CONTRIBUTION TO THE THEORY OF PARAMETRIC INTERACTION BETWEEN A HIGH FREQUENCY FIELD AND A MAGNETOACTIVE PLASMA

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The instability of a microwave plasma located in stationary magnetic and microwave electric fields with respect to excitation of nonpotential electron oscillations parametrically coupled with low frequency potential perturbations is considered. The threshold value of the microwave field strength for development of such instability is determined and expressions are obtained for the maximal increments in the case of weak or strong perturbation coupling. Nonresonance kinetic instability of a magnetoactive plasma due to the inverse Cerenkov effect for electrons is investigated. The theoretical results are compared with available experimental data.

1. Much attention has been paid recently to the theoretical and experimental study of nonlinear phenomena occurring when a microwave field interacts with a plasma[1-5]. It can now be regarded as established that many of the anomalous effects that appear in such an interaction are the consequence of plasma turbulization due to the development of parametric instabilities (see, for example,[1,2]). This indeed explains why it is important to determine theoretically the conditions under which the plasma becomes unstable against the excitation of small perturbations.

An appreciable share of the experimental research on the interaction of a microwave field with a plasma is carried out in the presence of a constant magnetic field B_0. To eliminate the possibility of linear transformation of the waves, the parameters of the plasma and of the external fields are chosen such that the frequency ω_0 of the microwave pump field satisfies the relation

$$\omega_{Le} < \omega_0 < |\Omega_e|, \tag{1.1}$$

where $\omega_{Le} = (4\pi e^2 n_e/m_e)^{1/2}$, and $\Omega_e = eB_0/m_e c$ are the Langmuir and gyroscopic frequencies of the electrons[3-5]. As is well known, parametric resonance with electron plasma oscillations at the fundamental harmonic of the external microwave field is impossible in this range of parameters[6]. Parametric resonance at higher harmonics is apparently responsible for the anomalous effects observed at $|\Omega_e|/\omega_0 \approx 1.5$ and $|\Omega_e|/\omega_0 \approx 2$.

The nonlinear character of the interaction of the microwave field with the plasma in the frequency region $1 < |\Omega_e| \lesssim 1.5$ may be due to the buildup of high-frequency nonpotential oscillations[7,8]. However, as shown by Gorbunov and Silin[7], in the case of nonresonant excitation of high frequency nonpotential oscillations and of the low frequency potential perturbations that are parametrically coupled with them, instability is possible only in a strong microwave field whose energy greatly exceeds the thermal energy of the plasma plasma. The instability investigated in[7], against the buildup of natural nonpotential and Langmuir oscillations under the conditions of (1.1), arises for relatively long-wave perturbations with wavelength $\lambda \gtrsim c/\omega_0$ (c is the speed of light), which exceeds the dimension of the plasma used in the experiments of[3-5]. Such instabilities can therefore hardly be excited in these experiments.

The second section of the present paper deals with parametric resonance at the natural mode of high-frequency nonpotential oscillations connected with low-frequency potential perturbations. The expressions obtained for the threshold value of the external microwave field intensity, for the builtup increments, and for the characteristic wavelengths of the perturbations show that such an instability can develop under the conditions of the experiments of Batanov and Sarksyan[4,5].

The conditions for parametric excitation of potential oscillations in the cyclotron resonance region $(|\omega_0 - |\Omega_e|| \ll \omega_0)$ were theoretically investigated by Gradov and Zyunder[9]. However, their conclusions, that potential perturbations can build up in the frequency region (1.1) when account is taken of the high-frequency perturbation dissipation due only to collisions, are in error. The reason is that in view of the nonresonant behavior of the real part of the longitudinal dielectric constant of the plasma in the frequency region (1.1) it is necessary to take into account more accurately the terms responsible for the absorption of the high-frequency oscillations. The mechanism that can lead to nonresonant excitation of potential perturbations in a microwave field is the inverse Cerenkov effect on the electrons. The kinetic instability of the plasma against the buildup of potential oscillations, brought about by this effect and first investigated by Silin[10] as applied to a plasma without a magnetic field, is considered in Sec. 3 of the present article. In the conclusion (Sec. 4), the theoretical results are discussed in light of the latest experimental data.

2. In view of the limited sizes of the plasma objects used in experiments, greatest interest attaches to an investigation of instability against the excitation of relatively short-wave nonpotential oscillations. For a magnetoactive plasma under conditions when the oscillation frequency is close to the electron cyclotron frequency, the largest refractive index will be possessed by the extraordinary wave propagating along the constant mag-

netic field. We consider below the instability against excitation of high-frequency perturbations of just such a wave.

We assume that the plasma and the external microwave electric field

$$\mathbf{E} = \mathbf{E}_0 \sin \omega_0 t$$

are homogeneous, and the vector $\mathbf{E}_0$ of the microwave electric field intensity is perpendicular to the homogeneous constant magnetic field $\mathbf{B}_0$. For perturbations with wave vectors $\mathbf{k}$ parallel to $\mathbf{B}_0$ we can write the following dispersion equation (see[7]):

$$\frac{1}{\delta\varepsilon_e(\omega,k)} + \frac{1}{1+\delta\varepsilon_i(\omega,k)} + \frac{1}{8}\frac{v_E^2}{c^2}\frac{\omega_0^2}{(\omega_0 - |\Omega_e|)^2} \times\left[\frac{n^2(\omega+\omega_0)}{\Delta^+(\omega+\omega_0)} + \frac{n^2(\omega-\omega_0)}{\Delta^-(\omega-\omega_0)}\right] = 0. \quad (2.1)$$

Here $\omega = \omega + i\gamma$; $\delta\epsilon_\alpha(\omega, k)$ is the contribution of particles of type α to the longitudinal dielectric constant of the plasma; $v_E = |e|E_0/m_e\omega_0$ is the velocity amplitude of the electron oscillations in the microwave field; $n(\omega) = kc/\omega$ is the refractive index; $\Delta^\pm(\omega) \equiv \Delta^\pm(\omega, k) = \epsilon_{11}(\omega, k) \mp i\epsilon_{12}(\omega, k) - n^2(\omega)$, $\epsilon_{ij}(\omega, k)$ are the components of the ordinary dielectric tensor of the plasma. In the derivation of (2.1) the electron motion was assumed to be nonrelativistic, i.e., it was assumed that $v_E^2\omega_0^2/c^2|\omega_0^2 - \Omega_e^2| \ll 1$.

This dispersion equation describes the excitation of both high-frequency nonpotential oscillation with frequencies $\omega \pm \omega_0$ and increment γ, and the low-frequency potential perturbations connected with them, with frequency ω ($\omega \ll \omega_0$) and with the same increment.

Assuming that the frequency of the external field does not lie in the cyclotron-absorption region ($v_{Te} = \sqrt{T_e/m_e}$ is the thermal velocity of the electrons):

$$|\omega_0 - |\Omega_e|| \gg kv_{Te},$$

and that the deviation from cyclotron resonance exceeds the frequency of the low-frequency perturbations and the effective frequency of the electron-ion collisions (ν_{eff}):

$$|\omega_0 - |\Omega_e|| \gg |\omega + i\gamma|, \quad \nu_{eff} = \frac{4}{3}\frac{\sqrt{2\pi}\, e^2 e_i^2 n_i L}{T_e^{3/2} m_e^{1/2}}, \quad (2.2)$$

we can simplify the high-frequency terms of (2.1) in the following manner:

$$\frac{n^2(\omega+\omega_0)}{\Delta^+(\omega+\omega_0)} + \frac{n^2(\omega-\omega_0)}{\Delta^-(\omega-\omega_0)} = \frac{n^2(\omega_0)}{\varkappa(\omega_0)}\frac{\omega_0\Delta\omega_0}{(\Delta\omega_0)^2 + (\gamma+\tilde{\gamma})^2 - \omega^2 - 2i\omega(\gamma+\tilde{\gamma})},$$

where

$$\varkappa(\omega_0) = \omega_0\frac{\partial}{\partial\omega_0}\mathrm{Re}\,\Delta^\pm(\pm\omega_0) \approx \frac{\omega_{Le}^2}{(\omega_0 - |\Omega_e|)^2} \text{ when } |\omega_0 - |\Omega_e|| < \omega_0,$$

$$\tilde{\gamma} = \pm\frac{\omega_0}{\varkappa(\omega_0)}\mathrm{Im}\,\Delta^\pm(\pm\omega_0) = \nu_{eff}, \quad (2.3)$$

$$\Delta\omega_0 \equiv \Delta\omega_0(k) = \frac{\omega_0}{\varkappa(\omega_0)}\mathrm{Re}\,\Delta^\pm(\pm\omega_0) \approx \approx (|\Omega_e| - \omega_0)\left[1 + \frac{|\Omega_e| - \omega_0}{\omega_0}\frac{\omega_0^2 - k^2c^2}{\omega_{Le}^2}\right] \ll \omega_0.$$

It is easy to establish here that the dispersion relation (2.1) has aperiodic solutions corresponding to growth of spatial oscillations with frequency $\omega \approx 0$ and high-frequency potential oscillations with increasing time, as well as almost-periodic solutions with a low-frequency perturbation frequency that exceeds the increment. We examine first the periodic solutions.

a) Being interested primarily in the threshold value of the intensity of the external microwave field, above which the plasma becomes unstable (i.e., the increment γ becomes positive), we assume that $\gamma \ll \omega$. We confine ourselves further to the case of relatively small dissipation, when the frequency ω exceeds the damping decrements of the natural high-frequency and low-frequency oscillations: $\omega \gg \tilde{\gamma}, \gamma_0$, where $\tilde{\gamma}$ is given by formula (2.3), and

$$\gamma_0 = -\left[\frac{\mathrm{Im}\,\delta\varepsilon_e(\omega)}{[\mathrm{Re}\,\delta\varepsilon_e(\omega)]^2} + \frac{\mathrm{Im}\,\delta\varepsilon_i(\omega)}{[1+\mathrm{Re}\,\delta\varepsilon_i(\omega)]^2}\right] \times\left\{\frac{\partial}{\partial\omega}\left[\frac{1}{\mathrm{Re}\,\delta\varepsilon_e(\omega)} + \frac{1}{1+\mathrm{Re}\,\delta\varepsilon_i(\omega)}\right]\right\}^{-1}.$$

It follows here from (2.1) that the threshold and the maximum of the increment are reached under conditions corresponding to the decay:

$$\Delta\omega_0(k_0) = \omega(k_0), \quad (2.4)$$

when the frequency ω differs little from the natural frequency determined by the equation $1 + \mathrm{Re}\,\delta\epsilon_e(\omega) + \mathrm{Re}\,\delta\epsilon_i(\omega) = 0$, and the high-frequency perturbations with frequency $\omega_0 - \omega$ satisfy the dispersion law for the natural nonpotential oscillations: $\mathrm{Re}\,\Delta^-(\omega - \omega_0) = \mathrm{Re}\,\Delta^+(\omega_0 - \omega) = 0$.

In a non-isothermal plasma with an electron temperature exceeding the ion temperature

$$\ln(e_i^2T_e^3m_i/e^2T_i^3m_e) < \left|\frac{e_i}{e}\right|\frac{T_e}{T_i} < \left|\frac{e}{e_i}\right|\frac{m_i}{m_e},$$

and for perturbations with a phase velocity larger than the thermal velocity of the ions but smaller than the thermal velocity of the electrons

$$v_{Ti} < \omega/k < v_{Te} \quad (2.5)$$

the low-frequency decrement is equal to

$$\gamma_0 = \sqrt{\frac{\pi}{8}}\frac{\omega_{Li}}{\omega_{Le}}\omega_s,$$

where $\omega_{Li} = \sqrt{4\pi e_i^2 n_i/m_i}$ and $\omega_s = kv_{Te}\omega_{Li}/\omega_{Le}$ are respectively the Langmuir ion and the ion-sound frequencies. In this case, the threshold value of the microwave field intensity is determined by the relation

$$\frac{v_{E\,thr}^2}{v_{Te}^2} = 16\frac{\tilde{\gamma}\gamma_0}{\omega_0\omega_s} = 2\sqrt{8\pi}\frac{\nu_{eff}}{\omega_0}\frac{\omega_{Li}}{\omega_{Le}}. \quad (2.6)$$

At not too large an excess of the microwave field intensity over the threshold value, when the increment does not exceed the largest of the damping decrements, i.e.,

$$\gamma < \Gamma \equiv \max(\tilde{\gamma}, \gamma_0) \gg \min(\tilde{\gamma}, \gamma_0),$$

the maximum increment is equal to

$$\gamma_{max} = \frac{1}{16}\frac{v_E^2 - v_{E\,thr}^2}{v_{Te}^2}\frac{\omega_0\omega_s(k_0)}{\Gamma}. \quad (2.7)$$

At larger external field intensities, when $\gamma_{max} > \Gamma$ (or $\Gamma \gg |\tilde{\gamma} - \gamma_0|$) we obtain from (2.1)

$$\gamma_{max} = \frac{1}{4}\frac{v_E - v_{E\,thr}}{v_{Te}}\sqrt{\omega_0\omega_s(k_0)} \ll \omega_s(k_0). \quad (2.8)$$

The increment maxima (2.7) in (2.8) are reached for perturbations with a wave number k_0 determined, according to (2.4), from the following relation:

$$k_0^2 = \frac{\omega_0^2}{c^2} + \frac{\omega_{Le}^2}{c^2} \frac{\omega_0}{|\Omega_e| - \omega_0}. \quad (2.9)$$

When the field E_0 is so strong that the increment (2.8) becomes much larger than Γ and becomes comparable with $\omega = \omega_s(k_0)$, i.e.,

$$v_E / v_{Te} \gtrsim 4\sqrt{\omega_s(k_0)/\omega_0}, \quad (2.10)$$

the weak-coupling approximation no longer holds, and the dissipative effects are obviously negligible. Then, assuming $|\omega + i\gamma| < \omega_{Li}$, we can obtain the following expression for the maximum increment of the buildup of the perturbations and frequency:

$$\gamma_{max} = \frac{\sqrt{3}}{4^{7/6}} \omega_0 \left[\frac{v_E}{v_{Te}} \frac{\omega_s(k_{max})}{\omega_0}\right]^{2/3}, \quad \omega = \sqrt{\frac{5}{3}} \gamma_{max}, \quad (2.11)$$

where $k_{max} \approx k_0$ (2.9). Since the formulas were derived on the basis of relation (2.2), the following limitation should be satisfied for the deviation from the cyclotron resonance

$$\frac{||\Omega_e| - \omega_0|}{\omega_0} > \left[\frac{v_E}{v_{Te}} \frac{\omega_s(k_{max})}{\omega_0}\right]^{2/3}. \quad (2.12)$$

If the frequency ω_0 is close to the gyroscopic frequency of the electrons, so that $k_{max} > \omega_0/c$, then the expression for the maximum increment can be written in the form

$$\gamma_{max} = \frac{\sqrt{3}}{4^{7/6}} \omega_0 \left[\frac{v_E}{c} \frac{\omega_{Li}}{\omega_0} \sqrt{\frac{\omega_0}{|\Omega_e| - \omega_0}}\right]^{2/3} \quad (2.13)$$

with the corresponding applicability conditions

$$\frac{1}{2}\sqrt{\frac{v_E}{c}\frac{\omega_{Le}}{\omega_0}} < \frac{|\Omega_e| - \omega_0}{\omega_0} < \frac{\omega_{Le}^2}{\omega_0^2}, \quad \frac{v_E}{v_{Te}} > 4\left[\sqrt{\frac{\omega_0}{|\Omega_e| - \omega_0}} \frac{v_{Te}\omega_{Li}}{c\ \omega_0}\right]^{1/2}. \quad (2.14)$$

b) We now turn to a study of the aperiodic perturbations. To determine the threshold value of the pump field intensity and the increment near the threshold, we consider Eq. (2.1) with $\gamma > kv_{Te}$, kv_{Ti}. Then, neglecting the imaginary parts of $\delta\epsilon_\alpha$ at $\omega = 0$, we find that the maximum of the increment is reached at $k_{max} \approx k_0$ (2.9) and is equal to

$$\gamma_{max} = \frac{1}{8} \frac{v_E^2 - v_{E\,thr}^2}{v_{Te}^2(1 + |e/e_i| T_i/T_e)} \omega_0, \quad (2.15)$$

where

$$\frac{v_{E\,thr}^2}{v_{Te}^2} = 8\left(1 + \left|\frac{e}{e_i}\right| \frac{T_i}{T_e}\right) \frac{\tilde{\gamma}}{\omega_0} = 8\left(1 + \left|\frac{e}{e_i}\right| \frac{T_i}{T_e}\right) \frac{\nu_{eff}}{\omega_0}. \quad (2.16)$$

The corresponding expression for the maximum increment at $k_{max}v_{Ti} < \gamma_{max} < \omega_s(k_{max})$ differs from (2.15) in the absence of a term $|e/e_i| T_i/T_e$ in the denominator. Finally, when $\gamma_{max} > \omega_s(k_{max})$ we get for the maximum increment the following relation:

$$\gamma_{max} = \frac{1}{2}\left[\frac{v_E}{v_{Te}} \frac{\omega_s(k_{max})}{\omega_0}\right]^{2/3}, \quad (2.17)$$

which is valid under conditions (2.10) and (2.12). This expression is approximately 1.5 times larger than the value of (2.11), and at $k_{max} > \omega_0/c$ it can be written in the form

$$\gamma_{max} = \frac{1}{2}\omega_0\left[\frac{v_E}{c}\frac{\omega_{Li}}{\omega_0}\sqrt{\frac{\omega_0}{|\Omega_e| - \omega_0}}\right]^{2/3} \quad (2.18)$$

with the applicability conditions (2.14).

3. As noted above, when the frequency of the external microwave field or its harmonic is not close to the frequencies of the electron plasma oscillations, the instability against excitation of potential perturbations can be due to the inverse Cerenkov effect on the electrons. To describe such a nonresonant kinetic instability we can use the following dispersion equation for the potential oscillations, obtained by Aliev et al.[6]

$$\frac{1}{\delta\varepsilon_i(\omega + i\gamma, \mathbf{k})} + \sum_{n=-\infty}^{\infty} \frac{J_n^2(a_B)}{1 + \delta\varepsilon_e(\omega + i\gamma + n\omega_0, \mathbf{k})} = 0. \quad (3.1)$$

Here $J_n(a_B)$ is a Bessel function of order n and of argument a_B:*

$$a_B^2 = \left(\frac{e(\mathbf{bE}_0)}{m_e\omega_0^2}\mathbf{b} + \frac{e[\mathbf{b}[\mathbf{E}_0\mathbf{b}]]}{m_e(\omega_0^2 - \Omega_e^2)}, \mathbf{k}\right)^2 + \left(\frac{e\Omega_e[\mathbf{bE}_0]}{m_e\omega_0(\omega_0^2 - \Omega_e^2)}, \mathbf{k}\right)^2, \quad \mathbf{b} = \frac{\mathbf{B}_0}{B_0}$$

and $\delta\epsilon_\alpha(\omega, \mathbf{k})$ is, as before, the partial contribution of the particles of type α to the longitudinal dielectric constant of the plasma.

We consider low-frequency perturbations with phase velocity exceeding the thermal velocity of the ions and with frequency higher than the ion gyroscopic frequency:

$$\omega \gg kv_{Ti}, \quad \Omega_i = e_i B_0 / m_i c. \quad (3.2)$$

Assuming the phase velocity of the perturbations to be small in comparison with the thermal velocity of the electrons

$$\omega \ll kv_{Te} \quad (3.3)$$

we obtain from the dispersion equation (3.1) the following expressions for the increment and for the frequency:

$$\gamma = \frac{\omega_{Li}^2}{\omega_0} k^2 r_{De}^2 \left\{\sum_{n=1}^{\infty} J_n^2(a_B) \frac{(n^2C_n^2 - B_n^2)E_n + 2B_nC_nD_n}{(B_n^2 + n^2C_n^2)^2} - \frac{1}{2} J_0^2(a_B)\frac{C_0}{B_0^2}\right\} - \sqrt{\frac{\pi}{8}} \frac{\omega^4}{k^3 v_{Ti}^3} \exp\left(-\frac{\omega^2}{2k^2v_{Ti}^2}\right) - \frac{4}{5}\frac{k^2v_{Ti}^2}{\omega^2}\nu_{ii}, \quad (3.4)$$

$$\omega = \omega_{Li} k r_{De} \left\{J_0^2(a_B)\frac{1}{B_0} + 2\sum_{n=1}^{\infty} J_n^2(a_B) \frac{B_n}{B_n^2 + n^2C_n^2}\right\}^{1/2}. \quad (3.5)$$

We have used here the following notation: $\nu_{ii} = 4\sqrt{\pi} e_i^4 n_i L / 3T_i^{3/2} m_i^{1/2}$ is the frequency of the ion-ion collisions, $r_{De} = v_{Te}/\omega_{Le}$ is the Debye radius of the electrons;

$$B_n = k^2r_{De}^2[1 + \operatorname{Re}\delta\varepsilon_e(n\omega_0, \mathbf{k})] = k^2r_{De}^2 + 1 - \sum_{s=-\infty}^{\infty} \frac{n\omega_0}{n\omega_0 - s\Omega_e} A_s(z) \operatorname{Re} J_+\left(\frac{n\omega_0 - s\Omega_e}{|k_z|v_{Te}}\right);$$

$$C_n = k^2r_{De}^2 \frac{1}{n} \operatorname{Im}\delta\varepsilon_e(n\omega_0, \mathbf{k}) = \sqrt{\frac{\pi}{2}} \frac{\omega_0}{|k_z|v_{Te}} \sum_{s=-\infty}^{\infty} A_s(z) \times \exp\left(-\frac{(n\omega_0 - s\Omega_e)^2}{2k_z^2v_{Te}^2}\right);$$

$$D_n = n\omega_0 \frac{\partial B_n}{\partial n\omega_0}; \quad E_n = n\omega_0 \frac{\partial C_n}{\partial n\omega_0};$$

$$\operatorname{Re} J_+(x) = x\exp\left(-\frac{x^2}{2}\right)\int_0^x \exp\left(\frac{t^2}{2}\right) dt,$$

$$A_s(z) = \exp(-z) I_s(z), \quad z = k_\perp^2 v_{Te}^2 / \Omega_e^2.$$

In the derivation of (3.4) and (3.5) it was assumed that

*$[\mathbf{b}[\mathbf{E}_0\mathbf{b}]] \equiv \mathbf{b} \times [\mathbf{E}_0 \times \mathbf{b}]$.

the increment is much smaller than the frequency ($\gamma \ll \omega$), and an expansion with respect to the frequency ω was carried out, by virtue of the inequality (3.3). The real parameters of such an expansion are the following quantities:

$$\left(\frac{\omega}{n\omega_0}\frac{D_n}{B_n}\right)^2 \ll 1, \quad \left(\frac{\omega}{n\omega_0}\frac{E_n}{C_n}\right)^2 \ll 1. \tag{3.6}$$

The vanishing of the increment (3.4) determines the limiting value $E_{0\,lim}(\mathbf{k})$ of the external microwave field intensity, above which the plasma becomes unstable against excitation of oscillations with a given wave vector $\mathbf{k}$. To obtain the minimum limiting field (the threshold) it is necessary to minimize the so-defined limiting value of the microwave field intensity with respect to the direction and magnitude of the wave vector $\mathbf{k}$ of the perturbations. Such a procedure was carried out with the aid of a numerical calculation.

Figure 1 shows, for hydrogen plasma, the dependence of the threshold microwave field intensity on the intensity of the constant magnetic field at different ion temperatures. The abscissas represent $|\Omega_0|/\omega_0$, and the ordinates

$$V_{E\,\mathrm{thr}} = \frac{v_{E\,\mathrm{thr}}}{v_{Te}} = \frac{|e|E_{0\,\mathrm{thr}}}{\omega_0\sqrt{m_eT_e}}.$$

The microwave electric field vector is perpendicular to the constant magnetic field ($\mathbf{E}_0 \perp \mathbf{B}_0$); $T_e = 6$ eV; $\omega_{Le}^2/\omega_0^2 = 0.4$, and $\omega_0 = 2 \times 10^{10}\ \mathrm{sec}^{-1}$. The decrease of the threshold pump field intensity with increasing non-isothermy of the plasma is due to the perturbation-dissipation decrease caused by the Cerenkov effect on the ions. At the selected plasma parameters, the absorption due to the ion-ion collisions becomes decisive at an ion temperature $T_i \lesssim 0.1$ eV ($T_e/T_i \gtrsim 60$), and consequently the threshold microwave field intensity ceases to decrease when the ratio of the electron and ion temperatures becomes larger than 100 ($T_e/T_i \gtrsim 100$). The perturbation wave vector for which the threshold is obtained (k_{thr}) depends on the magnetic field and on the non-isothermy of the plasma and its order of magnitude is $k_{thr} \approx \omega_0/v_{Te}$, $\theta_{thr} = \sphericalangle\, \mathbf{k}_{thr}, \mathbf{B}_0 \approx 1$. The oscillation frequency (3.5) differs little from the Langmuir ion frequency. Thus, for example, at $|\Omega_e|/\omega_0 = 1.1$ and $T_i = 0.06$ eV, $k_{thr} = 1.04\ \omega_0/v_{Te}$, $\theta_{thr} = 1.03$, and the frequency is $\omega_{thr} = \omega(\mathbf{E}_{0thr}, \mathbf{k}_{thr}) = 0.98\ \omega_{Li}$.

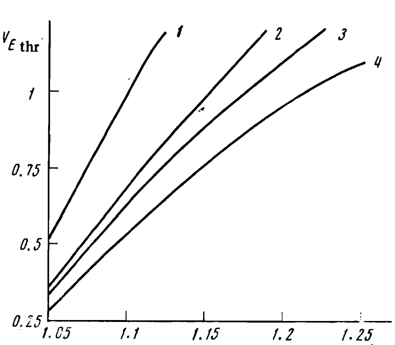

FIG. 1. Dependence of the threshold microwave electric field intensity on the constant magnetic field $|\Omega_e|/\omega_0 = |e|B_0/\omega_0 m_e c$; $V_{E\,thr} = v_{E\,thr}/v_{Te} = |e|E_{0\,thr}/\omega_0\sqrt{m_eT_e}$, $\mathbf{E}_0 \perp \mathbf{B}_0$; $\omega_{Le}^2/\omega_0^2 = 0.4$; $\omega_0 = 2 \times 10^{10}$; $T_e = 6$ eV. 1–$T_e/T_i = 10$; 2–$T_e/T_i = 20$; 3–$T_e/T_i = 40$; 4–$T_e/T_i = 100$.

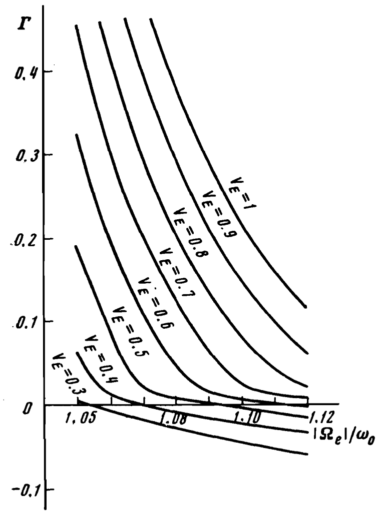

FIG. 2. Dependence of the maximum increment on the intensities of the microwave electric field and of the constant magnetic field: $\Gamma = (\gamma_{max}/\omega_{Li})(\omega_0/\omega_{Li})$; $V_E = v_E/v_{Te} = |e|E_0/\omega_0\sqrt{m_eT_e}$; $|\Omega_e|/\omega_0 = |e|B_0/\omega_0 m_e c$; $\mathbf{E}_0 \perp \mathbf{B}_0$; $\omega_{Le}^2/\omega_0^2 = 0.4$; $\omega_0 = 2 \times 10^{10}\ \mathrm{sec}^{-1}$; $T_e = 6$ eV; $T_i = 0.06$ eV.

To determine the maximum increment, expression (3.4) was maximized with respect to the direction and magnitude of the wave vector at a specified microwave intensity $\mathbf{E}_0$. The results of such a calculation are shown in Fig. 2, where the abscissas represent $|\Omega_e|/\omega_0$ and the ordinates represent the dimensionless increment $\Gamma = \gamma_{max}\omega_0/\omega_{Li}^2$. It should be noted that expression (3.4) is a complicated function of the wave vector $\mathbf{k}$ with several local extrema. At a microwave field intensity close to the threshold (for a given magnetic field and non-isothermy), the absolute maximum of the increment is reached at a wave vector $\mathbf{k}_{max}$ close to the threshold value $\mathbf{k}_{thr}$. In this case the frequency of the most rapidly growing perturbations is close to the corresponding value at the threshold ω_{thr}. With increasing microwave field intensity, the ion damping in (3.4) turns out to be negligible and the absolute maximum of the increment is reached in the vicinity of another local extremum, which leads to a jumplike change in the frequency and in the wave vector of the fastest growing perturbations. Thus, for example, for $|\Omega_e|/\omega_0 = 1.08$ and $T_i = 0.06$ eV at $v_E = 1.5v_{Te}$ we have:

$$k_{max} = 0.93\omega_0/v_{Te};$$
$$\theta_{max} = 0.97;$$
$$\omega_{max} = \omega(\mathbf{E}_0, \mathbf{k}_{max}) = 0.99\ \omega_{Li},$$

and for $v_E = 0.6\ v_{Te}$:

$$k_{max} = 0.42\ \omega_0/v_{Te};\quad \theta_{max} = 0.51;$$
$$\omega_{max} = 0.77\ \omega_{Li}.$$

The parameters of the expansion (3.6) were $\lesssim 10^{-2}$ in the presented calculations.

4. A large number of anomalous phenomena were observed in the experiments on the interaction of a microwave field with a magnetoactive plasma in the frequency region (1.1), such as anomalous heating of the electron[3], absorption of the microwave field[4], and nonthermal plasma radiation[5]. The dependence of the in-

tensity of the indicated processes on the magnetic field intensity has in this case a resonant character, revealing a correlation between these phenomena. Thus, at $1 < |\Omega_e|/\omega_0 \lesssim 1.08$ there is observed a maximum of absorption of the microwave field energy and of the radiation from the plasma at the frequency ω_0. These phenomena may be connected with a buildup of the nonpotential perturbations considered in Sec. 2 of the present article. For the plasma density used in the experiment $\omega_{Le}^2/\omega_0^2 \approx 0.4$ and $|\Omega_e|/\omega_0 \approx 1.08$ the wavelength of the fastest growing oscillations, given by formula (2.9), turns out to be half as large as the dimension of the system in the direction of the perturbation propagation (along $\mathbf{B}_0$).

The experimentally measured spectrum of nonthermal radiation at $|\Omega_e|/\omega_0 \approx 1.08$ depends essentially on the intensity of the external microwave field[5]. Thus, at $E_0 \lesssim 200$ V/cm ($v_E/v_{Te} \lesssim 0.2$) there is only a broadened fundamental line at the frequency ω_0, and when the intensity E_0 increases, satellites appear in the spectrum and are shifted relative to the fundamental line by an amount close to the ion Langmuir frequency. The saturation of the satellite intensity occurs at $v_E/v_{Te} \gtrsim 0.7$, and the shift decreases somewhat with increasing microwave field intensity. Such a spectrum may be connected with simultaneous development of instability against nonpotential perturbations (causing emission at the frequency ω_0) and the kinetic instability considered in Sec. 3 of the present article, in which low-frequency perturbations of frequency close to ω_{Li} are excited. The broadening of the fundamental emission line at the frequency ω_0 corresponds here to the frequency and increment values in (2.11) and (2.17). The theoretically determined threshold for the excitation of the kinetic instability at $|\Omega_e|/\omega_0 \approx 1.08$, however, is approximately double the threshold microwave field intensity required for the appearance of a satellite (measured in a waveguide outside the plasma). We note that for a more adequate comparison of the theory with experiment it is necessary to determine experimentally the threshold and the buildup time of the oscillations, and also to measure the spectra of the excited oscillations in the near-threshold region of the intensity of the external microwave field.

An experimental study of the anomalous dissipation of the external pump wave has shown that this phenomenon has a threshold with respect to the intensity of the external microwave field[4]. For $|\Omega_e|/\omega_0 \approx 1.08$ and $\omega_{Le}^2/\omega_0^2 \approx 0.4$ the corresponding threshold value is of the order of 60 V/cm ($v_E/v_{Te} \approx 0.06$). At such an external-field intensity, the time of instability development $\sim 1/\gamma_{max}$, determined from formulas (2.13) and (2.18), is smaller by approximately one order of magnitude than the time of the experiment[4], so that the instability with respect to the buildup of nonpotential oscillations may be responsible for the observed anomalous absorption.

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