FLUCTUATIONS OF THE RADIATION RISE TIME IN A GAS LASER WITH NONLINEAR RESONANT ABSORPTION

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Fluctuations of the transient time in a helium-neon laser at $\lambda = 0.63\ \mu$ with nonlinear resonant absorption are investigated experimentally and theoretically. An expression for the probability density function for the transient period in the "soft" and prehysteresis excitation regimes is obtained by means of the Fokker-Planck equation. The mean transient period and its dispersion are calculated. The theoretical calculations are in agreement with the experimental data. Transition phenomena in an ordinary laser and in a nonlinear resonant absorption laser are compared.

1. INTRODUCTION

THE dynamics of the electromagnetic field in a resonator is determined essentially by the dependence of the effective gain on the electromagnetic-field intensity. This dependence usually has the form of a monotonically decreasing curve, and can be converted into a curve with a maximum by introducing an absorbing cell into the laser resonator. In this case, as is well known,[1,2] a "hard" regime of oscillation excitation is possible. Therefore the transient processes in lasers with nonlinear resonant absorption (NRA) have their own distinguishing features.

It is known[3-8] that statistical phenomena occur in a usual helium-neon laser during the time of the transient processes and become strongly manifest near the excitation threshold. Consequently it is natural to expect statistical phenomena to accompany the unique features of the dynamics of generation in the nonstationary regime of a laser with NRA. In the stationary regime, the field fluctuations in a laser with NRA were investigated theoretically by Kazantsev and Surdutovich[9], who found significant differences between the photon statistics in an ordinary laser and in a laser with NRA.

In the present paper we study the statistical phenomena in the transient of a gas laser with nonlinear absorption. The theoretical analysis is based on the classical Fokker-Planck equation. Unlike other authors, we solve it by using the Laplace transformation[10]. Although it is impossible to obtain an exact solution in this case, too, such a solution method seems to us simpler and more lucid. The experiments were performed with a helium-neon laser ($\lambda = 0.63\ \mu$) with an absorbing cell in the resonator and in a "soft" excitation regime. We investigated the most interesting cases of small values of the unsaturated effective gain and the pre-hysteresis regime.

2. THEORY

We start with the theory developed by Baklanov, Rautian, et al.[5] for the case of an ordinary laser. Their formulas (2.1)–(2.5) remain unchanged in our case, but the polarizability χ should contain a second correction for the saturation effect both in the amplifying and in the absorbing medium[1,2,11]:

$$\chi_i = \chi_{0i} - \varkappa_i |\mathscr{E}|^2 + \theta_i |\mathscr{E}|^4.$$

(the subscripts i = 1, 2 refer all the quantities to the active and passive parts, respectively). In addition, it depends significantly on the difference Ω between the frequency ω_{mn} of the atomic transition and the generation frequency ω, and on the shifts Δ_1 of the maxima of the gain lines and Δ_2 of the absorption lines relative to ω_{mn}.

The equation of motion for the field amplitude $\mathscr{E}(t)$ then takes the form

$$\dot{\mathscr{E}} - {}^1/_2\{4\pi\omega[(\chi_{01} - \chi_{02}) - (\varkappa_1 - \varkappa_2)|\mathscr{E}|^2 + (\theta_1 - \theta_2)|\mathscr{E}|^4] - \omega/Q\}\mathscr{E} = 2i\pi\omega\mathscr{P}_s(t). \tag{1}$$

We express $\mathscr{E}(t)$ in units of $\sqrt{\kappa_1}$ and the time in units of ω/Q, so that we obtain from (1)

$$\dot{\mathscr{E}} - {}^1/_2\{\alpha + \beta|\mathscr{E}|^2 - \gamma|\mathscr{E}|^4\}\mathscr{E} = f(t). \tag{2}$$

The factor preceding $\mathscr{E}$ has here the meaning of the effective gain, and we have introduced the notation

$$f(t) = 2\pi i\varkappa_1^{1/2} Q\mathscr{P}_s(t);$$
$$\alpha = 4\pi Q(\chi_{01} - \chi_{02}) - 1, \quad \beta = 4\pi Q(\varkappa_2/\varkappa_1 - 1), \tag{3}$$

$$\gamma = \frac{4\pi Q}{\varkappa_1^2}(\theta_2 - \theta_1). \tag{4}$$

The coefficients α, β, and γ in (4) coincide, apart from the sign, with the coefficients c, b, and a respectively of[2], and have the same meaning.

We shall assume that the parameters of the amplifying and absorbing medium are chosen in such a way that the plot of the effective gain against the field has the form of a parabola, i.e., $\beta \geq 0$ and $\gamma > 0$. Following[5], we assume that the noise is δ-correlated:

$$\langle f^*(t)\ f(t')\rangle = I\delta(t - t'), \tag{5}$$

where I is the rate at which the photons begin to participate in the chosen oscillation mode. Then application of the standard method[12,13] to (2) leads to the Fokker-Planck equation

$$\frac{\partial W}{\partial t} + \frac{\partial}{\partial n}\{(\alpha + \beta n - \gamma n^2)nW\} = I\frac{\partial}{\partial n}\left(n\frac{\partial W}{\partial n}\right), \tag{6}$$

where the number of photons n is proportional to $\mathscr{E}^2$.

Our problem is to obtain a solution of (6) such as to satisfy the initial function W(n, 0) describing the photon distribution in the laser below the generation threshold.

The form of the function W(n, 0) can be easily obtained by assuming that the stationary conditions below the threshold had time to become established at[1] $\alpha = -\alpha_1$, i.e., $\partial W/\partial t = 0$. The solution of (6) can be simplified by using the obvious assumption that the number of induced photons far below the threshold is small, and the saturation of the media can be neglected. Then we obtain, in accord with[9]

$$W(n, 0) = (\bar{n}_1)^{-1} \exp\{-n/\bar{n}_1\}, \tag{7}$$

where $\bar{n}_1 = I_1/\alpha_1$ and I_1 is the value of I below the threshold.

Let us consider, further, the growth of the generation intensity when $\alpha = \alpha_2 > 0$ is turned on. (As shown in[2], the change of β and γ following the switching of α can be neglected.) It is impossible to solve exactly the equation (6) describing this process, and we shall therefore make the same assumptions as in[5,7], which, as will be shown later on, are perfectly realistic in the case of a laser with NRA. At sufficiently short times (in the low intensity region), the laser operates like a linear amplifier, and we can therefore disregard saturation and put

$$\alpha_2 \gg \gamma n^2 - \beta n. \tag{8}$$

Equation (6) then takes the form

$$\frac{\partial W}{\partial t} + \frac{\partial}{\partial n}(\alpha_2 n W) = I_2 \frac{\partial}{\partial n}\left(n \frac{\partial W}{\partial n}\right), \tag{9}$$

where I_2 is the value of I above the threshold.

To solve it, following Feller[10], we introduce the Laplace transformation

$$w(s,t) = \int_0 W(n,t)e^{-sn}\,dn.$$

We then obtain from (9)

$$\frac{\partial w}{\partial t} + s(I_2 s + \alpha_2)\frac{\partial w}{\partial s} = -I_2 s w + \varphi(t). \tag{10}$$

In our case the function

$$\varphi(t) = -\lim_{n\to 0}\left(I_2 \frac{\partial W}{\partial n} + sI_2 W - \alpha_2 W\right) n$$

the function is equal to zero by virtue of the properties of W(n, t).

The solution of (10) with allowance for the initial distribution (7) has the simple form

$$w = 1/(1 + s\bar{n}),$$
$$\bar{n} = [I_1/\alpha_1 + I_2(1 - e^{-\alpha_2 t})/\alpha_2]e^{\alpha_2 t}. \tag{11}$$

After taking the inverse Laplace transform, we obtain

$$W(n, t) = (\bar{n})^{-1} \exp\{-n/\bar{n}\}. \tag{12}$$

For times satisfying the condition $\exp(-\alpha_2 t) \ll 1$, we have

$$\bar{n}(t) = \bar{n}_0 e^{\alpha_2 t}, \; \bar{n}_0 = I_1/\alpha_1 + I_2/\alpha_2. \tag{13}$$

Then (12) becomes

$$W(n,t) = \frac{1}{\bar{n}_0 e^{\alpha_2 t}} \exp\left\{-\frac{n}{\bar{n}_0 e^{\alpha_2 t}}\right\}. \tag{14}$$

This formula can be obtained from the Fokker-Planck equation (9) without taking into account the diffusion term, if the initial distribution (7) is redefined by replacing $\bar{n}_1$ by $\bar{n}_0$ in (12). Consequently, at $t > 1/\alpha_2$ we can neglect the term with I_2 in (6), and the influence of the noise at $t > 0$ reduces to an effective increase of the initial priming signal by an amount I_2/α_2, just as in the case of an ordinary laser[5]. Thus, the initial period of the growth of the generation in a laser with NRA in the "soft" excitation regime seems to be the same as in an ordinary laser.

In the limiting case $\alpha_1 \gg \alpha_2$, when the priming signal for the development of the generation is the field radiated spontaneous at $t > 0$, assuming $I_1 \sim I_2$, formula (12) coincides, apart from the notation, with the result of Risken and Vollmer[3] for short times, under the condition that there are no photons in the resonator at the instant when the Q is switched on, i.e.,

$$W(n, 0) = \delta(n). \tag{15}$$

The considered linear part of the transient processes takes place when n satisfies the condition (8); the latter can be rewritten in the form

$$n \ll n_2.$$

Here n_2 is the number of photons in the stationary generation regime and is the largest root of the equation $\gamma n^2 - \beta n - \alpha_2 = 0$:

$$n_{1,2} = (\beta \mp \Delta)/2\gamma, \quad \Delta = (\beta^2 + 4\alpha\gamma)^{1/2}. \tag{16}$$

Thus, by virtue of (13), the linearity condition means

$$[I_1/\alpha_1 + I_2/\alpha_2]e^{\alpha_2 t} \ll n_2. \tag{17}$$

As in an ordinary laser, n_2 exceeds the priming number of photons by many orders: $n_2 \gg \bar{n}_0$. Therefore, there exists times such that

$$1/\alpha_2 \leqslant t < (1/\alpha_2)\ln(n_2/n_0), \tag{18}$$

for which the linear approximation is still justified.

We consider now the nonlinear period of development of generation, when effects of saturation are significant both in the amplifying and in the absorbing media. Since at values of t satisfying the condition (18) the second priming signal has already been fully formed, one can disregard the succeeding spontaneous emission. The reason is that the noise radiated prior to $t \sim 1/\alpha_2$ had time to become amplified by a factor $\exp(\alpha_2 t)$ and play a much larger role than the noise emitted after $t \sim 1/\alpha_2$[5]. We assume that the condition (18) is satisfied and we operate with equation (6) without allowance for the noise:

$$\frac{\partial W}{\partial t} + \frac{\partial}{\partial n}[(\alpha_2 + \beta n - \gamma n^2) n W] = 0. \tag{19}$$

In accordance with the foregoing, the initial distribution takes the form

$$W(n) = \frac{1}{\bar{n}_0}\exp\left\{-\frac{n}{\bar{n}_0}\right\}, \quad \bar{n}_0 = \frac{I_1}{\alpha_1} + \frac{I_2}{\alpha_2}. \tag{20}$$

The first integrals of (19) are

$$c_1 + t = \frac{1}{2\alpha_2}\ln\left|\frac{n^2}{\gamma(n+\tilde{n}_1)(n-n_2)}\left(\frac{n-n_2}{n+\tilde{n}_1}\right)^m\right|, \quad c_2 = \ln|W\gamma n(n+\tilde{n}_1)(n-n_2)|, \tag{21}$$

where $\tilde{n}_1 = |n_1|$ and $m = \beta/\Delta$.

The general form of W can be obtained from (21) only at m = 1, 1/2, 1/3, and 0. From the definition of m it is seen that at fixed β and γ, its value can range from 1

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[1]For concreteness, we put $\alpha_1 \geqslant \beta^2/4\gamma$, corresponding to placing the entire effective-gain curve below the $\mathcal{E}^2$ axis.

to 0, depending on α_2. The case m = 1 corresponds to the ideal condition $\alpha_2 = 0$, which is not realized experimentally because of the fluctuations of n. In addition, formulas (21) was obtained for arbitrary $\alpha_2 \neq 0$. Inasmuch as the essential singularities of the transient processes should become manifest at small α_2 in a laser with NRA, we consider the case m = 1/2. It is then easy to obtain from (20) and (21) the solution of (19).

To characterize the fluctuations of the time establishment of generation by means of the known relations, we proceed to another function, the probability density for the appearance of a given number of photons in the time interval t, t + Δt:

$$W(t)=\alpha_2\frac{n_2}{\bar{n}_0}\eta\exp\left\{-\alpha_2 t-\frac{n_2}{\bar{n}_0}\eta\, e^{-\alpha_2 t}\right\}, \tag{22}$$

$$\eta=2n/(n_2+3n)^{3/4}(n_2-n)^{1/4}.$$

This result does not differ in form from the corresponding formula in[5]. The maximum value of the probability density in both cases is the same:

$$W_m=\alpha_2/e, \tag{23}$$

but it is now reached at a different value of the time than in an ordinary laser:

$$t_m=\frac{1}{\alpha_2}\ln\frac{n_2}{\bar{n}_0}\eta. \tag{24}$$

The use of (23) and (24) transforms (22) into

$$W(t)=W_m\exp\{-\alpha_2(t-t_m)+[1-\exp\{-\alpha_2(t-t_m)\}]\}. \tag{25}$$

The average time and variance, calculated with (25), are

$$\bar{t}=C/\alpha_2+t_m,\quad \sigma_t^2=\pi^2/6\alpha_2^2, \tag{26}$$

where C is the Euler constant and t_m is defined in (24).

In conclusion, we consider the fluctuations of the transient time of a laser with NRA in the pre-hysteresis regime ($\beta = 0$)[2]. In this case the solution of the problem (19) and (20) can be carried out in the general form. As a result we obtain

$$W(t)=\frac{n_\infty}{\bar{n}_0}\alpha_2\eta\exp\left\{-\alpha_2 t-\frac{n_\infty}{\bar{n}_0}\eta\, e^{-\alpha_2 t}\right\}, \tag{27}$$

$$n_\infty=(\alpha_2/\gamma)^{1/2},\quad \eta=n/(n_\infty^2-n^2)^{1/2}.$$

Introducing

$$W_m=\frac{\alpha_2}{e},\quad t_m=\frac{1}{\alpha_2}\ln\frac{n_\infty}{\bar{n}_0}\eta, \tag{28}$$

we obtain for W(t) a formula that coincides with (25). The expressions for the average time and the variance are in this case analogous to (26).

Thus, the expression for the distribution density of the growth time of the generation W(t) in a laser with NRA has the same form as in an ordinary laser. They differ essentially in the parameter t_m.

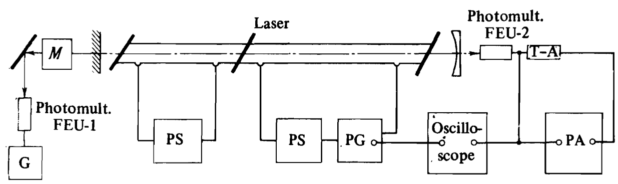


FIG. 1. Diagram of experimental setup. PS—power supply, PG—pulse generator, M—monochromator, G—Galvanometer, T-A—time-amplitude converter, PA—pulse analyzer.

3. EXPERIMENT

We investigated experimentally the transient process in a laser with NRA with the gain turned on, and the accompanying statistical phenomena, using the setup described in Fig. 1.

The laser construction was the same as that used by Lisitsyn and Chebotaev[1,11]; there were slight differences in the geometrical dimensions. Measures were taken to eliminate vibrations. A dc discharge was excited in both the absorbing and the amplifying tubes. We used a natural mixture of neon isotopes. The neon pressure was 0.3 Torr in the absorbing tube and ~ 0.1—0.2 in the amplifying tube, while the helium pressure was 2—3 Torr. At these pressures, the discharge currents were adjusted to be able to observe current hysteresis[1].

To switch the effective gain, an additional rectangular voltage pulse from pulse generator PG was applied to the supply circuit of the amplifying tube in accordance with the scheme described in[5]. The pulse amplitude and the discharge direct current used in the gain could be chosen such that the plot of the effective gain against $\mathscr{E}^2$, shown in Fig. 2, changed jumpwise from position 6 to position 2 (the figure is taken from[2]). The growth of the generation intensity was registered with an FÉU-15 photomultiplier and was observed on the screen of an oscilloscope synchronized with the pulse generator. The corresponding oscillogram is shown in Fig. 3a. Approximately 25 switchings were performed during the exposure time.

Figure 3b shows an oscillogram of the growth of the generation in the same laser, but with the absorbing cell turned off. The amplitude of the pulse from the pulse generator and the direct current of the discharge were then chosen such that the average intensities of the spontaneous priming signal prior to turning on the

FIG. 2. Dependence of the effective gain on the square of the electric field amplitude for different values of the unsaturated gain and unsaturated absorption. Curve 1 corresponds to complete absence of absorption. f is the loss level in the resonator.

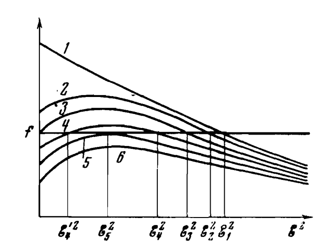
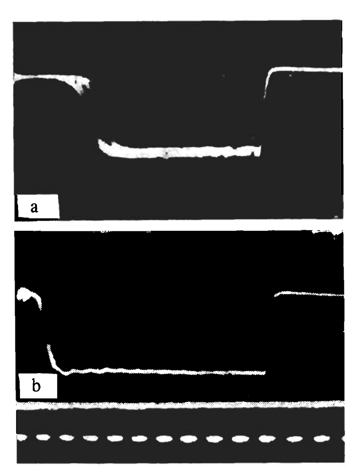


FIG. 3. Growth of the generation intensity in a laser with NRA (a) and in an ordinary laser (b) (the intensity increases in the downward direction). The interval between points is 20 μsec.



gain and of the stationary generation were equal in both lasers. The former was monitored with monochromator M, photomultiplier FEU-15, and galvanometer G.

When operating with a laser without an absorbing cell, the generation frequency was tuned to the center of the gain line, and when operating with the cell it was tuned to the generation power peak.

The generation in a laser with absorption builds up differently from an ordinary laser. In the former both the average transient time of the oscillations and its variance are much larger than in the latter, and the nonlinear growth is slower. This agrees with formula (26) and is connected with the fact that at the same stationary generation intensity the unsaturated gain in a laser with absorption is smaller than in an ordinary laser (the latter is obvious from a comparison of curves 2 and 3 with curve 1 on Fig. 2, when there intersection with the line f occurs at the same point). Thus, it can be assumed that the main assumptions of the theory of[5], which we have chosen as our basis, are confirmed also in the case of a laser with NRA in the "soft" excitation regime.

To determine experimentally the form of the distribution W(t), the signal from the photomultiplier FEU-2 was simultaneously applied to a time-amplitude converter (T—A) with a threshold device, where the scatter in the generation growth time relative to a chosen (adjustable) level of the amplitude of the stationary generation was converted uniquely into an amplitude scatter of standard electric pulses. Their distribution was determined by an AI-100-1 pulse analyzer, so that the channel numbers correspond to a time scale $\sim 1\ \mu$sec per channel. The converter operating threshold and the amplitude of the stationary generation were monitored directed with an oscilloscope. To obtain each distribution, we chose approximately 10^6 pulses. The characteristic form of the distribution of the transient time W(t) at the $0.5n_2$ level, obtained from the analyzer screen, is shown in Fig. 4a. For a comparison, Fig. 4b shows photographs of a similar distribution of the establishment time in an ordinary laser. The average intensity of the first primer and the time scale of both cases are the same, but for an approximate alignment of the maxima of the distributions the amplitude of the stationary generation level was decreased in the latter case by a factor of 10. It should be noted that the track density on Fig. 3 represents quite fully the form of the distributions in Fig. 4.

To verify formula (22), we counted the number of pulses in each channel.

A complete comparison of theory and experiment presupposes an independent measurement of the parameters α, β, γ, and $\bar{n}_0$. Estimates show that when the generation frequency is tuned to the generation peak power, and at the pressures chosen by us, it is possible to disregard the dependences of the coefficients α, β, and γ on Ω and Δ, and take them in the form given by (2.1) of[2]. (The resultant error in the values of $\bar{t}$ and σ_t^2 is not more than 20%, which is within the accuracy limit of their experimental determination.) They can be then easily determined by finding the threshold current of the hysteresis quenching and ignition of the generation, the generation quenching current without a cell, and the above-threshold discharge current corresponding to the amplitude of the voltage pulse from the pulse generator (for the chosen value of the discharge current in the absorbing cell). The latter parameter was determined by measuring the generation-pulse amplitude on the oscilloscope screen, after which the modulation pulses were removed and the discharge current was varied to establish a constant generation such that the oscilloscope beam was deflected by an amount equal to the pulse amplitude. The ratio of the saturation parameters σ_1/σ_2 (see (1.6) of[2]) was determined from the gas pressures. Allowance for the field configuration in the resonator results in an additional coefficient ~ 1.5. The total nonresonant loss is $f \sim 0.2\%$, so that the time of "cold" resonator is of the order of $1\ \mu$sec.

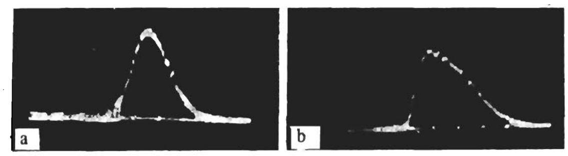


FIG. 4. Distribution of the establishment time W(t) in a laser with NRA (a) and in an ordinary laser (b).

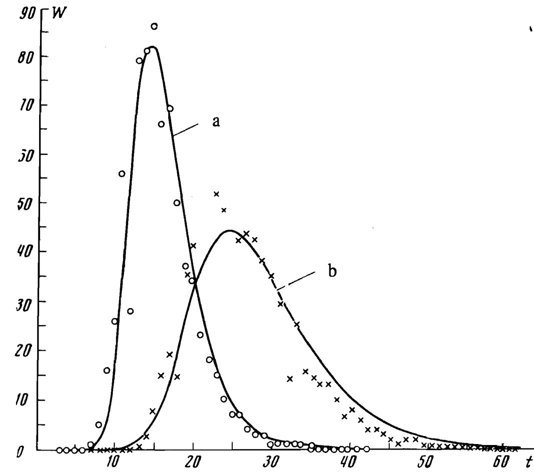


FIG. 5. Distribution of transient time in a laser with NRA at different values of α, β, γ_1 and t_m. Solid curve—theory, points—experiment.

The parameter $\bar{n}_0$, just as in[5], cannot be measured directly, yet all the more the interesting features of a laser with NRA appear at small α_2, and the hysteresis quenching of the generation occurs at sufficiently large α_1. Thus, the role of the second priming signal is most important in this case. Therefore the parameter t_m (24), which contains $\bar{n}_0$, was used for adjustment purposes.

The numerical values of the parameters under the conditions of our experiments were the following:

a) $\alpha_2 = 0.23,\ \beta = 5.4,\ \gamma = 147,\ t_m = 6.82;$

b) $\alpha_2 = 0.13,\ \beta = 5.5,\ \gamma = 147,\ t_m = 11,$

corresponding to the case m = 1/2 in formula (21). Figures 5a and 5b show the results of a calculation by formula (22) at these values of the parameter and at the normalized probability density of the transient time, obtained by counting the number of pulses in each analyzer channel. The solid curve corresponds to calculation by formula (22), and the points are experimental.

Under the conditions of the accuracy assumed by us in the calculation of the parameters α_2, β, and γ by means of formula (2.1) of[2], and the accuracy of their experimental determination, the agreement between theory and experiment can be regarded as satisfactory. It should be noted that the relative error in the determination of the parameter α_2, to the variation of which the shape of the W(t) plot is particularly sensitive, increases with decreasing α_2. On the other hand, as already noted, to observe the most interesting features of the transient processes in a laser with an absorbing cell, it would be necessary to operate precisely at small values of α_2.

The performed experiments have demonstrated the validity of our theoretical analysis of statistical phenomena in the transients of a helium-neon laser with NRA.

We did not perform detailed investigations of the fluctuations of the radiation growth time in a laser with NRA in the pre-hysteresis regime. The point is that an exact experimental detection of the presence of pre-hysteresis regime ($\beta = 0$) is difficult, and preliminary experiments near this regime, by the procedure described above, have shown that the dynamics of the generation growth in this regime, in accordance with (27) and (28) did not differ significantly from the dynamics in an ordinary laser.

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