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EXCITATION OF PARAMETRIC INSTABILITIES IN A MAGNETOACTIVE PLASMA BY A MICROWAVE PUMPING WAVE

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An investigation is made of the nonthermal radiation from a magnetoactive plasma interacting with a microwave pumping wave whose frequency ω_0 exceeds the electron Langmuir frequency ω_{Le} but is less than the electron cyclotron frequency ω_{He}. Radiation is observed at a frequency which is shifted to lower frequencies by an amount of the order of the ion-sound oscillation frequency ω_{s}. The dependence of the radiation with a frequency equal to that of the second harmonic of the microwave pumping frequency, $2\omega_0$, on plasma concentration is obtained. The relation between the radiation observed and development of parametric instabilities is discussed.

THE induced Raman scattering of microwaves and the generation of radiation at the second harmonic of the pump wave in a magnetoactive plasma, which were observed in[1], have led to the conclusion that the primary reason of the previously observed[2,3] anomalous dissipation of waves in the frequency range $\omega_{\mathrm{Le}} < \omega_0 < \omega_{\mathrm{He}}$ (ω_0 is the angular frequency of the microwave field, while $\omega_{\mathrm{Le}} = 4\pi ne^2/m_e)^{1/2}$ and $\omega_{\mathrm{He}} = eB/m_ec$ are the Langmuir and gyroscopic frequencies of the electrons) are nonlinear mechanisms of interaction between the microwave field and the plasma, particularly parametric instabilities[4-6], since it is known that linear transformation of waves is impossible when $\omega_{\mathrm{Le}} < \omega_0 < \omega_{\mathrm{He}}$[7]. From among the parametric instabilities that occur in this frequency region, we note primarily the following two types. The first is the parametric instability that develops under conditions of resonance between the higher harmonics of the external microwave fields and the frequency of the natural mode of the electronic potential oscillations, particularly the second harmonic of the microwave pump field with the upper hybrid frequency $2\omega_0 \approx (\omega_{\mathrm{Le}}^2 + \omega_{\mathrm{He}}^2)^{1/2}$[6]. The second is the instability of the plasma against excitation of nonpotential oscillations[5] (the "nonpotential" instability).

The conditions for the excitation of a high-frequency nonpotential wave of frequency ω_0' and of a low-frequency plasma wave of frequency ω_{LF} were investigated in[5]. The excitation threshold of the instability in question is reached under conditions corresponding to the decay $\omega_0 = \omega_0' + \omega_{\mathrm{LF}}$, and the frequency ω_0' of the nonpotential wave that builds up is consequently lower by an amount ω_{LF} than the frequency ω_0 of the external microwave field. The wavelength of the oscillations for which the growth increment is maximal is given by

$$k_{max}^2 \approx \frac{\omega_{Le}^2}{c^2}\frac{\omega_0}{|\omega_{He}| - \omega_0} + \frac{\omega_0^2}{c^2}.$$

For a comparison of the experimental and theoretical results, it is of interest to identify the types of the oscillations excited in the plasma.

As shown earlier[1,2], the plot of the radiation intensity and of the fast-electron current against the magnetic field intensity have a number of resonant peaks. The presence of a peak of nonthermal plasma radiation at $\omega_{\mathrm{He}}/\omega_0 = 1.08$ may be due to the buildup of nonpotential oscillations, and the radiation and the peaks of the fast electrons near $\omega_{\mathrm{He}}/\omega_0 \approx 2$ may be due to resonance of the second harmonic of the pump wave with the natural mode of the electron longitudinal plasma oscillations. Such a correspondence alone, however, is insufficient to identify the type of instability.

The most direct way of obtaining the relation between the experimental results and the linear theory of parametric instabilities of the plasma would be a direct experimental determination of the excitation thresholds of the plasma oscillations. Unfortunately, in most cases this is made quite difficult by the need of having recording apparatus of high sensitivity and measuring the threshold field with high accuracy. There is, however, another possibility of determining the nature of the excitation of various oscillations, namely by establishing the dependence of the spectrum and of the oscillation intensity on the charged-particle density, since theoretical considerations show that the oscillation growth increments should depend on the plasma concentration. We therefore attempt in this article to establish the dependence of the spectrum and of the intensity of nonthermal radiation of a magnetoactive plasma on the charged-particle concentration, and to determine in some cases the oscillation excitation thresholds and to tie in the obtained relations with the conclusions of the theory of parametric instabilities.

1. The experiments were performed with the same setup as in[1-3], which is shown schematically in Fig. 1. The plasma produced by the spark gun 1 was injected into a quasistationary homogeneous magnetic field produced by a system of coils 10. The cylindrical plasma current, whose cross section was limited by a diaphragm 2 of 1.5 cm diameter, passed through a rectangular waveguide 5 through stubs 6 operating beyond cutoff and welded to the narrow walls of the waveguide. The plasma concentration, which was controlled by the voltage on the discharge gap of the spark gun, was 10^{11} cm^{-3} at $\omega_{\mathrm{Le}}^2/\omega_0^2 = 1$. The plasma density was measured with a multielectrode differential probe 12. The

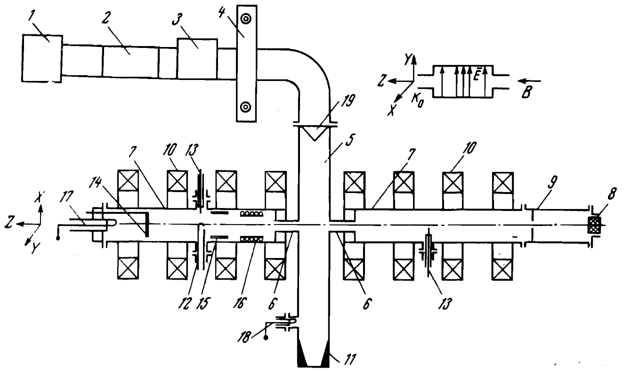

FIG. 1. Experimental setup: 1—magnetron generator; 2—ferrite switch, 3—attenuator; 4—directional coupler; 5—vacuum waveguide; 6—stubs operating beyond cutoff; 7—vacuum chambers; 8—spark plasma injector; 9—diaphragm; 10—solenoid; 11—matched load; 12—multielectrode probe; 13—Langmuir probe; 14—plasmascope; 15—electrostatic probe; 16—magnetic probe; 17—loop antenna outside waveguide; 18—loop antenna in waveguide; 19—dielectric microwave window.

electron temperature was $T_e \approx 6$ eV, and the nonisothermy corresponded to $T_e/T_i \approx 100$. The TE_{10} mode was excited in the rectangular waveguide, so that the microwave electric-field vector **E** was perpendicular to the quasistationary magnetic field **B**. The setup operated in a pulsed regime. The plasma radiation was registered with two loop antennas, one located outside the waveguide in a vacuum chamber 17, and the other placed directly in waveguide 18 at a distance $3\lambda_g$ from the plasma column (λ_g is the wavelength of the microwave of frequency ω_0 in the waveguide). The nonthermal radiation of the plasma was investigated in the frequency ranges ω_0 and $2\omega_0$. The microwave radiation signals were amplified and frequency-analyzed with an S-4-14 single-pulse spectrum analyzer (bandwidth 250 kHz) and high-sensitivity standard receivers P-5-5B and P-5-7B (bandwidths 5 and 10 MHz, respectively).

2. The radiation near the frequency ω_0 was investigated only outside the waveguide. Figure 2 shows the emission spectra measured with the S-4-14 at different values of the plasma and magnetic field concentrations. The frequency scale in Fig. 2a is indicated in units of the linear frequency. On all the presented spectrograms, the frequency increases from left to right. From a comparison of the plasma emission spectra with the emission spectrum of the microwave generator (Fig. 2b) it is seen that the plasma emission spectrum broadens in comparison with the pump wave spectrum. Thus, if $\omega_{He}/\omega_0 = 2$ (Fig. 2f) the broadening reaches a value $\Delta f \approx 2$ MHz, which may be evidence of the buildup in the plasma of low-frequency oscillations in a band $\Delta\omega = 2\pi\Delta f \approx 12$–$14$ MHz. At $\omega_{He}/\omega_0 = 1.08$ and $\omega_{Le}^2/\omega_0^2 = 0.35$ one observes clearly an emission-peak spectrum shifted relative to the microwave pump frequency towards lower frequencies (Fig. 2d). The value of the shifts, determined from the maximum of the radiation intensity, is $\Delta\omega \approx 4$ MHz, and the broadening of the spectrum is of the same order of magnitude. With increasing plasma concentration, the shift of the emission peak at $\omega_{He}/\omega_0 = 1.08$ increases somewhat (Fig. 2c), as well as the broadening of the spectrum at the shifted frequency. To the contrary, with decreasing plasma concentration the emission spectrum near the frequency ω_0 and $\omega_{He}/\omega_0 = 1.08$ (Fig. 2e) becomes narrower, and no emission is observed, within the limits of the resolution of the spectrum analyzer at the shifted frequency.

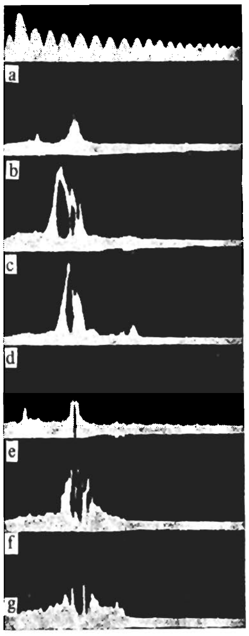

FIG. 2

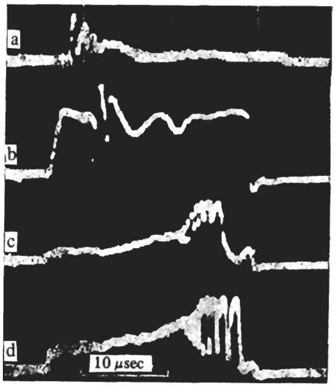

FIG. 3

FIG. 2. Emission spectrum near the frequency ω_0; E = 2.5 kV/cm ($v_E/v_{Te} \approx 2.5$). a—Frequency scale 1 MHz per marker; b—microwave generator emission spectrum, c-g—emission spectra: c—$\omega_{He}/\omega_0 = 1.08$, $\omega_{Le}^2/\omega_2^0 = 0.65$; d—$\omega_{He}/\omega_0 = 1.08$, $\omega_{Le}^2/\omega_0^2 = 0.35$; e—$\omega_{He}/\omega_0 = 1.08$, $\omega_{Le}^2/\omega_0^2 = 0.20$; f—$\omega_{He}/\omega_0 = 2$, $\omega_{Le}^2/\omega_0^2 = 0.65$; g—$\omega_{He}/\omega_0 = 2$, $\omega_{Le}^2/\omega_0^2 = 0.2$.

FIG. 3. Oscillograms of emission envelope at the frequency ω_0 with $\omega_{Le}^2/\omega_0^2 = 0.35$: a—$\omega_{He}/\omega_0 = 1$, E = 30 V/cm; b—$\omega_{He}/\omega_0 = 1.08$, E = 10 V/cm; c—$\omega_{He}/\omega_0 = 1.65$, E = 10 V/cm; d—$\omega_{He}/\omega_0 = 2$, E = 10 V/cm.

Unlike the emission at $\omega_{He}/\omega_0 = 1.08$, the emission spectrum near the frequency ω_0 at $\omega_{He}/\omega_0 = 2$ is practically independent of the plasma concentration, as follows from a comparison of the spectrograms of Figs. 2f and 2g, obtained at different values of the plasma density.

Measurements of the emission intensity near the frequency ω_0, using a P-5-5B receiver at a low microwave pump power level ($E \approx 30$ V/cm and $v_E/v_{Te} \approx 0.03$, where v_{Te} is the thermal velocity of the electrons and $v_E = eE/m_e\omega_0$ is the electron oscillation velocity in the microwave field) have made it possible to register at $\omega_{Le}^2/\omega_0^2 = 0.35$ the low-frequency oscillations on the oscillograms of the emission envelope (Fig. 3). The oscillogram of Fig. 3b ($\omega_{He}/\omega_0 = 1.08$) shows low-frequency oscillations whose frequency, determined from the first period of the oscillations, is $\omega_{LF} \approx 4$–5 MHz, i.e., it corresponds to the value $\Delta\omega$ by which the emission peak on the spectrogram of Fig. 2d is displaced. The oscillation frequency at $\omega_{He}/\omega_0 = 1$, 1.65, and 2 increases with increasing magnetic field (the oscillograms of Figs. 3a, c, and d, respectively), and it turns out that the frequency of these oscillations is smaller than the corresponding values of the ion cyclotron frequency by a factor 1.5–2.

When the microwave field intensity decreases from $E \approx 10$ V/cm to $E \approx 3$ V/cm, the low-frequency oscillations are no longer excited so that one can speak of a threshold of their excitation (with respect to the microwave field). We note that measurements at a low microwave pump level ($E \lesssim 10$ V/cm) were performed at practically maximum gain of the P-5-5B receiver, so that the low-frequency oscillations were observed on the oscillograms of Figs. 3b, c, and d against the background of uncontrollable induced noise.

The measurement results for the case $\omega_{He}/\omega_0 = 1.08$ can be compared with the predictions of the theory of parametric instability of a magnetoactive plasma relative to excitation of nonpotential oscillations[5]. It must be borne in mind here that this study, like most investigations on the theory of parametric instabilities[4], describes only the initial linear stage of perturbation development, whereas many of the experimentally observed effects are in essence a consequence of strongly developed plasma turbulence. However, such experimental data as threshold value of the microwave field, and the time of development and the characteristic frequencies of the excited oscillations, can be set in correspondence with the theoretical results.

The threshold value of the electric microwave field intensity above which the plasma becomes unstable against the excitation of nonpotential high-frequency oscillations and the potential ion-acoustic oscillations that are parametrically coupled with them is, according to formula (2.6) of[5], equal to $E_{thr} = 1$ V/cm for $\omega_{Le}^2/\omega_0^2 = 1.4$ and $\omega_{He}/\omega_0 = 1.08$.

The time of development of the oscillations in the experiment at $E \approx 10$ V/cm, determined from the oscillogram of Fig. 3b, is 3×10^{-6} sec, i.e., the experimental growth increment is $\gamma_{exp} \approx 3 \times 10^5$ sec^{-1}. According to (2.8) of[5], at this value of the microwave field the theoretically determined maximum growth increment is $\gamma = 7 \times 10^5$ sec^{-1}.

The frequency of the experimentally-observed oscillations (Fig. 3b) agrees with the frequency of the most rapidly growing ion-acoustic oscillations, whose wavelength is determined by formula (2.9) of[5]. The emission at a frequency shifted by $\Delta\omega \approx 4$–5 MHz relative to ω_0 (Figs. 2d and 2e) is apparently due to the excitation of nonpotential high-frequency oscillations.

3. As shown earlier[1], the plot of the intensity of the nonthermal emission at the second harmonic of the microwave field $2\omega_0$ in the waveguide revealed two maxima, one at $\omega_{He}/\omega_0 = 1.9$ and the other at $\omega_{He}/\omega_0 = 2$.

It was proposed that this emission is connected with parametric instability that develops when the second harmonic of the external microwave field is at resonance with the frequency of the natural mode of the electronic potential oscillations of the plasma[6]. For oscillations of sufficiently large wavelength, when $k\rho_e \ll 1$ (ρ_e is the Larmor radius of the electron), this condition takes the form

$$2\omega_0 \approx 2^{-1/2}\{\omega_{He}^2 + \omega_{Le}^2 + [(\omega_{He}^2 + \omega_{Le}^2)^2 - 4\omega_{He}^2\omega_{Le}^2\cos^2\theta]^{1/2}\}^{1/2},$$

where θ is the angle between the wave vector **k** of the perturbations and the direction of the external magnetic field **B**. The peak of the emission at $\omega_{He}/\omega_0 = 1.9$ may be due to excitation of oscillations that propagate perpendicular to **B**, when the second harmonic of the external microwave field is close to the upper hybrid frequency, $2\omega_0 \approx (\omega_{Le}^2 + \omega_{He}^2)^{1/2}$ (for $\omega_{Le}^2/\omega_0^2 = 0.35$, this resonance takes place at $\omega_{He}/\omega_0 = 1.91$).

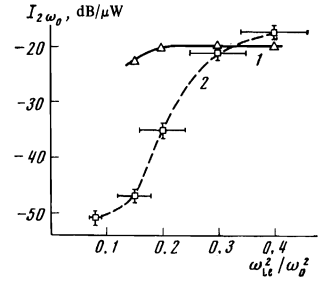

FIG. 4. Intensity of emission at frequency $2\omega_0$ against the plasma concentration, $E = 2.5$ kV/cm. 1—$\omega_{He}/\omega_0 = 2$, 2—$\omega_{He}/\omega_0 = 1.9$.

The emission peak at $\omega_{He}/\omega_0 = 2$ can correspond to parametric excitation of long-wave oscillations propagating at acute angles θ to **B** (but not $\theta \neq 0$) and also due to excitation of short-wave oscillations ($k\rho_e \gtrsim 1$) propagating perpendicular to **B**[8].

To verify the proposed connection between the emission at the frequency $2\omega_0$ with the parametrically-excited high-frequency plasma oscillations, we investigated the dependence of the emission intensity at the frequency $2\omega_0$ in a waveguide on the plasma concentration. Figure 4 shows the results of the measurements performed with the aid of a P-5-7B receiver. The emission intensity $I_{2\omega_0}$ is plotted in a logarithmic scale (dB/μW), and the measurement error did not exceed 2dB/μW.

From a comparison of curve 1 ($\omega_{He}/\omega_0 = 2$) with curve 2 ($\omega_{He}/\omega_0 = 1.9$) it is obvious that there is a substantial difference between the dependences of the emission intensity at the frequency $2\omega_0$ on the plasma concentration at $\omega_{He}/\omega_0 = 1.9$ and $\omega_{He}/\omega_0 = 2$. At a fixed value of the magnetic field $\omega_{He}/\omega_0 = 1.9$ the deviation from the hybrid resonance $\Delta = [(\omega_{He}^2 + \omega_{Le}^2)^{1/2}/2\omega_0]^2 - 1$ increases with the decreasing ω_{Le}^2/ω_0^2. As shown by Aliev et al.[6] (see formulas (4.7) and (4.8) of their paper), with increasing Δ the instability increment decreases sharply. In particular, for $\omega_{He}/\omega_0 = 1.9$, the sharp drop in the instability increment should begin with $\omega_{Le}^2/\omega_0^2 = 0.3$, which agrees with the experimentally observed dependence of $I_{2\omega_0}$ on ω_{Le}^2/ω_0^2. It should be borne in mind here that no account was taken in[6] of the thermal motion of the particles, which, as shown in[9], extends the limits of the instability region. In the case of resonance at the cyclotron frequency $2\omega_0 \approx \omega_{He}$ (curve 1), as expected, the dependence of the emission intensity $I_{2\omega_0}$ on the plasma concentration is weak.

The experimental results reported here on nonthermal radiation from a magnetoactive plasma interacting with a microwave field at frequencies $\omega_{Le} < \omega_0 < \omega_{He}$, and the comparison of these results with the theory of plasma parametric instabilities, enable us to relate the experimentally observed nonthermal plasma emission with the development of nonpotential instability at $\omega_{He}/\omega_0 = 1.08$ and with parametric resonance at the second harmonic of the microwave pump frequency at a natural mode of the potential oscillations at $\omega_{He}/\omega_0 = 1.9$–$2$.

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[1] G. M. Batanov and K. A. Sarksyan, Zh. Eksp. Teor. Fiz. **62**, 1721 (1972) [Sov. Phys.-JETP **35**, 895 (1972)].

[2] G. M. Batanov and K. A. Sarksyan, ZhETF Pis. Red. **13**, 539 (1971) [JETP Lett. **13**, 384 (1971)].

[3] G. M. Batanov, L. M. Gorbynov, and K. A. Sarksyan, Kratkie soobshcheniya po fizike (FIAN) **8**, 60 (1971).

[4] V. P. Silin, A Survey of Phenomena in Ionized Gases, IAEA, Vienna, 1968, p. 205—237.

[5] N. E. Andreev, Zh. Eksp. Teor. Fiz. **63**, 1283 (1972) [Sov. Phys.-JETP **36**, No. 4 (1973)].

[6] Yu. M. Aliev, V. P. Silin, and H. Watson, Zh. Eksp. Teor. Fiz. **50**, 943 (1966) [Sov. Phys.-JETP **23**, 626 (1966)].

[7] V. E. Golant and A. D. Piliya, Usp. Fiz. Nauk **104**, 413 (1971) [Sov. Phys.-Uspekhi **14**, 413 (1972)].

[8] Yu. M. Aliev and V. P. Silin, Preprint FIAN, No. 71, 1972.

[9] N. E. Andreev, A. Yu. Kiriĭ, and V. P. Silin, Izv. vuzov (Radiofizika) **13**, 1321 (1970).

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