0^i\beta \quad (i=v,in), \qquad \beta=\frac{a}{4\pi}\frac{\omega G}{(1-\nu)kT}\ln\frac{R_0}{a}$$

a is the lattice constant, $\omega = a^3$, G is Young's modulus, and ν is the Poisson coefficient. The dimension distribution functions f^v and f^{in} for the vacancy and interstitial loops are obtained by solving the corresponding kinetic equations (i = v, in):

$$\frac{\partial f^i}{\partial t}+\frac{\partial}{\partial R}\left(f^i\frac{dR^i}{dt}\right)=0.\qquad(2)$$

After the external load is removed, the sources that produce new loops cease to operate, since the excesses Δ^i are equal to zero.

The functions f^i satisfy the initial conditions[1]

$$f^i|_{t=0}=f_0^i(R),$$

with

$$f_0^i(R)=\int_{R_{cr}}^{R}\rho^i(z)\frac{dz}{z} \qquad \text{for } R>R_{cr},$$
$$f_0^i(R)=0 \qquad \text{for } R<R_{cr},\ R>R_0^i,$$

where ρ^i is the density of the sources of the dislocation loops, R_0^i is the maximum dimension attained by the loop during the process of prior steady-state flow, and $R_{cr} \cong aG/\sigma$ is the critical loop dimension starting with which the loops can increase at the load σ.

If the average dimension l of the priming loops produced under load is such that $R_{cr} \ll l \ll R_0$, then f_0^i = const, i.e., the crystal contains at the instant of load removal equal numbers of loops of a given type for all dimensions up to R_0^i. On the other hand, if the load was such that $l \sim R_0$ and $\rho(l)$ is a smooth function in this region, then f_0 can also be regarded as a constant quantity, neglecting the logarithmic dependence $f_0(R) \sim \ln R$, which is not very noticeable at large values of R.

The change of the loop dimensions following removal of the external load is described by the equations

$$\frac{dR^v}{dt}=\frac{2\pi D^*}{a\ln(8R_0/a)}\left(\Delta^*-\frac{\beta}{R^v}\right),\qquad \frac{dR^{in}}{dt}=\frac{2\pi D^*}{a\ln(8R_0/a)}\left(-\Delta^*-\frac{\beta}{R^{in}}\right),$$
$$D^*=c_0^vD^v+c_0^{in}D^{in},\qquad \Delta^*=(\Delta^vD^v-\Delta^{in}D^{in})/D^*,\qquad(3)$$

from which we see that the proposed aftereffect mechanism, which provides for the dissolution of the loops of both types, operates only when $|\Delta^*| < \beta/R^i$. Let us assume that $|\Delta^*| \ll \beta/R^i$. Then, substituting

$$\frac{dR^i}{dt}=-\frac{\alpha}{R^i},\quad \alpha=\frac{2\pi D^*\beta}{a\ln(8R_0/a)},\qquad(4)$$

in (2) and solving the latter, we obtain

$$f^i(R,t)=\frac{R^i}{(R_i^2+2\alpha t)^{1/2}}f_0^i\theta(R_{i0}^2-R_i^2-2\alpha t),\qquad(5)$$
$$\theta(x)=\begin{cases}1 & x>0\\ 0 & x<0\end{cases}.$$

The function θ is introduced to take into account the initial condition, namely the maximum loop dimension at t = 0 is R_{i0}.

The distribution function (5) describes the relaxation of an ensemble of prismatic loops produced in the sample by the external load. The change of the dislocation-loop distribution is accompanied by a matched rapid readjustment of the excesses Δ^i. This holds true the

dimensions of the loops do not change significantly during this readjustment time τ:

$$\Delta R / R_0 \sim D^*\Delta^*\tau / aR_0 \ll 1 \tag{6}$$

(as shown in[1], $\Delta^* = \sigma\omega/3kT = \kappa/3$). We shall subsequently see that this condition is usually satisfied.

Let us verify the consistency of the system of assumptions made in the analysis of the aftereffect process. The speed with which the excesses adjust themselves to a given distribution of the loops makes it possible to equate to zero the derivatives $d\Delta^i/dt$ in (1). Subtracting one equation of the system (1) from the other, we obtain

$$\Delta^* = \beta\left[\frac{1}{3^{1/2}}\int_0^\infty f^v dR - \left(1-\frac{1}{3^{1/2}}\right)\int_0^\infty f^{in} dR\right] \Big/ \Big/ \left[\frac{1}{3^{1/2}}\int_0^\infty f^v R\, dR + \left(1-\frac{1}{3^{1/2}}\right)\int_0^\infty f^{in} R\, dR\right].$$

integrating with the functions (5) and putting

$$R_0^v = R_0^{in} = R_0, \quad f_0^v = f_0^{in} = f_0,$$

we obtain

$$\Delta^* = \frac{2\beta}{R_0}\left(\frac{2}{3^{1/2}}-1\right)(1-x^{1/2}) \Big/ \left[(1-x)^{1/2} - x\ln\left(\frac{1+(1-x)^{1/2}}{x^{1/2}}\right)\right], \tag{7}$$

where we have introduced the dimensionless time $x = 2\alpha t/R_0^2$.

Let us compare the value of (7) with the term β/R^i for a loop that becomes resolved from a maximum possible dimension R_0. Taking the introduced notation into account, we obtain for this loop from (4)

$$R_{max}^i = R_0(1-x)^{1/2}, \tag{8}$$

so that

$$\beta/R_{max}^i = \beta/R_0(1-x)^{1/2}.$$

A direct numerical comparison shows that for all $0 < x \ll 1$ we have

$$\Delta^* \approx \frac{1}{4}\frac{\beta}{R^i}.$$

Thus, the assumption $|\Delta^*| \ll \beta/R^i$ is satisfied for a g given sample more accurately the smaller the number of loops of radius $\sim R_0$ among all the already present loops.

Formula (8) enables us to estimate the time T_0 of termination of the aftereffect

$$\frac{1}{T_0} = \frac{2\alpha}{R_0^2} = \frac{4\pi D^*\beta}{aR_0^3\ln(8R_0/a)}. \tag{9}$$

During this time, loops of both types become completely dissolved. We note, however, that the result is valid only in first approximation, when $|\Delta^*| \ll \beta/R^i$. Actually, however, the dissolution of the loops does not proceed in uniform manner, and loops of some particular type (interstitial, if the prior deformation was tension), disappear more rapidly after which the aftereffect terminates, and then the loops of the remaining type coalesce without causing a directed change in the shape of the sample. The time of the aftereffect therefore turns out to be shorter than the time calculated from (9), a fact that must be borne in mind in the comparison with experiment.

It is now easy to calculate the diffusion aftereffect (the change $\Delta\epsilon$ in the strain) due to the dissolution of the residual dislocation loops from initial dimensions R_0 to dimensions R. We use for this purpose the general formula for ϵ (see[1]). As a result we have

$$\Delta\varepsilon = \varepsilon_0 - \varepsilon(t) = 4\pi a\left\{-\int_R^{R_0}\int_0^{3^{-1/2}} \pi R^2 f u^2\, du\, dR + \int_R^{R_0}\int_{3^{-1/2}}^{1} \pi R^2 f u^2\, du dR\right\}.$$

Allowance was made here for the fact that the distribution of the dislocation loops with the initial dimensions relative to the directions of the Burgers vectors is determined by the previously applied load: $\cos^2\varphi_0 = \Delta^*/\kappa = 1/3$. Hence, calculating the integrals and substituting the expression for R from (8), we obtain

$$\Delta\varepsilon = \Delta\varepsilon_0(2-\sqrt{2})^{-1}\left[\left(1+\frac{t}{T_0}\right)^{3/2} + 3\frac{t}{T_0} - 3\frac{t}{T_0}\left(1+\frac{t}{T_0}\right)^{1/2} - 1\right], \tag{10}$$

$$\Delta\varepsilon_0 = {}^4/_9\pi^2 a f_0(2-2^{1/2})(1-2/3^{3/2})R_0^3. \tag{11}$$

Here $\Delta\epsilon_0$ is the total strain change due to the aftereffect.

We can now write the condition (6) in the form

$$\Delta R / R_0 \sim c_0\sigma\omega / kT\Delta\varepsilon_0 \ll 1.$$

Physically this condition becomes obvious. It means that the residual deformation due to the excess point defects present in the crystal at the initial instant of time is much smaller than that due to the residual dislocation loops.

The value of R_0, which is the maximum dislocation-loop dimension, is determined, as shown earlier[2,3] by the dimensions of the blocks or grains when the load is sufficiently large and the loops do not interact elastically. In this case R_0 does not depend on the load and should satisfy the condition

$$R_0^2 \ll \frac{\varkappa^{1+m}}{\pi h_0 n}\left(\frac{l}{h_0}\right)^m, \quad h_0 = a\frac{G\omega}{2\pi(1-\nu)kT}p,$$

where n is the number of sources of loops per unit volume, $p \approx 1$, and $0 \le m \le 2$. At sufficiently small loads, R_0 is determined by the elastic interaction of the dislocation loops[2,3] and is given by

$$R_0^2 = \frac{\varkappa^{1+m}}{\pi h_0 n}\left(\frac{l}{h_0}\right)^m. \tag{12}$$

The number m depends on the number of atomic planes in which dislocation loops can grow, with m = 0 and m = 2 corresponding to the anisotropic and isotropic cases, respectively. The block dimensions should now satisfy the condition

$$R^2 \gg \frac{\varkappa^{1+m}}{\pi h_0 n}\left(\frac{l}{h_0}\right)^m. \tag{13}$$

We note that expressions for R_0 were obtained[2,3] in an approximation in which the priming dimension of the dislocation loops l was much smaller than their average dimension R_0. In this case $n \cong fl$ = const. On the other hand, if the load is such that $l \sim R_0$ and ρ is a smooth function in the integration region, then

$$n = \int_{R_{cr}} \rho\, dR \cong \rho R_0$$

under the condition $R_0 d\rho/\rho dR_0 \ll 1$ and m = 0. Then the range of angles in which the loops interact elastically is of the order of unity. This means that

$$\rho R_0^3 = \varkappa / \pi h_0. \tag{14}$$

We thus obtain for $\Delta\epsilon_0$, as a function of the previously applied load and of the structural properties of the crystal, the following expression at $l \ll R_0$:

$$\Delta\varepsilon_0 = \frac{4\pi^2}{9} a(2-2^{1/3})\left(1-\frac{2}{3^{3/2}}\right)\frac{\varkappa_\theta^{(1+m)/2}}{\pi^{3/2}h_0^{3}f^{1/2}}\left(\frac{l}{h_c}\right)^{3(m-1)/2}, \quad 0 \leqslant m \leqslant 2; \tag{15}$$

and at $l \cong R_0$

$$\Delta\varepsilon_0 = \frac{4\pi^2}{9} a(2-2^{1/3})\left(1-\frac{2}{3^{3/2}}\right)\frac{\varkappa}{\pi h_0}. \tag{16}$$

Substituting the corresponding values of R_0, we obtain the following expressions for T_0 and the rate of establishment of the creep in the case of uniaxial load

$$T_0 = \frac{\ln(8R_0/a)}{4\pi}\frac{a}{D^*\beta}R^2 = \text{const} \tag{17}$$

$$\text{for } R^2 = \text{const} \ll \frac{\varkappa^{1+m}}{\pi h_0 n}\left(\frac{l}{h_0}\right)^m,$$

$$T_0 = \frac{\ln(8R_0/a)}{4\pi}\frac{a}{D^*\beta}\frac{1}{\pi h_0 n}\left(\frac{l}{h_0}\right)^m \varkappa^{1+m} \tag{18}$$

$$\text{for } R^2 \gg R_0^2 = \frac{\varkappa^{1+m}}{\pi h_0 n}\left(\frac{l}{h_0}\right)^m,$$

$$T_0 = \frac{\ln(8R_0/a)}{4\pi}\frac{a}{D^*\beta}\frac{1}{(\pi h_0\rho)^{2/3}}\varkappa^{2/3} \tag{19}$$

$$\text{for } l \cong R_0, \quad \rho R^3 \gg \frac{\varkappa}{\pi h_0},$$

where R is the average dimension of the satellite;

$$\dot{\varepsilon}_{zz} = \frac{8\pi^2}{45}\ln^{-1}\frac{8R_0}{a}D^*\frac{R^2}{l}n\varkappa, \tag{20}$$

$$\dot{\varepsilon}_{zz} = k\frac{8\pi^2}{h_0 l}\ln^{-1}\frac{8R}{a}D^*\left(\frac{l}{h_0}\right)^m \varkappa^{2+m}, \quad l \ll R_0, \tag{21}$$

$$\dot{\varepsilon}_{zz} = P\frac{8\pi^2}{9}\frac{\ln(R_0/R_{cr})}{\ln(8R_0/a)}D^*\frac{\rho}{(\pi h_0\rho)^{2/3}}\varkappa^{5/3}, \quad l \cong R_0. \tag{22}$$

Here k is a number that depends on the value of m; $k \cong 1/7$ at $m = 0$; P is a numerical factor of the order of unity. We have taken into account the fact that

$$\int_{R_{cr}}^{R_0}\rho\frac{dR}{R} \cong \rho\ln\frac{R_0}{R_{cr}}.$$

Thus, a study of the rate of the steady-state creep simultaneously with a study of the process of diffusion aftereffect in a given sample can not only identify uniquely the mechanism producing the diffusion flow of the crystal, but also determine quantitatively all the structural microscopic parameters (D^*, l, ρ) on which these processes depend.

We note that for the case of compression it is necessary to replace $\varkappa$ everywhere by the absolute value $|\varkappa|$ and to reverse the sign in the expressions for $\dot{\epsilon}_{zz}$ and $\Delta\epsilon_0$.

2. The theory of diffusion aftereffect was tested experimentally on polycrystalline samples of copper (average grain dimension ~ 0.5 mm) 60 mm long with cross section 1.5 × 1.5 mm. The tests were made with previously described setups[4], using a paired set of induction-differential displacement pickups, so that the creep and diffusion-aftereffect parameters could be measured with high accuracy even under conditions of instability or with continuous variation of the temperature. The accuracy with which the absolute displacements was measured was ~ 0.2 μ. The samples were tested (in accordance with a procedure described in[5]) at a constant temperature T = 850 ± 1°C and at different values of the stress σ. The stress was varied in a range of values at which, according to the proposed theory, the plastic deformation of the crystals is determined by the diffusion-dislocation flow mechanism that ensures, as in[6], a power-law dependence of the creep rate. Here, however, the exponent was not 3—5, as follows from Weertman's theory[6], but n = 5/3.[1)] The value of n is determined in the given range of loads by the microstructure of the crystal. Under our conditions, relation (14) is satisfied.

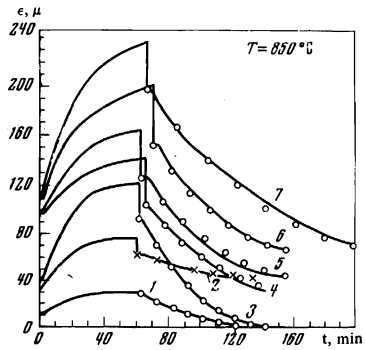

FIG. 1. Total-strain curves copied from the potentiometer chart. The points on the sections of the aftereffect strain correspond to the calculated values of $\Delta\epsilon$. The values of σ for the curves (in gf/mm^2) are as follows: 1—30, 2—60, 3—110, 4—170, 5—220, 6—280, 7—320.

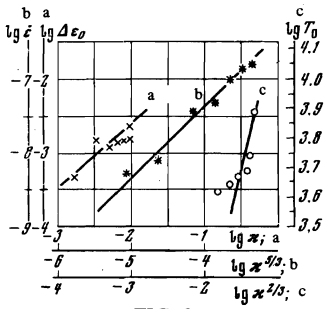

FIG. 2

FIG. 2. Plots of log $\Delta\epsilon_0$, log $\dot{\epsilon}$, and log T_0 vs. log κ.

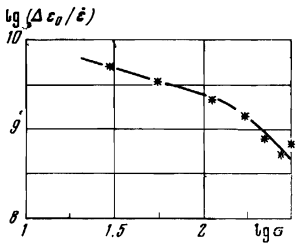

FIG. 3

FIG. 3. Plot of log $(\Delta\epsilon/\dot{\epsilon})$ vs. log σ.

Figure 1 shows the total-strain curves, copied from the chart of the EPP-09 potentiometer, for seven values of the load: 30, 60, 110, 170, 220, 280, and 320 gf/mm^2. The points on the sections of the strain aftereffect represent the values of $\Delta\epsilon$ calculated from formula (10). The calculation of $\Delta\epsilon$ is based on the values, taken from the corresponding strain curves, of the maximum aftereffect $\Delta\epsilon_0$ and of the time T_0 during which the total aftereffect strain $\Delta\epsilon_0$ is reached. Figure 2 shows plots of log $\Delta\epsilon_0$(log κ), log $\dot{\epsilon}$(log κ), and also log T_0(log κ). According to the experimental data, $\Delta\epsilon_0 \sim \kappa$ (Fig. 2a); the exponent in the expression for the creep rate $\dot{\epsilon}$ is equal to 1.7 (Fig. 2b); $T_0 \sim \kappa^{2/3}$ (Fig. 2c), and the ratio $\Delta\epsilon_0/\dot{\epsilon}$ at small loads is $\sim \kappa^{-0.6}$ (see Fig. 3).

For our samples and loads, the microparameters of the theory (h_0, D^*, ρ, R_0) should be determined by using the relations (14), (16), (19), and (22), which describe

[1)] The value of n changes appreciably when the experimental conditions are changed. It has a clear-cut tendency to decrease with decreasing σ and with increasing T (compare the various data in [6-9]), i.e., with increasing role of the diffusion processes in the plastic flow of the crystals.

sufficiently well the experimental curves of Figs. 1–3. In addition, from the experimental values of $\Delta\epsilon_0$ obtained for different κ (see Fig. 2a) it is possible, by using (16), to determine $h_0 = 9.7 \times 10^{-9}$ cm, while the plots of $\log \epsilon = f(\frac{5}{3}\log\kappa)$ (22) and $\log T_0 = f(\frac{2}{3}\log\kappa)$ (19) (see Figs. 2b and 2c) enable us to find the quantities

$$\alpha_1 = \frac{\dot\varepsilon}{\kappa^{5/3}} = \frac{8\pi^2}{9}\frac{\ln(R_0/R_{cr})}{\ln(8R_0/a)} D^* \frac{\rho^{1/3}}{(\pi h_0)^{2/3}} = 4.16\cdot 10^{-4}\ \text{sec}^{-1},$$

$$\alpha_2 = \frac{T_0}{\kappa^{2/3}} = \frac{\ln(8R_0/a)}{\ln(R_0/a)}\frac{(1-\nu)kT}{\omega G}\frac{1}{(\pi h_0\rho)^{2/3}D^*} = 2.24\cdot 10^5\ \text{sec},$$

from which we determine

$$D^* = \frac{r}{\rho^{1/3}}, \quad r = \frac{9}{8\pi^2}\frac{\ln(8R_0/a)\,\alpha_1(\pi h_0)^{2/3}}{\ln(R_0/R_{cr})} = 0{,}17\cdot 10^{-7};$$

$$D^* = \frac{q}{\rho^{2/3}}, \quad q = \frac{\ln(8R_0/a)}{\ln(R_0/a)}\frac{(1-\nu)kT}{\omega G}\frac{1}{(\pi h_0)^{2/3}\alpha_2} = 10^{-4}.$$

From these relations we readily obtain $D^* = r^2/q = 3 \times 10^{-12}$ cm^2/sec and $\rho = (q/r)^3 = 2 \times 10^{11}$ cm^{-4}.

Using (14), we obtain the mean value $R_0 = 5 \times 10^{-3}$ cm. We obtain also the number of sources of dislocation loops per unit volume: $n = \rho R_0 = 10^7$ cm^{-3}. We see thus that all these calculated parameters have reasonable values.

Let us note an effect observed in the investigation of the creep and diffusion aftereffect of the samples. When the load is applied and removed, a shortening and lengthening of the sample is observed, respectively, for several minutes. The absolute magnitude of such an anomalous change of shape amounts to several microns (at a sample working length of 40 mm). A detailed study has shown that the effect cannot be attributed either to thermal expansion connected with the possible radial displacement of the sample in the furnace, or to the thermoelastic effect in the adiabatic expansion or contraction of the sample at the instant of loading or unloading. It remains to assume that an anomalous aftereffect is registered in this case. In similar earlier experiments[10] it was impossible to observe the anomalous aftereffect directly, possibly because the absolute displacements were not registered with sufficient accuracy. The anomalous aftereffect was then manifest by a decrease of the normal aftereffect under definite experimental conditions.

In the proposed theory of diffusion aftereffect it is impossible to provide a natural explanation for this anomalous effect. Indeed, application of the load starts immediately a transient process producing a supersaturated point-defect solution corresponding to the applied load. The source of the point defects is mainly the dislocation "forest" inside the sample, including the dislocations in the block boundaries. The equations describing the changes of the supersaturation of the vacancies Δ^v and of the interstitial atoms Δ^{in} are the same as obtained in[1]. Here, however, F_v and F_{in} must be taken to mean the perimeter of the dislocation segments per unit volume, $F_v + F_{in} = L^{-2} \approx N$, where N is the dislocation density and L is the characteristic dimension of the cell of the dislocation "forest." For estimates we assume that the Burgers vectors of such segments are uniformly distributed among the directions. Then

$$\frac{d\Delta^v}{dt} = -\frac{2\pi}{\ln(8L/a)} D^v N \int_0^\pi (\Delta^v - \varkappa\cos^2\varphi)\sin\varphi\, d\varphi,$$

$$\frac{d\Delta^{in}}{dt} = -\frac{2\pi}{\ln(8L/a)} D^u N \int_0^\pi (\Delta^{in} + \varkappa\cos^2\varphi)\sin\varphi\, d\varphi.$$

we see therefore that within a time

$$\tau' \sim \ln(8L/a)\, c_0 / 2\pi D^* N$$

the supersaturation takes on the value $\Delta^v = -\Delta^{in} = \frac{1}{3}\kappa$, and $\Delta^* = \frac{2}{3}\kappa$.

We can easily obtain equations for the diffusion sags of the vacancy and interstitial segments h^v and h^{in}, by using relations (4) from[1] and expressing the curvature of a segment with chord L in terms of the sag h ($1/R = 8h/L^2$):

$$\frac{dh^v}{dt} = \frac{3\pi D^*}{a\ln(8L/a)}[\Delta^* - \varkappa\cos^2\varphi - 8\beta N h^3], \qquad (23)$$

$$\frac{dh^{in}}{dt} = \frac{3\pi D^*}{a\ln(8L/a)}[-\Delta^* + \varkappa\cos^2\varphi - 8\beta N h^u].$$

We have used here the expression for the volume of a flat segment $V = Sa = \frac{2}{3}Lha$. Consequently, within a time

$$\tau = a\ln(8L/a)/24\pi\beta D^* N \gg \tau'$$

the values of h^v and h^{in} become

$$h^v = \frac{\Delta^* - \varkappa\cos^2\varphi}{8\beta N} > 0, \quad h^{in} = \frac{-\Delta^* + \varkappa\cos^2\varphi}{8\beta N} > 0,$$

corresponding to $\Delta^* = \frac{2}{3}\kappa$.

From the law of matter conservation it follows that the areas of the vacancy and interstitial segments are approximately equal (as already shown, the excess number of point defects in the solution can be neglected). Thus, the maximum value of the anomalous deformation after loading, $\Delta\epsilon_L^{an}$, can be easily calculated from the general formula (see[1])

$$\Delta\varepsilon_L^{an} = \frac{V}{L^3}\Big[-\int_{\varphi_0}^{\pi/2}\cos^2\varphi\sin\varphi\, d\varphi + \int_0^{\varphi_0}\cos^2\varphi\sin\varphi\, d\varphi\Big] \cong -\frac{V}{L^3}\frac{1}{3}\Big[2\Big(\frac{2}{3}\Big)^{3/2} - 1\Big] \cong -0.03\frac{V}{L^3},$$

where $\cos^2\varphi_0 = \Delta^*/\kappa = \frac{2}{3}$, and V/L^3 is the relative volume of the dislocation segments per unit volume:

$$\frac{V}{L^3} = \frac{Sa}{L^3} = \frac{2}{3}\frac{Lha}{L^3} = \frac{a}{12\bar R} \cong \frac{a}{12\beta}\varkappa\overline{\cos^2\varphi} \cong \frac{a}{24\beta}\varkappa.$$

We have substituted here the equilibrium value of the curvature of the segment $\bar R \cong \beta/\kappa\cos^2\varphi$.

After the time τ required to complete the transient process and to operate the dislocation-loop sources, Δ^* becomes equal to $\kappa/3$ (see[1]), the dislocation segments assume equilibrium shapes and cease to be sources of point defects, and a stationary creep rate is established. If the load is now removed, these segments become sources of point defects that are absorbed by the dislocation loops previously produced by the external load. This anomalous aftereffect $\Delta\epsilon_U^{an}$ can also be easily calculated:

$$\Delta\varepsilon_U^{an} = \frac{V}{L^3}\Big[-\int_{\varphi_0'}^{\pi/2}\cos^2\varphi\sin\varphi\, d\varphi + \int_0^{\varphi_0'}\cos^2\varphi\sin\varphi\, d\varphi\Big] = \frac{V}{L^3}\frac{1}{3}\Big[2\Big(\frac{1}{3}\Big)^{3/2} - 1\Big] \cong 0.18\frac{V}{L^3},$$

$\cos^2\varphi_0' = \frac{1}{3}$. To obtain the equations for the dissolution

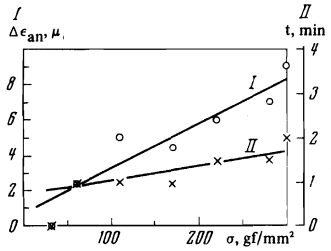

FIG. 4. Strain and time of anomalous aftereffect vs. prior stress σ (in gf/mm^2).

of the dislocation segments it is necessary to put $\Delta^* = \kappa = 0$ in (23).

We see thus that the characteristic time of the anomalous aftereffect after the unloading is also equal to τ.

The experimental curves of Fig. 4 provide a fair confirmation of the linear dependence of the anomalous aftereffect following application and removal of the applied stress; the equality of the corresponding characteristic times and their independence of the external load are also confirmed. We note also that the diffusion aftereffect starts to decrease starting with a certain load or with a certain temperature at a fixed load, because the nonequilibrium number of dislocation loops previously produced by the applied load decreases as a result of the appearance of additional mechanical motion. Such a behavior was clearly revealed by experiment[10].

We note that in the formulas we used for the diffusion creep the stresses inside the grains are assumed to coincide with the external stresses. In a polycrystalline sample, however, inhomogeneously stressed states (stresses of the second kind) are actually produced. The creep rate and the diffusion aftereffect must therefore be calculated by averaging over all the grain dimensions, recognizing that the grain creep rate depends on the grain dimension R if $R < R_0(\kappa)$ and is independent if $R > R_0(\kappa)$. It is necessary here to use the condition that the creep rates of all the grains be equal regardless of their dimensions (the condition that there be no loss of continuity), and also the equality of the stressed state, averaged over the grain dimensions, to the externally applied stress.

In the calculation of the rate of the diffusion aftereffect of a polycrystal it is necessary also to average over the maximum dimensions of the dislocation loops, assuming that this dimension coincides with the grain dimension when $R < R_0(\kappa)$ and with $R_0(\kappa)$ for all grains with $R > R_0(\kappa)$. In our sample, the distribution of the grain dimensions was such that almost all the grains had dimensions $R > R_0(\kappa)$ in the investigated range of loads. It is therefore unnecessary to average in our case.

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