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NONLINEAR DAMPING OF SHORT-WAVE SOUND IN A CONDUCTOR LOCATED IN A MAGNETIC FIELD

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A nonlinear theory is constructed for the absorption of short-wave sound with wavelength much smaller than the mean free path of the conduction electrons in a conductor located in an external magnetic field. The cases of both classically strong and quantizing magnetic fields are considered. In a nonquantizing magnetic field of moderate strength, a strong sound wave may significantly distort the electron trajectories near the points at which the trajectories touch the wave front. Since it is just these parts of the trajectories which are responsible for sound absorption, such a distortion leads to strong nonlinear effects. Estimates are obtained for the sound absorption coefficient and its dependence on the magnetic field strength is determined for the case of strong nonlinearity. It is shown that the amplitude of giant absorption oscillations in a quantizing magnetic field decreases with increase in the sound intensity, whereas their width increases. The shape of the oscillation lines is analyzed for various limiting cases. The possibilities of experimental observation of the effects are considered.

It is well known that the absorption of a short-wave sound[1)] in a conductor placed in a magnetic field depends distinctively on the value of the magnetic field. Thus, the sound absorption coefficient Γ can experience oscillations, depending on the value of the magnetic field. A large number of theoretical and experimental researches have been devoted to the investigation of such oscillations (see, for example,[5-7]). In these researches, it has been shown that the oscillations have essentially a different nature in classical and quantizing magnetic fields. In either case, the form of the oscillation picture is closely connected with the topology of the Fermi surface of the conductor, and the study of sound absorption is a convenient method of study of the Fermi surface and also of the relaxation mechanisms of the conduction electrons.

However, in the theories previously constructed, a linear approximation in the intensity of the sound field has been used, and therefore the sound was assumed to be sufficiently weak. At the same time, it is known[1,2] that nonlinear effects appear in the propagation of short-wave sound in the absence of a magnetic field at intensities much less than in the hydrodynamic regime ($ql \ll 1$). The nonlinearity here is connected with the distortion of the distribution of the electrons that move in phase with the sound wave. A nonlinear theory of propagation of short-wave sound in the absence of a magnetic field, under the assumption that the sound intensity is such that it satisfies the condition

$$e\tilde{\varphi}_0 / \varepsilon_F \ll 1, \tag{1}$$

has been constructed in[2-4]. Here e is the charge on the electron, ϵ_F the Fermi energy, and $\tilde{\varphi}_0$ the amplitude of the potential of the effective field which accompanies the sound wave. It has been shown that the parameters that determine the nonlinear effects are

$$d = e\tilde{\varphi}_0 / \hbar\tau^{-1} \tag{2}$$

for

$$\hbar^2q^2 / m \gg \max\,(\hbar / \tau, e\tilde{\varphi}_0) \tag{3}$$

and

$$a^{-2} = e\tilde{\varphi}_0(ql)^2 / \varepsilon_F \tag{4}$$

for

$$\hbar^2q^2 / m \ll \max\,(e\tilde{\varphi}_0, \hbar / \tau). \tag{5}$$

Here τ is the relaxation time of the electrons, and m is the effective mass. Modern experimental techniques allow us to introduce into the crystal sound of such intensity that these parameters become large.

The purpose of our research was the construction of a nonlinear theory of propagation of short-wave sound in a conductor placed in an external magnetic field **H** under conditions when the only limitation on the sound intensity is the relation (1).

1. CLASSICAL THEORY

We consider sound absorption in a transverse, nonquantizing magnetic field **H** ($\mathbf{q} \perp \mathbf{H}$) of such a value that the conditions

$$qv_F \gg \Omega \gg 1 / \tau. \tag{6}$$

are satisfied[2)]. Here $\Omega = eH/mc$ is the cyclotron frequency, c the speed of light, and v_F the velocity of the electron on the Fermi surface. The conditions (6) mean that the electrons move along orbits whose characteristic dimensions are much greater than the sound wavelength. The sound absorption coefficient in this situation, according to Pippard[5] and Gurevich,[6] is significantly greater (by a factor of about $\Omega\tau$) than the absorption coefficient Γ_0 in the absence of the magnetic

[1)]By short-wave sound we mean a sound whose wavelength $2\pi/q$ is much smaller than the free path length of the electron $l(ql \gg 1)$.

[2)]We note that the case of a longitudinal nonquantized magnetic field does not have anything new in comparison with the case H = 0.

field. Furthermore, this quantity has a periodic dependence on the magnetic field intensity H, while the period is determined by the diameter of the extremal intersection of the Fermi surface with the plane perpendicular to the direction of the magnetic field. This phenomenon, sometimes called geometric resonance, has been well studied and is a convenient method for studying the topology of the Fermi surface.

In the presence of a magnetic field, the parts of the trajectories on which the electrons interact effectively with the sound wave are important for the sound absorption. These are the parts where the projections of the electron velocity on the sound vector are small. On these parts, according to[2-4], the trajectories of the electron can be severely distorted by the effective field of the sound wave even upon satisfaction of the condition (1). We shall show that this leads to a decrease in the absorption and also to a "glossing over" of the oscillations of geometric resonance.

We shall consider qualitatively the influence of the effective field on the shape of the trajectory of the electron, which we shall assume to be closed. In the case of a limitingly small sound intensity, the electron's trajectory is a closed plane curve with a characteristic diameter 2R, where $R = v_F/\Omega$ is the Larmor radius (Fig. 1). The effective interaction with the sound wave takes place on the portions AA′ and BB′; on the remaining portions of the trajectory, the velocity of the electron in the direction of the sound wave vector (the x axis) is large, and the electron quickly "senses" the changing field of the wave. Correlation of the phases of the wave on the portions AA′ and BB′ also leads to geometric resonance. The influence of the effective field of the wave increases with increasing sound intensity. The characteristic velocity transferred to the electron by this field over a single wavelength is of the order of $\tilde{v} = (e\tilde{\varphi}_0/m)^{1/2}$. On the other hand, near the turning point on the x axis, the magnetic field transfers a velocity v_x of the order of $v_F(\Omega/qv)^{1/2}$ to the electron (over the same distance). Thus the distortion of the trajectory near the turning point is determined by the parameter

$$b = (v_F(\Omega/qv_F)^{1/2}\tilde{v}^{-1})^2 = mv_F\Omega/qe\tilde{\varphi}_0.$$

If $b \gg 1$, the effect of the sound wave on the trajectory can be neglected; this corresponds to the linear theory. If $b \ll 1$, the wave strongly distorts the trajectory near the tuning points. In this situation, there is singled out a group of trapped electrons, i.e., electrons that execute finite motion in the potential wells created by the wave. The condition $b \ll 1$ indicates the impossibility of the "removal" of the electrons from the well. The formation of the two groups of electrons is clear from Fig. 2, which shows the dependence of the energy of the longitudinal motion of the electron, $E_1(x) = \frac{1}{2}mv_x^2 + e\tilde{\varphi}(x)$, on the x coordinate. The boundary points of the curves correspond to the classical turning points along x.

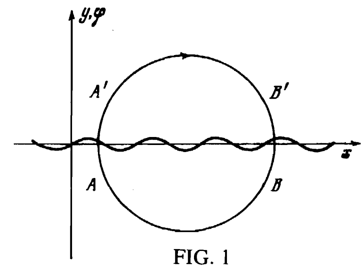

FIG. 1

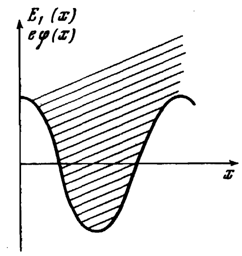

FIG. 2. The slant lines correspond to the set of curves $E_1(x)$ for various trajectories of the motion of the electron; the graph $e\varphi(x)$ describes the values of the energy E_1 at the turning points along the x coordinate.

A. The Electron Distribution Function

In the calculation of the absorption coefficient, we follow the method of[4]. According to that paper, when condition (1) is satisfied, the higher harmonics of the potential $\tilde{\varphi}$ of the effective field of the wave can be neglected, and one has

$$e\tilde{\varphi} = e\tilde{\varphi}_0 \cos(\mathbf{qr} - \omega t),$$

where $e\tilde{\varphi} = e\varphi + \Lambda_{ik}u_{ik}$ for deformation interaction, and $e\tilde{\varphi} = e\varphi$ for piezoelectric interaction; Λ_{ik} is the deformation potential tensor, u_{ik} is the deformation tensor; and φ is the potential of the wave of the self-consistent electric field. The amplitude $\tilde{\varphi}_0$ changes slowly in space.

The kinetic equation for the electron system is[3)]

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + \frac{e[\mathbf{vH}]}{c}\frac{\partial f}{\partial \mathbf{p}} - \frac{\partial e\tilde{\varphi}}{\partial x}\frac{\partial f}{\partial p_x} + \hat{I}f = 0, \qquad (7)^*$$

where f is the electron distribution function, and $\hat{I}$ is the collision operator. We seek a solution of (7) in the form $f = F_0(\epsilon + e\tilde{\varphi}) + g(qx - \omega t)$, where F_0 is the equilibrium distribution function of the electrons. Transforming to the Onsager parameters η and v in the (v_x, v_y) plane (these parameters have the meaning of the phase and modulus of the electron velocity on the trajectory), and also keeping only the lowest order in the ratio of the sound velocity w to v_F, we obtain

$$-\frac{\partial g}{\partial \eta}\Omega + \frac{1}{m}\frac{\partial e\tilde{\varphi}}{\partial x}\left(\sin\eta\frac{\partial}{\partial v} + \frac{\cos\eta}{v}\frac{\partial}{\partial \eta}\right)g - v\sin\eta\frac{\partial g}{\partial x} - \hat{I}g = -w\frac{\partial e\tilde{\varphi}}{\partial x}\frac{\partial F_0}{\partial \varepsilon}. \qquad (8)$$

In deriving (8), we have also used the assumption that the spectrum is quadratic. However, inasmuch as we are interested in the small parts of the trajectories near the turning points, the result is valid in order of magnitude for any energy spectrum. The angle η in (8) is reckoned from the y direction. Similar to[4], we can show that to calculate the sound damping, the collision operator can be represented in the form $\hat{I}g = g/\tau$, i.e., we neglect the arrival term.

The equations of the characteristics, which satisfy (8), are

$$\frac{d\eta}{A} = \frac{dv}{B} = \frac{dx}{C} = \frac{dg}{D},$$

$$A = -\Omega + \frac{\cos\eta}{mv}\frac{\partial e\tilde{\varphi}}{\partial x} \qquad B = \frac{\sin\eta}{m}\frac{\partial e\tilde{\varphi}}{\partial x}, \qquad (9)$$

$$C = -v\sin\eta, \qquad D = -\frac{1}{\tau}g + w\frac{\partial e\tilde{\varphi}}{\partial x}\frac{\partial F_0}{\partial \varepsilon}.$$

*[vH] ≡ v × H.

3)We shall assume here, that condition (5) which allows us to use the ordinary Boltzmann equation is satisfied.

The first integral of the system, $\frac{1}{2}mv^2 + e\tilde{\varphi}(x) = E_0$, corresponds to the law of energy conservation. We shall find the equation for the electron trajectory. Setting $\cos\eta = \vartheta$, we have, from the system (9),

$$-\frac{d\vartheta}{dx}+\frac{\vartheta(x)}{2\Phi(x)}\frac{\partial e\tilde{\varphi}}{\partial x}+\frac{\Omega}{\sqrt{2\Phi(x)}}=0,$$
$$\Phi(x)=E_0-e\tilde{\varphi}(x). \qquad (10)$$

In the derivation of this equation, we have chosen the direction of motion of the electron along the trajectory, assuming $v_x > 0$. It is easy to see that the trajectory is symmetric relative to the OX axis; the second branch of the trajectory is obtained by the substitution $\eta \to -\eta$, i.e., by changing the sign of v_x. The solution of Eq. (10) is

$$\vartheta(\xi)=\frac{\Phi(\xi_1)}{\Phi(\xi)}\left[\vartheta(\xi_1)-\int_{\xi_1}\frac{\Phi(\xi')}{\Phi(\xi_1)}\frac{\Omega\,d\xi'}{q\sqrt{2\Phi(\xi')}}\right]. \qquad (11)$$

Here $\xi = qx - \omega t$ is the wave coordinate.

According to (1), we can expand the right side of (11) in powers of the parameter $e\tilde{\varphi}_0/\epsilon$, limiting ourselves to the lowest order. Near the left turning point ($\eta \ll 1$), we have

$$\frac{\eta^2v^2}{2}-\frac{\xi\Omega v}{q}+\frac{e\tilde{\varphi}(\xi)}{m}=\text{const}=E. \qquad (12)$$

The equation of the trajectory near the right turning point differs from (12) by the replacement of η by $\pi-\eta$, $\xi \to -\xi$. The quantity $E_1 = E + \xi\Omega v/q$ has the meaning of motion along the direction of sound propagation. The dependence of this quantity on ξ is shown in Fig. 2. Thus the problem of the motion of the electron near the turning point reduces to a one-dimensional one. On these portions, the motion can be described by the introduction of some one-dimensional energy E, which is conserved according to (12). The formation of groups of trapped and untrapped particles then follows. It is easy to establish the fact that for $b \gg 1$ the term $e\tilde{\varphi}/m$ can be neglected, and we get the equation of the trajectory near the turning point in a magnetic field, which corresponds to the linear theory. We shall be interested below in the opposite limiting case ($b \ll 1$).

For $b \ll 1$, the motion of the trapped electrons is completely described by Eq. (12), inasmuch as their trajectories are bounded by the limits of the potential well of the wave and $\eta \ll 1$. For analysis of the motion of the transiting electrons, we use the more general equation (11). We note that the turning points of the untrapped electrons are distributed on the "crests" of the potential contour of the wave since, for $b \ll 1$, the slope of the curve E_1 is not large (see Fig. 2). We denote the left turning point by ξ_1. Then, by virtue of the circumstance noted above, $\xi_1 \ll 1$ if we take the vortex of the nearest crest as the origin of the coordinate ξ. Using this, we obtain for the untrapped electrons

$$\cos\eta=\frac{e\tilde{\varphi}_0}{mv^2}(u(\xi)-1)+1-\frac{\Omega\xi}{qv}+\frac{e\tilde{\varphi}_0}{mv^2}\frac{\xi_1^2}{2}+\frac{\Omega}{qv}\xi_1$$
$$-\frac{\Omega}{qv}\frac{e\tilde{\varphi}_0}{mv^2}(\xi-\xi_1)u(\xi), \qquad (13)$$

where $u(\xi)$ is the dimensionless potential of the wave, $u(\xi) = e\tilde{\varphi}(\xi)/e\tilde{\varphi}_0$.

We have the following equation for the function g:

$$\frac{d\xi}{-qv\sin\eta(\xi)}=\frac{dg}{D(\xi)},\quad D(\xi)=-\frac{1}{\tau}g(\xi)+wq\frac{\partial e\tilde{\varphi}}{\partial\xi}\frac{\partial F_0}{\partial\varepsilon}, \qquad (14)$$

where $\eta(\xi)$ is determined by the expressions (13) for the untrapped electrons and (12) for the trapped electrons. Imposing the conditions of reflection at the turning points ξ_1 and ξ_2 as boundary conditions (similar to[4]) we obtain the solution of Eq. (14):

$$g(\eta(\xi),\xi,v)=2\,\mathrm{sh}^{-1}\left(\int_{\xi_1}^{\xi_2}\frac{d\xi' G}{q\tau}\right)\left[\mathscr{E}(\xi_1,\xi_2)\int_{\xi_1}^{\xi}\frac{GU\,d\xi'}{\mathscr{E}(\xi',\xi)}\right.$$
$$\left.+\frac{1}{\mathscr{E}(\xi_1,\xi_2)}\int_{\xi}^{\xi_2}GU\mathscr{E}(\xi,\xi')d\xi'+\frac{\mathscr{E}(\xi_1,\xi_2)}{\mathscr{E}(\xi_1,\xi)}\int_{\xi_1}^{\xi_2}\frac{GU\,d\xi'}{\mathscr{E}(\xi,\xi')}\right], \qquad (15)$$
$$G=\frac{1}{v\sin\eta}\qquad U=\frac{w}{q}\frac{\partial e\tilde{\varphi}}{\partial x}\frac{\partial F_0}{\partial\varepsilon},\qquad \mathscr{E}(\xi_1,\xi_2)=\exp\left\{\int_{\xi_1}^{\xi_2}\frac{G\,d\xi}{q\tau}\right\}.$$

We now consider the expression in the arguments of the exponentials of (15). For the untrapped electrons, $\xi_2-\xi_1 \sim qR$; therefore, there is a quantity of the order of $1/\Omega\tau$ in the corresponding argument; for the trapped electrons, $\xi_2-\xi_1 \sim 1$, $v\sin\eta \sim \tilde{v}$; therefore, the argument is of the order of $a = 1/q\tau\tilde{v}$. The condition $b \ll 1$ requires that the inequality $a \ll 1$ be satisfied. Therefore, one can expand of the exponentials up to the first nonvanishing term in this parameter. As a result, we obtain

$$g=\int_{\xi_1}^{\xi_2}GU\,d\xi'\Big/\frac{1}{q\tau}\int_{\xi_1}^{\xi_2}G\,d\xi'. \qquad (16)$$

This is the state of affairs for untrapped electrons; for the trapped electrons, the corresponding term in the expansion vanishes after summation over the trajectories. Therefore, to calculate the absorption by the trapped electrons, we must carry out an expansion up to the next, nonvanishing order. As a result, we obtain an expression of the type given in[4]; the sound absorption by the trapped electrons is $\sim a\Gamma_0$.

B. The Sound Absorption Coefficient

Let us estimate the contribution made to the sound absorption by the untrapped electrons, which are determined by the distribution function (16). For the calculation of the absorption, we transform to the variables ξ_1 and ξ, i.e., we characterize the state of the electron by the trajectory and by the location of the electron on the trajectory. According to (13), the Jacobian of the transformation is equal to

$$\frac{d\eta}{d\xi_1}=-\left(\frac{\Omega}{qv}+\frac{e\tilde{\varphi}_0}{m}\xi_1\right)\Big/\sin\eta(\xi_1,\xi). \qquad (17)$$

The reactive part of the function of the response of the electron concentration to the sound excitation, which determines the absorption, can be found from the formula[4)]

$$K_q''(\omega)=2\frac{2m^3}{(2\pi\hbar)^3}\int_0^\infty dv_z\int_0^\infty v\,dv\int_0^{\xi_0}\frac{d\xi_1}{\pi}\int_{\xi_1}^{\xi_2}d\xi(-\sin\xi)g(\xi_1,\xi,v,v_z)\frac{d\eta}{d\xi_1}d\xi_1. \qquad (18)$$

Here ξ_0 is the boundary value of the coordinate of the turning point (between the untrapped and trapped electrons), measured from the vertex of the crest, the coefficient 2 is connected with the account of the motion of the electrons along the trajectory in the opposite direction.

4) The active response K′ determines the screening. As is shown in [4], upon satisfaciton of (1), we have $4\pi e^2K'/\epsilon_0 = \kappa^2$, where κ is the inverse of the Debye screening radius; ϵ_0 is the dielectric permittivity.

Actually, the reactive part of the electron response is expressed in terms of the Fourier component of the function $\sum_{\xi_1} g(\xi_1, \xi, v)$, where summation over ξ_1 is carried out over all trajectories, while the electrons corresponding to them may turn out to be on the part $(0, 2\pi)$. This function, as can easily be seen, is periodic in ξ, which leads to (18).

At $b \gg 1$, near the turning points, $\sin\eta$ behaves like $\sqrt{\xi - \xi_1}/qR$; in this case, it is easy to obtain the results of the linear theory from (16) and (18), and particularly geometric resonance. In our case, $b \ll 1$, the reactive response is of the form

$$K_q''(\omega) = 2\frac{2m^3}{(2\pi\hbar)^3}\int_0^\infty dv_z \int_0^\infty v\,dv \int_0^{\xi_0} \frac{d\xi_1}{\pi}\left(-\frac{e\tilde{\varphi}_0}{mv^2}\xi_1 - \frac{\Omega}{qv}\right)\frac{mwv}{G_{12}}\frac{\partial F_0}{\partial \varepsilon} J^2(\xi_1), \tag{19}$$

where

$$J = \frac{\tilde{v}}{v}\int_{\xi_1}^{\xi_2}\left(\frac{\partial u}{\partial \xi}\Big/ \sin\eta(\xi')\right) d\xi', \qquad G_{12} = \frac{1}{q\tau}\int_{\xi_1}^{\xi_2} G\, d\xi'. \tag{20}$$

For ξ_0 it is easy to get the estimate $\xi_0 \sim \sqrt{4\pi b}$ by using (12). In the calculation of J, the vicinities of the turning points are important, where relations of the type (12) are applicable. After cumbersome but essentially simple calculations, we obtain the result that $J \sim [u(\xi_1) - u(\xi_2)]$. The integral G_{12} in the denominator of (19) does not change in comparison with the linear theory, since the function G decreases sufficiently slowly on moving away from the turning point.

Thus, the absorption coefficient is equal to

$$\Gamma \sim a\Gamma_0 + \Omega\tau b^3\Gamma_0 \tag{21}$$

in order of magnitude, where the first and second terms are the contributions of the trapped and untrapped electrons. Here it is easy to see that the result of integration over ξ_1 in (19) actually does not have a resonant dependence on the magnetic field. The physical reason for this is the distortion of the trajectories, leading to a disruption of the correlation of the phases of the wave at the turning points.

We now discuss the dependence of the sound absorption coefficient on its intensity. For low sound intensities, a linear theory can be used; the absorption coefficient exceeds Γ_0 by a factor of about $\Omega\tau$ and geometric resonance takes place. With increase in the sound intensity for $b \sim 1$, a group of trapped electrons is formed, the contribution to the absorption of which is of the order of $a\Gamma_0$. Upon further increase in the sound intensity, the contribution of the untrapped electrons falls off sharply (as $S^{-3/2}$), which leads to a notable decrease in the absorption. Simultaneously, as a result of the violation of the phase correlation, the resonant dependence of the absorption coefficient on the magnetic field vanishes. Nonlinear effects appear more strongly the weaker the magnetic field, inasmuch as the latter prevents the formation of a group of trapped electrons. In the case of a very strong magnetic field, when the characteristic dimension of the trajectory is smaller than the wavelength, the situation becomes hydrodynamic. In this case, the nonlinearity has a concentration character and is determined by the parameter $e\tilde{\varphi}_0/\epsilon_F$. At the present level of experiment it is not possible to obtain sound intensities at which a concentration nonlinearity can appear in metals. To the contrary, estimates show that at $qR \gg 1$ nonlinear effects can be observed for sound intensities accessible to experimental study.

2. QUANTUM THEORY

As is well known,[7] the absorption coefficient of short-wave sound in a conductor placed in a quantizing magnetic field experiences giant oscillations as a function of H.

We consider the qualitative picture of the generation of giant oscillations. It follows from the energy-momentum conservation laws that the only electrons that can absorb a sound quantum are those with momentum projection in the direction of the magnetic field **H** (**H** ∥ **q**)

$$p_z = \frac{m}{q}\left(\omega - \frac{\hbar q^2}{2m}\right). \tag{22}$$

Here ω is the sound frequency; for simplicity, we have assumed that the sound propagates along the magnetic field. On the other hand, electrons close to the Fermi surface participate in the sound absorption. Quantization of the transverse energy in the magnetic field leads to the result that their longitudinal momentum takes on the discrete values

$$p_{zn} = [2m(\varepsilon_F - \hbar\Omega n)]^{1/2} \tag{23}$$

(n is a natural number). For coincidence of one of the p_{zn} with p_z, determined from (22), the absorption coefficient has a maximum. Here the line shape is determined both by the thermal smearing of the Fermi level and by the indeterminacy of the energy entering in the conservation law. In the case of weak sound, the latter is determined by collisions and is equal to $\hbar/\tau_p$ in order of magnitude (τ_p is the relaxation time of the electronic momentum). The line shape in this case has been studied in detail in[8], where the necessary and sufficient condition for the existence of giant oscillations has been obtained:

$$ql\hbar\Omega / \sqrt{T\varepsilon_F} \gg 1. \tag{24}$$

In the case of intense sound, the energy indeterminacy associated with account of the interaction of this electron with the sound wave becomes essential. We show that this circumstance can lead to a change in the line shape of the giant oscillations, of their amplitude and the per cent of modulation, and also to their complete disappearance.

A. The Kinetic Equation for Electrons in the Field of the Wave

We shall assume that the sound wave propagates along the direction of the magnetic field, the value of which is such that the inequality

$$\varepsilon_F \gg \hbar\Omega \gg T \tag{25}$$

is satisfied. In view of the fact that transitions under the action of the sound wave cannot take place between states with different transverse quantum numbers, we introduce the one-dimensional Wigner density

$$f_W = \sum_\varkappa e^{i\varkappa z}\,\mathrm{Sp}\left(\hat{\rho}\,\hat{a}^+_{\alpha_\perp, p_z - \hbar\varkappa/2}\,\hat{a}_{\alpha_\perp, p_z + \hbar\varkappa/2}\right), \tag{26}$$

constructed from the eigenfunctions of the unperturbed Hamiltonian,

$$\psi_\alpha=\varphi_n(x-x_\alpha)\exp\left[i(p_y y+p_z z)/\hbar\right],\qquad(27)$$

where φ_n is the normalized wave function of the harmonic oscillator, $\alpha = (\alpha_+, p_z, \sigma)$, $\alpha_\perp = n$, x_α is the set of Landau quantum numbers, $x_\alpha = cp_y/eH$, σ is the spin quantum number. The function f_W depends on the transverse quantum numbers as parameters. The reactive part of the concentration response, which determines the sound absorption, is expressed directly in terms of f_W:

$$K_q''(\omega)=\frac{1}{e\bar{\varphi}_0}\int_0^{2\pi}\left(\frac{-\sin\xi}{\pi}\right)\sum_\alpha\left[f_W(\xi,p_z,\alpha_\perp)-F_0(\varepsilon_\alpha)\right]d\xi,\qquad(28)$$

where F_0 is the Fermi distribution function, which depends on the energy of the electron in the magnetic field

$$\varepsilon_\alpha=n\hbar\Omega+p_z^2/2m.\qquad(29)$$

One can show (similar to[9]) that upon satisfaction of the condition

$$\hbar\Omega\ll\max\left[\varepsilon_F\cdot(T,e\bar{\varphi}_0,\hbar/\tau)\right]^{1/2}\qquad(30)$$

in the case of scattering by acoustical phonons and neutral point impurities, the kinetic equation for f_W has the form

$$\frac{\partial f_W}{\partial t}+\frac{p_z}{m}\frac{\partial f_W}{\partial z}+\frac{1}{q\hbar}\frac{\partial e\bar{\varphi}}{\partial z}\left\{f_W\left(p_z-\frac{\hbar q}{2}\right)-f_W\left(p_z+\frac{\hbar q}{2}\right)\right\}+\hat{I}f_W=0.1)$$

where, for $\hbar q \ll p_z$, the operator I represents the classical collision operator. We shall seek the Wigner density f_W in the form

$$f_W=F_0(\varepsilon_\alpha+e\bar{\varphi})+g_W(t,z,p_z,\alpha_\perp),\qquad(32)$$

where g_W satisfies the equation

$$\frac{\partial g_W}{\partial t}+\frac{p_z}{m}\frac{\partial g_W}{\partial z}+\frac{1}{q\hbar}\frac{\partial e\bar{\varphi}}{\partial z}\left\{g_W\left(p_z-\frac{\hbar q}{2}\right)-g_W\left(p_z+\frac{\hbar q}{2}\right)\right\}+\hat{I}g_W$$
$$=w\frac{\partial e\bar{\varphi}}{\partial z}\frac{\partial F_0}{\partial\varepsilon}(\varepsilon_\alpha+e\bar{\varphi}).\qquad(33)$$

Inasmuch as the reactive part of the concentration response is expressed in terms of the function g_W

$$K_q''(\omega)=\int_0^{2\pi}\left(\frac{-\sin\xi}{\pi}\right)\frac{eH}{(2\pi)^2\hbar^2ce\bar{\varphi}_0}\sum_{n\sigma}g_W(\xi,p_z,n,\sigma)\,d\xi,\qquad(34)$$

the study of the damping reduces to the solution of (33).

We now consider in more detail the expression for the collision operator entering into (33). This problem is important in connection with the discussions which developed several years ago relative to the conditions for the existence and line shape of the giant oscillations. The reason for the discussion was the difference in results obtained by the Green's function method[8,9] and by the solution of the kinetic equations for the density matrix.[10-12] In our opinion, the divergence of the results was connected with the fact that the relaxation time approximation was incorrectly used in[10-12].

For quasi-elastic scattering, which takes place in most cases of interest, the collision operator can be represented in the form

$$\hat{I}f=\sum_{\alpha'}(W_{\alpha'\alpha}f_\alpha-W_{\alpha\alpha'}f_{\alpha'})=\frac{1}{\tau_\alpha}f_\alpha-\sum_{\alpha'}W_{\alpha\alpha'}f_{\alpha'},\qquad(35)$$

where $W_{\alpha\alpha'}$ is the transition probability and τ_α the lifetime of state α. The relaxation time approximation used in a number of researches including[10-12] reduces to representing the distribution function in the form $f = F_0(\epsilon) + f^{(1)}$, and replacing the collision integral by a multiplication operator, i.e., we assume

$$\hat{I}f^{(1)}=f^{(1)}/\tau=(f-F_0)/\tau.\qquad(36)$$

However, by virtue of the quasi-elasticity of the scattering the relaxation time of the part of the distribution function $f^{(1)}$ (averaged over the surface of constant energy) significantly exceeds the relaxation time of the anisotropic part (see, for example,[13]), which is of the order of τ_α. Therefore the approximation (36) is suitable only in the case in which the part of the nonequilibrium contribution $f^{(1)}$ (averaged over the surface of constant energy) is negligibly small, for example, in the problem of electrical conductivity in a weak electric field. In the problem of sound absorption, the part of the contribution $f^{(1)}$ averaged over the surface of constant energy is seen to be very important and determines the screening. In such a situation, Eq. (36) is shown to be incorrect, which evidently leads also to error in the results of[10-12]. Writing the Wigner density in the form (32), we include the isotropic part of the nonequilibrium Wigner density in the first term. Here, in the problem of the calculation of the damping, we can neglect the next term of the collision operator in the action of it on g_W (just as in the classical theory), i.e., we set

$$\hat{I}g_W=g_W/\tau_p.\qquad(37)$$

The rigorous establishment of this possibility was performed in[4]. Thus, solving Eq. (33) with account of (37), we can calculate $K_q''(\omega)$ and then the absorption coefficient Γ:

$$\Gamma=\chi q\frac{4\pi e^2}{\varepsilon_0 q^2}\frac{K_q''(\omega)}{(1+\varkappa^2/q^2)^2},\qquad(38)$$

where χ is the nondimensional coupling constant, equal to $\Lambda^2\epsilon_0 q^2/4\pi e^2 c$ for deformation interaction and $4\pi\beta^2/\epsilon_0 c$ for piezoelectric interaction.

B. Giant Absorption Oscillations

The solution of Eq. (33) has an essentially different form in regions (3) and (5). The condition (3) denotes the smallness of the indeterminacy of the momentum of the electron in comparison with the transfer of momentum in the interaction with the sound quantum. Therefore, the situation corresponds to quantum mechanical perturbation theory.[3] In case (5), the inverse relation holds and an expansion can be carried out in (33) in terms of $\hbar q$, which corresponds to the theory constructed in[4]. We note that Eq. (33) depends on the magnetic field as a parameter, owing to the dependence of the right side on ϵ_α. Therefore, we can use the solutions obtained in[3,4] in the absence of a magnetic field, and then carry out summation over the transverse quantum numbers to obtain (34).

a) Region (3). According to[3], the part of the Wigner density

$$g_W{}^1=\frac{q/\tau_p}{(qv-\omega)^2+\tau_p^{-2}(1+2d^2)}w\frac{\partial F_0}{\partial\varepsilon}\sin\xi,\qquad v=\frac{p_z}{m}.\qquad(39)$$

makes the contribution to the sound damping. Using (39), (34) and (38), we get the following expression for the ratio of the nonlinear absorption coefficient Γ to Γ_0:

$$\frac{\Gamma}{\Gamma_0}=\frac{\hbar\Omega}{8T}\frac{1}{\pi}\int_{-\infty}^{\infty}dv\,D_1(v)D_2(v),\qquad(40)$$

$$D_1 = \frac{q\tau_p^{-1}}{(qv-\omega)^2 + \tau_p^{-2}(1+2d^2)}, \qquad (41)$$

$$D_2 = \sum_{n\sigma} \mathrm{ch}^{-2}\left(\frac{\varepsilon_F - n\hbar\Omega - \mu_0 H\sigma - {}^1/_2 mv^2}{2T}\right). \qquad (42)$$

The expression for Γ/Γ_0 differs from that obtained in[8] by the coefficient $(1 + 2d^2)^{-1/2}$ and by the replacement of τ_p^{-1} by $\tau_p^{-1}(1 + 2d^2)^{1/2}$. Thus, the increase in sound intensity leads to a decrease in the amplitude of the oscillations and, upon satisfaction of the condition $(e\tilde{\varphi}_0)^2/\hbar\tau_p^{-1} \gtrsim 1$, to an increase in their width. Correspondingly, the necessary and sufficient condition for the existence of giant oscillations takes the form

$$\frac{\hbar\Omega}{\sqrt{Tm(e\tilde{\varphi}_0/\hbar q)^2}} \gg 1 \quad \text{or} \quad \frac{e\tilde{\varphi}_0}{\hbar\Omega} \ll \frac{\hbar q}{\sqrt{mT}}. \qquad (43)$$

b) Region (5). Assuming the condition of strong nonlinearity to be satisfied and using the expression obtained for g_W in[4], we have

$$\frac{\Gamma}{\Gamma_0} = a\frac{\hbar\Omega}{8T}\frac{1}{\pi^2}\int_{-1}^{\infty} dE\, D_3 D_4, \qquad (44)$$

where

$$D_3 = \Theta(E-1)\int_0^{2\pi} \frac{\sin\xi}{\sqrt{2(E-\cos\xi)}}\,\frac{\sqrt{E+1}\,d\xi}{2K(\sqrt{2/(E+1)})}\int_0^{2\pi}\frac{\xi'\,d\xi'}{\sqrt{E-\cos(\xi+\xi')}}$$

$$+\,\Theta(1-E)\int_{(C)} \frac{\xi\sin\xi}{\sqrt{2(E-\cos\xi)}}\,d\xi,$$

$$\Theta(E) = \begin{cases} 1 & \text{for } E>0 \\ 0 & \text{for } E<0 \end{cases}; \qquad (45)$$

$$D_4 = \sum_{n\sigma} \mathrm{ch}^{-2}\left(\frac{\varepsilon_F - n\hbar\Omega - m\tilde{v}^2 E - \mu_0 H\sigma}{2T}\right), \qquad (46)$$

$\tilde{v} = \sqrt{e\tilde{\varphi}_0/m}$ is the characteristic velocity of the trapped electrons, $E = v^2/2\tilde{v}^2 + e\tilde{\varphi}(\xi)/e\tilde{\varphi}_0$ their dimensionless energy, K the complete elliptic integral. The region of integration C over ξ in the second integral is selected from the condition of finiteness of the motion of the electrons in the field of the wave $E = \cos\xi \geq 0$. The function D_3 is a continuous function of E; its approximate plot is given in Fig. 3.

It is not difficult to establish the fact that the giant oscillations exist in the case in which the effective width of the function D_3 does not exceed the distance between peaks of the function D_4. This condition can be written in the form

$$\hbar\Omega / e\tilde{\varphi}_0 \gtrsim 1. \qquad (47)$$

It is seen that the condition is both necessary and sufficient.

The behavior of the oscillation picture with increase in sound intensity is determined essentially by the experimental situation. We consider several typical examples.

1. The case $\hbar^2 q^2/m \ll \hbar/\tau_p$. For $e\tilde{\varphi}_0(ql)^2/\epsilon_F \ll 1$, the linear theory is valid. If $ql\sqrt{\hbar\Omega/\epsilon_F} \gg 1$ and $m/(q\tau_p)^2 \ll T$, then a decrease of the amplitude of the oscillations is first observed with increase in the sound intensity; for $e\tilde{\varphi}_0 \sim T$, the oscillation peaks begin to broaden and for $e\tilde{\varphi}_0 \gg T$, their shape is determined by the function D_3. Thus, the distribution of the resonance electrons can be decided from the shape of the oscillations. If the condition $m/(q\tau_p)^2 \gtrsim T$ is satisfied, then the broadening begins simultaneously with the decrease in the amplitude. For $e\tilde{\varphi}_0 \sim \hbar\Omega$, the oscillations vanish and the absorption coefficient goes over to the value obtained in[4].

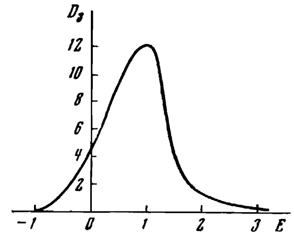

FIG. 3

If $ql\sqrt{\hbar\Omega/\epsilon_F} \lesssim 1$, then the oscillations vanish and the absorption coefficient reaches the value obtained in[4] simultaneously with satisfaction of the condition of strong nonlinearity.

2. The condition $\hbar^2 q^2/m \gg \hbar/\tau_p$. If $e\tilde{\varphi}_0 \ll \hbar^2 q^2/m$, then the absorption coefficient is described by the formulas obtained in Item a). The growth of the intensity in this region leads to a decrease in the amplitude and increase in the width of the oscillation peaks. If $e\tilde{\varphi}_0 \gg \hbar^2 q^2/m$, then the behavior of the oscillation picture is completely analogous to that described in Item 1.

In conclusion, we estimate the possibility of experimental observation of the obtained dependences. The giant sound absorption oscillations can be observed both in metal and also in degenerate semiconductors with high mobility. Thus, in n-type indium antimonide, with electron concentration 10^{15} cm^{-3} and mobility 2×10^5 cm^2/V-sec at T = 1°K and $\hbar\Omega/T = 5$, sound absorption oscillations, of frequency 9 GHz, which corresponds to the condition $q = \kappa$, should be observed at low sound intensities and disappear for $S \sim 2$ W/cm^2. A more favorable situation for the observation of nonlinearity in the regime of giant oscillations apparently exists in semimetals, since in them these effects should appear at very low sound frequencies. Thus, in a semimetal with a free path length of the order of 10^{-3} cm and constant deformation potential of 6 eV, observation of the disappearance of the sound absorption oscillations of frequency 100–200 MHz at T = 1°K and $\hbar\Omega/T = 5$, a sound intensity of the order of 1 W/cm^2 is required.

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