PRODUCTION OF TRIPLETS BY POLARIZED PHOTONS

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In the limit of high photon energy we have obtained the differential cross section $d\sigma/d\varphi$ in the laboratory system as a function of the azimuthal angle for production of triplets by linearly polarized photons, taking into account all triplet-production events with recoil-electron momenta q greater than some value q_0. The recoil-electron momentum distribution in the unpolarized-photon case agrees with the Suh-Bethe formulas. The distribution in azimuthal angle, taking into account all recoil-electron momenta, agrees with that obtained previously by Boldyshev et al. The distribution in the polar emission angle of the recoil electron is discussed. The expression obtained for $d\sigma/d\varphi$ can be used to determine the degree of linear polarization of a photon beam in experiments with track detectors.

INTRODUCTION

THE theoretical investigation of photoproduction of an electron-positron pair in the field of an electron (a triplet) involves great computational difficulties, since this question is described by eight third-order diagrams. Only expressions for the differential cross sections [1,2] have been obtained in analytical form with inclusion of all eight diagrams. It has not been possible to obtain distributions of any kind without extremely crude approximations.

The work of Borsellino [3] corresponds to inclusion of the diagrams of Fig. 1a. He obtained expressions for the differential cross section and distribution in the recoil-electron momentum q. This expression is very awkward, and only after simplification of it by Suh and Bethe [4] did it take a form suitable for comparison with experiment. For photon energies $\omega > 100$ MeV and recoil-electron momenta in the region $q \ll \omega$, $\omega q \gg 1$ (the electron mass is taken as unity; h = c = 1) the Suh-Bethe distribution does not depend on photon energy. The logarithmic rise of the total cross section with photon energy is due to the contribution of events with small recoil-electron momenta, close to the minimum value.

The results of Mork [5] were obtained by numerical integration of the differential cross section, taking into account all eight diagrams [2]. It was shown that for $\omega > 8$ MeV the main contribution is from the diagrams of Fig. 1a. The contribution of the remaining diagrams and their interference is negligible. The numerically calculated recoil-electron momentum distribution given by Mork is in good agreement with the results of Suh and Bethe and with the results of experiments [6-8].

Kopylov [9] showed by numerical integration by the Monte Carlo method that in the laboratory system, beginning already at comparatively low photon energies $\omega \lesssim 60\ mc^2$, the recoil electron has preferentially a momentum of the order of the electron mass and is emitted at a large polar angle θ to the incident-photon direction. The experimental distributions in the polar angle θ [7,8], like the distribution in recoil-electron momentum, do not depend on photon energy and have a broad maximum in the vicinity of 50°. There are no

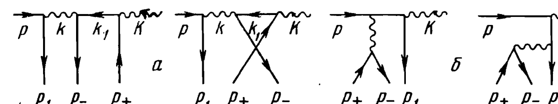


FIG. 1

theoretical calculations of these distributions in the literature.

Production of triplets by linearly polarized photons has been studied by Boldyshev and Peresun'ko [10]. It was shown that the recoil-electron yield has an azimuthal asymmetry, which it is proposed to use to measure the degree of polarization of a beam of linearly polarized photons. However, in experiments with track detectors where electrons with momenta less than tenths of mc are not recorded, use of Boldyshev's results is impossible, since they contain the contributions of undetected electrons with low momenta.

In the present work we have used the method of Sudakov's variables [11] in the high-photon energy limit to calculate asymptotically the main contribution (with accuracy to terms $1/\omega$) to the distribution of triplets produced by linearly polarized photons as a function of the perpendicular component of recoil-electron momentum for recoil momenta which are small $(q \sim 1/\omega)$ and finite $(q \sim 1)$. Using the relation between the value of recoil-electron momentum and the polar angle at which it is emitted, we have obtained expressions for $d^2\sigma/dqd\varphi$ and $d^2\sigma/d\theta d\varphi$ (φ is the azimuthal angle of recoil-electron emission).

By integration of the distribution $d^2\sigma/dqd\varphi$ over q we have obtained cross sections differential in φ for production of triplets with recoil-electron momenta greater than a certain value q_0. The expressions given for the azimuthal asymmetry coefficient can be used to measure the degree of polarization of linearly polarized photon beams. The expression for the azimuthal-angle distribution taking into account all recoil momenta agrees with the asymptotic expression obtained by Boldyshev and Peresun'ko.

1. KINEMATICS

We will carry out the calculations in the rest system of the initial electron. Following Sudakov [11], we will expand the momenta of the virtual particles in the momenta of the initial particles[1)]:

$$k = \alpha K + \beta p' + k_\perp, \quad Kk_\perp = p'k_\perp = 0, \quad p' = p - s^{-1}K, \tag{1}$$
$$1 + s = (K+p)^2 = 1 + 2\omega, \quad d^4k = {}^1/_2 s\, d\alpha\, d\beta\, d^2k_\perp, \quad p'^2 = K^2 = 0, \ k_\perp{}^2 \leqslant 0.$$

We will systematically discard terms proportional to $1/s$. When the z axis is chosen as the direction of the incident photon, these vectors have the following form in terms of components:

$$p = (p_0, p_z, \mathbf{p}_\perp) = (1, 0, 0, 0), \ p' = ({}^1/_2, -{}^1/_2, 0, 0), \ K = (\omega, \omega, 0, 0), \ k_\perp = (0, 0, \mathbf{k}_\perp).$$

Using the fact that the recoil-electron 4-momentum lies on the mass shell (Fig. 1a):

$$(p-k)^2 = 1 + k_\perp{}^2 - \beta - s\alpha(1-\beta) = 1, \ s\alpha = (k_\perp{}^2 - \beta) / (1-\beta), \tag{2}$$

we have rigorous relations for the value of its spatial part:

$$(\mathbf{p} - \mathbf{k})^2 = k_0{}^2 - \mathbf{k}^2 = (\alpha\omega + {}^1/_2\beta)^2 - \mathbf{k}^2 = {}^1/_4\mathbf{k}^4 - \mathbf{k}^2, \quad \mathbf{k}^2 = (k_\perp{}^2 - \beta^2) / (1-\beta) \tag{3}$$

and for the polar angle (Fig. 2)

$$\sin^2\theta = \mathbf{k}_\perp{}^2 \Big/ \left[\frac{\mathbf{k}_\perp{}^2 + \beta^2}{1-\beta} + \left(\frac{\mathbf{k}_\perp{}^2 + \beta^2}{2 - 2\beta} \right)^2 \right]. \tag{4}$$

The quantity β is related to the invariant mass of the pair produced:

$$(k+K)^2 = k^2 + 2kK = k^2 + s\beta = s_1, \quad \beta = (s_1 - k^2) / s. \tag{5}$$

In an overwhelming majority of pair-production events in the field of an electron, the invariant mass of the pair, like the three-momentum of the recoil electron, is bounded (does not increase with increasing ω). Therefore the quantity β is small:

$$\beta \sim s_1/s \sim 1/\omega \ll 1.$$

For pair-production events with a finite recoil-electron momentum $q \gg 1/\omega$, Eq. (4) can be simplified:

$$\sin^2\theta = \frac{4}{4 + \mathbf{k}_\perp{}^2} = \frac{4}{4 + q^2 \sin^2\theta}, \quad \cos\theta = \frac{\varepsilon - 1}{q}, \quad \varepsilon = (q^2+1)^{1/2}. \tag{6}$$

Equation (6), rewritten in the form $q = 2\cos\theta/\sin^2\theta$, was given earlier by Benaksas and Morrison [8]. In the case of small recoil momenta $k_\perp^2 \ll 1$ we obtain from (4)

$$\sin^2\theta = \mathbf{k}_\perp{}^2 / (\mathbf{k}_\perp{}^2 + s_1{}^2/s^2). \tag{7}$$

2. CONTRIBUTION OF FINITE RECOIL MOMENTA ($q \gtrsim q_0$, $q_0 \gg 1/\omega$)

The square of the modulus of the matrix element corresponding to the diagram of Fig. 1a, summed over final-particle spin states and averaged over initial electron spins, can conveniently be represented as twice the sum of the matrix elements corresponding to the diagrams of Fig. 3, where the cancelled lines correspond to real particles. The δ functions associated with it we include in the element of phase space of the final particles:

$$\begin{aligned}(2\pi)^4\delta(p + K - p_+ - p_-)&\frac{d^3\mathbf{p}_1}{2\varepsilon_1(2\pi)^3}\frac{d^3\mathbf{p}_+}{2\varepsilon_+(2\pi)^3}\frac{d^3\mathbf{p}_-}{2\varepsilon_-(2\pi)^3} = \\ = (2\pi)^{-5}\frac{s}{2}d\alpha\, d\beta\, d^2k_\perp\, &\delta[k_\perp{}^2 - \beta - s\alpha(1-\beta)]\theta(1-\beta)\, d^2k_{1\perp} \\ \times\frac{s}{2}d\alpha_1\, d\beta_1\, \theta(\alpha_1)\theta(1-\alpha_1)&\delta[s\alpha_1(\beta_1+\beta) + (k+k_1)_\perp{}^2 - 1]\delta[s(\alpha_1 - 1)\beta_1 \\ + k_{1\perp}{}^2 - 1] = (2\pi)^{-5}(4s)^{-1}&d^2k_{1\perp}\, d^2k_\perp\, \delta(z + z_1 + A)\delta(z - z_2 + B)\frac{dz_1\, dz_2\, dz}{z}\end{aligned} \tag{8}$$

where we have introduced the variables [12]

$$z = s\alpha_1\beta_1, \quad z_1 = s\alpha\beta_1, \quad z_2 = s\beta_1,$$

and the designations

$$A = k_{1\perp}{}^2 - 1, \quad B = (k + k_1)_\perp{}^2 - 1.$$

The recoil-electron momentum distribution of interest to us takes the form

$$\frac{2\pi\, d^2\sigma}{d\mathbf{k}^2\, d\varphi} = (8\pi s^2\mathbf{k}^4)^{-1}\int_0^{-\infty}\frac{dz}{z} \times \int dz_1\, dz_2\, \delta(z_1 + z + A)\delta(z - z_2 + B)\frac{d^2k_{1\perp}}{2(2\pi)^3}J_{\mu\nu}f_{\mu\nu\rho\sigma}e_\rho e_\sigma{}^*, \tag{9}$$

where

$$J_{\mu\nu} = \frac{1}{2}e^2\sum_{\lambda,\lambda'}\bar{u}^\lambda(p)\gamma_\mu u^{\lambda'}(p_1)\bar{u}^{\lambda'}(p_1)\gamma_\nu u^\lambda(p) = 2\pi\alpha\, \mathrm{Sp}(\hat{p}_1 + 1)\gamma_\nu(\hat{p} + 1)\gamma_\mu \approx \approx 16\pi\alpha p_\mu' p_\nu' \tag{10}$$

is the initial electron-current tensor, e is the photon-polarization vector, $\alpha = e^2/4\pi$, and $f_{\mu\nu\rho\sigma}$ is the square of the matrix element, summed over the spin states of the e^+e^- pair, for production of a pair by two photons, one of which is virtual.

The denominators of the electron propagators in the diagram of Fig. 3, with inclusion of Eq. (8), take the form

$$\begin{aligned} k_1{}^2 - 1 &= s\alpha_1\beta_1 + k_{1\perp}{}^2 - 1 = z + A, \\ (k_1 + k - K)^2 &= s(\alpha_1 - 1)(\beta_1 + \beta) + (k + k_1)_\perp{}^2 - 1 \\ &= (z + z_1)(z - z_2)/z + B = B(A + z)/z. \end{aligned} \tag{11}$$

Making the change of variables

$$z = xA / (1 - x), \tag{12}$$

we rewrite Eq. (9) in the form

$$\frac{2\pi\, d^2\sigma}{d\mathbf{k}^2\, d\varphi} = \frac{2\alpha}{s^2\mathbf{k}^4}\int_0^1\frac{dx}{x(1-x)}\int\frac{d^2k_{1\perp}}{\pi}\left\{-\frac{N_1(1-x)^2}{A^2} - \frac{N_2 x(1-x)}{AB}\right\}; \tag{13}$$

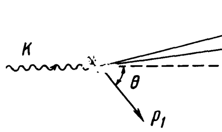
$$\begin{aligned} N_1 &= \mathrm{Sp}\hat{p}'(\hat{k}_1 + 1)\hat{e}(\hat{k}_1 - \hat{K} + 1)\hat{e}^*(\hat{k}_1 + 1)\hat{p}'(\hat{k}_1 + \hat{k} + 1) = \\ &= 2s^2x\left[4x(\mathbf{e}, \mathbf{k}_{1\perp})(\mathbf{e}^*, \mathbf{k}_{1\perp}) - \frac{A}{1-x}\right], \end{aligned} \tag{14}$$

$$N_2 = \mathrm{Sp}\hat{p}'(\hat{k}_1 + 1)\hat{e}(\hat{k}_1 - \hat{K} + 1)\hat{p}'(\hat{k}_1 + \hat{k} - \hat{K} + 1)\hat{e}^*(\hat{k}_1 + \hat{k} + 1) =$$
$$2s^2[(1-2x)^2(\mathbf{e}, \mathbf{k}_{1\perp})(\mathbf{e}^*, \mathbf{k}_{1\perp} + \mathbf{k}_\perp) - (\mathbf{e}^*, \mathbf{k}_{1\perp})(\mathbf{e}, \mathbf{k}_{1\perp} + \mathbf{k}_\perp) + \mathbf{k}_{1\perp}^2 + (\mathbf{k}_\perp, \mathbf{k}_{1\perp}) - 1].$$

Combining in the second term in Eq. (13) the denominators:

$$(AB)^{-1} = \int_0^1\frac{dy}{[Ay + B(1-y)]^2} = \int_0^1\frac{dy}{[(k_1 + ky)_\perp{}^2 + k_\perp{}^2 y(1-y) - 1]^2}, \tag{15}$$

making in Eq. (14) the displacement $\mathbf{k}_1 \to \mathbf{k}_1 - \mathbf{k}y$, in-



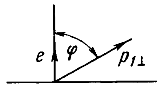


FIG. 2

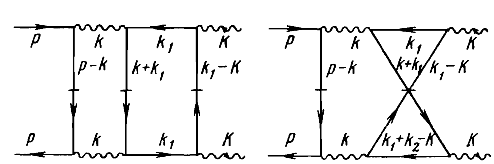


FIG. 3

[1)]The metric is $ab \equiv (a, b) = a_0b_0 - (\mathbf{ab})$.

tegrating over $k_{1\perp}^2$, and, finally, transforming to Euclidean vectors, we obtain[2)]

$$\pi\frac{d^2\sigma}{dk_\perp^2}=\frac{4\alpha r_0^2}{k_\perp^4}\int_0^1 dx\int_0^1 dy\int_0^\Delta dk_1^2\left\{\frac{k_1^2+1-2ak_1^2}{(k_1^2+1)^2}-\right.$$
$$\left.-\frac{-2ak_1^2+k_1^2+1+4ab(\mathbf{e}k_\perp)(\mathbf{e}^*k_\perp)-bk_\perp^2}{[k_1^2+k_\perp^2 b+1]^2}\right\}=$$
$$=\frac{4\alpha r_0^2}{3k_\perp^2}[\Phi_1(k_\perp^2)+(\mathbf{en})(\mathbf{e}^*\mathbf{n})\Phi_2(k_\perp^2)],$$
$$\Phi_1(z)=\int_0^1\frac{dy(1+2b)}{zb+1},\quad \Phi_2(z)=\int_0^1\frac{dy(-2b)}{zb+1}, \tag{17}$$

where

$$a=x(1-x),\ b=y(1-y),\ \mathbf{n}=\mathbf{k}_\perp/|\mathbf{k}_\perp|.$$

Using the correlation of the recoil momentum and the polar angle (6) and introducing the polarization density matrix of the photon

$$\rho_{\mu\nu}=\overline{e_\mu e_\nu^*}=(1/2+1/2\sigma\xi)_{\mu\nu},$$

we rewrite (16) in the following forms:

$$2\pi\frac{d^2\sigma}{d\theta\,d\varphi}=\frac{2\alpha r_0^2}{3}\frac{\sin\theta}{\cos^3\theta}\left\{1-\frac{1-5\cos^2\theta}{\cos\theta}\ln\operatorname{ctg}\frac{\theta}{2}\right.$$
$$\left.-(\xi_3\cos 2\varphi+\xi_1\sin 2\varphi)\left(1-\frac{\sin^2\theta}{\cos\theta}\ln\operatorname{ctg}\frac{\theta}{2}\right)\right\}. \tag{18}$$

$$2\pi\frac{d^2\sigma}{dq\,d\varphi}=\frac{2\alpha r_0^2}{3}\frac{q}{\varepsilon(\varepsilon-1)^2}\left\{1+\frac{2\varepsilon-3}{q}\ln(\varepsilon+q)\right.$$
$$\left.-(\xi_3\cos 2\varphi+\xi_1\sin 2\varphi)\left(1-\frac{1}{q}\ln(\varepsilon+q)\right)\right\}. \tag{19}$$

We note that integration of (19) over φ leads to an expression for the distribution of the recoil-electron momentum q for the case of an unpolarized photon and agrees with the Bethe-Suh formula [4] for finite recoil-electron momenta ($q \sim 1$).

3. CONTRIBUTION OF SMALL RECOIL MOMENTA ($q \ll q_0 \sim 1$)

The photon in diagram 3 can be taken on the mass shell. Using the condition of gradient invariance of the block $f_{\mu\nu\rho\sigma}$:

$$k_\mu f_{\mu\nu\rho\sigma}=k_\nu f_{\mu\nu\rho\sigma}=(p'\beta+k_\perp)_\mu f_{\mu\nu\rho\sigma}=0,$$

we transform the expression entering into Eq. (9)

$$\int d\Phi_{+-}f_{\mu\nu\rho\sigma}J_{\mu\nu}e_\rho e_\sigma^*$$

to the form

$$J_{\mu\nu}\int d\Phi_{+-}f_{\mu\nu\rho\sigma}e_\rho e_\sigma^*=16\pi\alpha\int d\Phi_{+-}f^{0\perp}_{\mu\nu\rho\sigma}\frac{k_\perp^\mu k_\perp^\nu}{\beta^2}e_\rho e_\sigma^*$$
$$=\frac{16\pi\alpha s^2 k_\perp^2}{s_1^2}\int d\Phi_{+-}f^{0\perp}_{\mu\nu\rho\sigma}n_\mu n_\nu e_\rho e_\sigma^*, \tag{20}$$

where

$$s_1=s\beta,\ d\Phi_{+-}$$

is the element of phase space of the pair, and $f^{0\perp}_{\mu\nu\rho\sigma}$ is the square of the matrix element for production of an e^+e^- pair by two real photons, summed over the spin states of the pair. Using the optical theorem we write

$$\int d\Phi_{+-}f^{0\perp}_{\mu\nu\rho\sigma}n_\mu n_\nu e_\rho e_\sigma^*=\frac{s_1}{4\pi}\sigma^{(n,e)}(s_1,\varphi), \tag{21}$$

where φ is the angle between n and e, $\sigma^{(n,e)}$ is the total cross section for production of a pair by two linearly polarized photons with polarization vectors n and e [15]:

$$\sigma^{(n,e)}=\frac{r_0^2 v_0}{4\varepsilon_0^2}\int_{-1}^1 dz\int_0^{2\pi}d\varphi\{\tau^{-1}[1-\tau(\mathbf{ne})^2+4(\mathbf{v}_0\mathbf{n})(\mathbf{v}_0\mathbf{e})(\mathbf{en})]$$
$$-4(\mathbf{v}_0\mathbf{n})^2(\mathbf{v}_0\mathbf{e})^2\tau^{-2}\}, \tag{22}$$

$\tau = 1 - v_0^2z^2$; v_0 and ϵ_0 are the velocity vector and the energy of the electron in the c.m.s. of the pair, and $s_1 = 4\epsilon_0^2$.

Equations (20)—(22) permit us to write down the distribution in the perpendicular component of the recoil-electron momentum:

$$2\pi\frac{d^2\sigma}{dk_\perp^2 d\varphi}=\alpha\int_4^\infty\frac{ds_1}{s_1}\frac{k_\perp^2}{(k_\perp^2+s_1^2/s^2)^2}\sigma^{(n,e)}(s_1,\varphi). \tag{23}$$

We note that if both sides of (23) are integrated over φ, the quantity $\sigma^{(n,e)}$ occurring in the integrand is replaced by the cross section for production of a pair by two unpolarized photons. Integration of the expression obtained over s_1 gives the distribution in the recoil-electron momentum $q \approx |k_\perp|$ for the case of small q, close to the minimum value [4]:

$$\frac{d\sigma}{dq}=\frac{4\alpha r_0^2}{q}\left\{\frac{2}{3}(1-x)^{1/2}\left(\frac{7}{6}+\frac{25}{12}x-\frac{1}{2}x^2\right)-\left[x-\frac{1}{4}x^2\left(1-\ln\frac{4}{x}\right)\right.\right.$$
$$\left.+\frac{1}{6}x^3\right]\ln\frac{1+(1-x)^{1/2}}{1-(1-x)^{1/2}}-\frac{1}{2}x^2\left[L\left(\frac{1+(1-x)^{1/2}}{2}\right)-\right.$$
$$\left.\left.-L\left(\frac{1-(1-x)^{1/2}}{2}\right)\right]\right\},$$

$$x=\frac{2}{\omega q},\quad L(y)=\int_0^y\frac{dx}{x}\ln(1-x),$$

4. DISTRIBUTION IN AZIMUTHAL ANGLE, TAKING INTO ACCOUNT ALL RECOIL-ELECTRON MOMENTA

Let us choose some small value of σ:

$$s^{-\varepsilon}\ll\sigma\ll 1,\quad \varepsilon>0.$$

The number of triplet-production events with a recoil-electron perpendicular momentum component greater than $\sigma^{1/2}$ is obtained by integrating Eq. (16) over $k_\perp^2$ from σ to ∞:

$$2\pi\frac{d\sigma}{d\varphi}=\alpha r_0^2\left\{-\frac{14}{9}\ln\sigma+\frac{82}{27}+\left(\frac{2}{9}\ln\sigma-\frac{10}{27}\right)(\xi_3\cos 2\varphi+\xi_1\sin 2\varphi)\right\}. \tag{24'}$$

Integration of (23) over $k_\perp^2$ from 0 to σ gives the number of events with $k_\perp^2 < \sigma$:

$$2\pi\frac{d\sigma}{d\varphi}=\alpha r_0^2\left\{(\ln s^2\sigma-1)\left[\frac{14}{9}-\frac{2}{9}(\xi_3\cos 2\varphi+\xi_1\sin 2\varphi)\right]-\frac{86}{9}\right.$$
$$\left.+\frac{8}{9}(\xi_3\cos 2\varphi+\xi_1\sin 2\varphi)\right\}. \tag{24''}$$

Adding (24') and (24") we obtain for the distribution in azimuthal angle, taking into account all recoil momenta, the following expression:

$$2\pi\frac{d\sigma}{d\varphi}=\alpha r_0^2\left\{\frac{28}{9}\ln s-\frac{218}{27}-\left(\frac{4}{9}\ln s-\frac{20}{27}\right)\right.$$
$$\left.\times(\xi_3\cos 2\varphi+\xi_1\sin 2\varphi)\right\}. \tag{25}$$

This expression agrees with the result of Boldyshev and Peresun'ko [10].

[2)] Equation (16) can be obtained from the Compton-scattering amplitude, using for the photon impact factor the expression found by Cheng and Wu [13]; it is also possible to use the results of BaÏer et al. and of Constantini et al. [14].

Unpolarized part J_2 of the distribution in azimuthal angle (26), the part J_1 characterizing the photon-beam polarization, and the asymmetry coefficient $I = J_1/J_2$, as a function of the lower limit q_0 for momentum of detected recoil electrons

	q_0 (in units of mc)												
	0.02	0.2	0.5	0.8	1	1.5	2	2,5	3	3,5	4	5	10
J_1	3.62	2.09	1.5	1.23	1,13	0.95	0.848	0.781	0.734	0.7	0,67	0.63	0.55
J_2	26.04	15.32	11,16	9.16	8,54	7.24	6.48	5,98	5,624	5.35	5,147	4.84	4,16
$I = J_1/J_2$	0.139	0.136	0,135	0.134	0.133	0.131	0.130	0.130	0.130	0,130	0.130	0.131	0.132

5. DISCUSSION OF RESULTS AND COMPARISON WITH EXPERIMENT

In experiments with track detectors, events with electron momenta greater than some value q_0 are recorded. In order that Eq. (19) can be used to determine the degree of polarization of photon beams from the azimuthal asymmetry of recoil electron emission, we will integrate it over q from q_0 to ∞. The resulting distribution in azimuthal angle, taking into account all triplet-production events with recoil-electron momentum greater than q_0, has the form

$$\frac{d\sigma}{d\varphi}=\frac{2\alpha r_0^2}{3}\left[J_2(q_0)+J_1(q_0)(\xi_3\cos 2\varphi+\xi_1\sin 2\varphi)\right],$$

$$J_1(q_0)=\int_{q_0}^{\infty}\frac{dq}{\varepsilon(\varepsilon-1)^2}(q-\ln(q+\varepsilon))=\frac{\operatorname{sh}t-t\operatorname{ch}^3 t}{3\operatorname{sh}^3 t}+t\frac{\operatorname{ch}t}{\operatorname{sh}t}-\frac{2}{3}\ln\operatorname{sh}t,$$

$$J_2(q_0)=\int_{q_0} dq\,\varepsilon^{-1}(\varepsilon-1)^{-2}[q+(2\varepsilon-3)\times\ln(\varepsilon+q)]=J_1(q_0)+$$

$$+4t\frac{\operatorname{ch}t}{\operatorname{sh}t}-4\ln\operatorname{sh}t,\qquad t={}^1/_2\varphi_0,\qquad q_0=\operatorname{sh}\varphi_0. \tag{26}$$

Values of J_1, J_2, and $I = J_1/J_2$ are given in the table for various values of q_0. The ratio $I = J_1/J_2$ determines the asymmetry of the recoil-electron yield.

As can be seen from the table, the value of I changes slowly with increasing q_0 from 0.139 at $q_0 = 0.2$ to 0.132 at $q_0 = 10$. This makes possible the use of the recoil-electron azimuthal asymmetry to measure the degree of polarization of beams of linearly polarized photons, having chosen a value $q_0 \geq 1$ MeV/c, as follows from the reasoning given above.

Let us compare the results obtained with the experimental data. We will discuss the first terms in (18) and (19), which correspond to the case of an unpolarized photon, since there are not as yet any experimental data for polarized photons. The momentum distribution given by the first term of (19) agrees with the Suh-Bethe formula, which is in good agreement with experiment. The polar angle distribution given by Eq. (18) is in good agreement with the experimental data [7,8] (see Fig. 4) only for the range of angles $0 < \theta < 50°$, which correspond, according to Eq. (6), to the range of momenta $q > 1$ MeV/c. For larger values of θ corresponding to $q < 1$ MeV/c, the curve of (18) continues to rise, whereas a dropoff is observed experimentally. This difference is apparently due to the rather large (~30% of the events) deviation from Eq. (6) which, according to the data of Benaksas and Morrison [8], is observed in the region 0.4 MeV/c $< q < 1$ MeV/c. This may be the result of multiple scattering of recoil electrons with low momenta. We note only that Eq. (18)

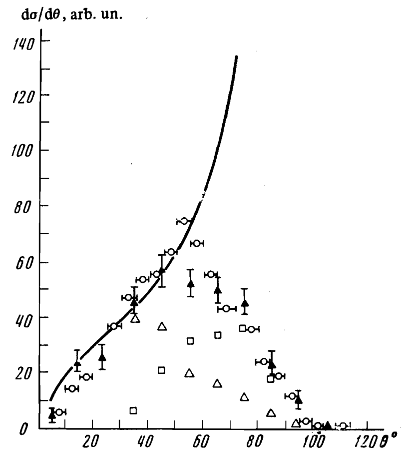


FIG. 4. Distribution of recoil-electron emission with respect to the polar angle θ: solid curve—calculation from Eq. (18). Triangles and squares represent data from the experiment of Benaksas and Morrison [8]: □—in the region 0.4 MeV/c < q < 1 MeV/c; Δ—q > 1 MeV/c; ▲—> 0.4 MeV/c. O—data of Gates et al. [7].

agrees on the average with the experimental data of Benaksas et al. in the region of angles greater than the value corresponding to the maximum ($\theta \sim 55°$). This follows from the equality of the areas under the experimental curve taking into account events with $q > 0.4$ MeV/c and the curve of Eq. (18) in the region of angles $0 < \theta < 70°$ which, according to Eq. (6), corresponds to this momentum interval.

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