IMPEDANCE OF TIN IN A STRONG MAGNETIC FIELD AND SCATTERING OF CONDUCTIVITY ELECTRONS BY THE METAL SURFACE

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Submitted April 6, 1972

Zh. Eksp. Teor. Fiz. **63**, 1324—1336 (October, 1972)

The surface impedance of tin single crystals is investigated at liquid helium temperatures in the radio-frequency range. In magnetic fields up to 60 kOe the penetration depth of the electromagnetic field δ changes by about two orders of magnitude and reaches a value of 1 mm. The magnitude of the impedance is found to be significantly different upon orientation of the magnetic field along the plane of the sample ($\mathbf{H} \perp \mathbf{n}$) or perpendicular to it ($\mathbf{H} \parallel \mathbf{n}$) ($\mathbf{n}$ is the normal to the sample plane). A comparison with calculations in which the static skin effect is taken into account shows that practically all electrons colliding with the surface are diffusely scattered. On the other hand, the dependence of the surface layer conductivity on H and T indicates that a small part of the carriers are reflected specularly and these carriers make a significant contribution to the excessive surface current. The number of such electrons in a 50 kOe field is estimated to be $\sim 10^{-3}$.

IT is customary to characterize the scattering of conduction electrons from a conductor-vacuum boundary by a diffuseness coefficient q, which is the fraction of the particles that change arbitrarily their momentum upon reflection from the surface.

It is customarily assumed that q is close to unity. The reason is that the conduction-electron wavelength $\lambda_e = p/h$ (p is the electron momentum and h is Planck's constant) is of the same order of magnitude as the interatomic distance in the metal. Only a small fraction of the electrons, called "glancing" electrons, which are incident on the surface at a small angle, can become specularly reflected. Connected with these electrons are the magnetic surface levels[1]. Investigations of the electric conductivity of filamentary crystals (whiskers) at liquid-helium temperatures[2,3] have revealed that $q \approx 0.5$, i.e., approximately half the electrons are specularly reflected. The reasons for the high degree of specularity of the reflection have not yet been explained.

At the same time, a theoretical analysis of the influence of surface scattering on the properties of real conductors leads to a large number of new interesting effects, which have not yet been observed or investigated in sufficient detail[4-6]. In particular, in strong magnetic fields such that $\omega_c \gg \nu$ (ω_c is the cyclotron frequency and ν is the electron scattering frequency) the current distribution over the cross section in metals having equal numbers of electrons and holes ($n_1 = n_2$) in a static field should become uneven, owing to the surface scattering; this is the static skin effect[4]. For the static skin effect to exist it is necessary to have only an unlimited increase of the resistance in the magnetic field, so that it will take place also for metals with open Fermi surfaces, but only for those current directions that are not parallel to the open sections of the Fermi surface. An experimental investigation of this phenomenon should yield new information on the character of reflection of the electrons from the surface.

The main difficulty in the experimental study of the static skin effect lies in obtaining thin plates of perfect single crystals with $d < l$ (d is the thickness of the crystal and l is the electron mean free path)[6]. This difficulty can be overcome by using the high-frequency skin effect, in which case the role of the plate thickness is assumed by the penetration depth δ.

To observe surface scattering of the conduction electrons, we have investigated the surface impedance of tin at liquid-helium temperatures in magnetic fields up to 60 kOe. Tin satisfies the conditions for observing the static skin effect. Its Fermi surface has an equal number of electrons and holes, thereby ensuring an unlimited increase of the resistivity in a magnetic field, $\rho(H) \sim H^2$. The open Fermi surface of tin complicates somewhat the experiment, since it becomes necessary to choose only the high-frequency field polarizations that correspond to current directions for which $\rho(H) \sim H^2$.

As one of the impedance-measuring methods, we used a comparison of δ with the length of the acoustic wave excited in the metal by the electromagnetic wave itself. The observed difference in the depth of penetration of the electromagnetic field at $\mathbf{H} \parallel \mathbf{n}$ and $\mathbf{H} \perp \mathbf{n}$ is due to the influence of the scattering of the conduction electrons by the surface.

I. METHOD

Measuring apparatus. This consisted of a self-oscillator whose tank circuit included a coil with the sample. The magnetic field was varied and the changes of the self-oscillator frequency f and of its high-frequency voltage U, due to the modulation of the external magnetic field, were measured. The low-frequency voltage was amplified, synchronously detected, and fed to the Y input of an automatic X-Y recorder. The magnetic field was produced by a superconducting solenoid; a voltage proportional to the current through the solenoid was fed to the X input of the X-Y recorder. The field was modulated by a separate coil placed in the solenoid channel at a modulation frequency 30 Hz.

The acoustic oscillations excited in the metal by the

Sample No.	d, mm	$\widehat{n[100]}$	H, E Orientation	$\frac{10^5\rho(4.2°)}{\rho(300°)}$	Sample No.	d, mm	$\widehat{n[100]}$	H, E Orientation	$\frac{10^5\rho(4.2°)}{\rho(300°)}$
1	0.55	0	$H \parallel n$, $E \parallel c_2$	1.8	4	0.6	0	$H \parallel n$, $E \parallel c_2$	—
2	1.0	0	$H \parallel n$, $E \parallel c_2$	1.9	5	1.0	22°	$H \parallel n$, $H \perp n$ $E \parallel c_4$	2.0
3	1.0	0	$H \parallel n$, $E \parallel c_2$	1.5					

electromagnetic wave were revealed by the resonant singularities of the impedance, corresponding to excitation of standing waves in the sample[7]. The signal, proportional to $\partial U/\partial f$, is the derivative of the characteristic of the acoustic resonator of the plates, and is produced by periodically varying the frequency f of the autodyne generator.

Samples in the form of disks 18 mm in diameter and up to 1.0 mm thick were grown in a dismountable quartz mold[8]. The purity of the samples was monitored by measuring the resistivity ratio $\rho(4.2°)/\rho(300°)$ of the fraction of the single crystal remaining in the pouring spout of the mold. As a rule, this was $(1.5-2.0) \times 10^{-5}$ (see the table), corresponding to an electric conductivity $\sigma(4.2°) \approx 3 \times 10^{21}$ cgs esu at 4.2°K. This conductivity, according to[9], corresponds to an electron mean free path $l \approx 0.5$ mm.

Reduction of measurement data. The change Δf of the generator frequency is determined by the corresponding change ΔL of the tank-circuit inductance; at not too large Δf we can write

$$\frac{\Delta f}{f} = -\frac{1}{2}\frac{\Delta L}{L}\left(1 - \frac{3}{2}\frac{\Delta L}{L}\right).$$

The change ΔU of the amplitude is determined by the change ΔR_t of the resonant resistance of the tank circuit. To determine the connection between the properties of the sample and the parameters of the coils in which it is placed it is necessary, generally speaking, to know the distribution of the high-frequency electric and magnetic fields e(z) and h(z) in the sample. For the normal skin effect we have e(z), h(z) $\sim \exp(-z/\delta_n)$, where $\delta_n = c(2\pi\omega\sigma)^{-1/2}$ (c is speed of light, ω is the angular frequency of the high-frequency field, and σ is the electric conductivity), and for a thin plate with $D \gg d$ (D is the width and d is the thickness of the plate), we obtain

$$\Delta L \sim \int_0^d h^2(z)\,dz \sim \delta_n \frac{\operatorname{sh}\xi + \sin\xi}{\operatorname{ch}\xi + \cos\xi} \quad (1)$$

$$\Delta R_t \sim \sigma\int_0^d e^2(z)\,dz \sim \delta_n \frac{\operatorname{sh}\xi - \sin\xi}{\operatorname{ch}\xi + \cos\xi}, \quad (2)$$

$\xi = d/\delta_n$. It follows from these expressions that at $\delta_n/d \approx 1$ the changes produced in the tank circuit by the sample reach the maximum values ΔR_t^{max} and ΔL^{max}; more accurately, ΔR_t^{max} takes place at $\delta_n/d = 0.46$, in which case $\Delta L = 0.61\,\Delta L^{max}$. It follows also from (1) and (2) that at $\delta_n/d < 0.3$ the changes of ΔR_t and ΔL are proportional, accurate to $\sim 3\%$, to the changes of the surface impedance of the half-space

$$Z_\infty = R_\infty + iX_\infty,\ \Delta R_t \sim \Delta R_\infty \text{ and } \Delta L \sim \Delta X_\infty.$$

We note that the value of the impedance does not determine uniquely the depth of penetration δ of the high-frequency field into the metal. Formally it is possible to introduce separately for the real and imaginary parts of Z parameters with dimensions of length, which in general are not equal to each other

$$\delta_R = \frac{c^2}{2\pi\omega}R_\infty, \qquad \delta_i = \frac{c^2}{2\pi\omega}X_\infty,$$

but in the case of the normal skin effect $\delta_R = \delta_i = \delta_n$. Therefore the experimental plots of $\Delta f/f$ at $\delta_n/d < 0.3$ are similar to the plots of $\delta_i(H)$ or $X_\infty(H)$.

The transformation of electromagnetic waves into acoustic waves at the boundary between vacuum and a conductor was considered theoretically in[10,11]. In the case of the normal skin effect, the transformation coefficient reaches a maximum when

$$\frac{c^2}{s_t^2}\frac{\omega}{4\pi\sigma} = 1, \quad (3)$$

where s_t is the speed of sound, corresponding to $\lambda \approx 4.4\delta_n$. Excitation of acoustic waves in the plate can lead to establishment of standing waves at a resonant frequency $f_{res} = s_t(2n+1)/2d$, where n is an integer. The impedance change registered in this case is proportional to the intensity of the acoustic oscillations and also reaches a maximum when relation (3) is satisfied.

Measurement errors. Quantitative information on the change of δ_i in a magnetic field are obtained from the relations $\Delta f(H)/f_0$, $\Delta f(H) = f(H) - f_0$, $f_0 = f(0) + \Delta f_{sc}$ at $T > T_c$, $f_0 = f(0)$ at $T < T_c$, and $T_c = 3.7°$ is the temperature of the superconducting transition of tin. The value of $\Delta f^{max}(H)/f_0$ in the experiments did not exceed 0.015, so that neglect of the term quadratic in $\Delta L/L$ in the expression

$$\frac{\Delta f}{f_0} = -\frac{1}{2}\frac{\Delta L}{L}\left(1 - \frac{3}{2}\frac{\Delta L}{L}\right)$$

leads to an inaccuracy in the determination of $\Delta L/L$, and consequently to a value of δ not exceeding 3%. The frequency jump Δf_{sc} at the superconducting transition amounts to $\approx 1\%$ of $\Delta f^{max}/f_0$, so that the depth of penetration of the field in the superconductor, $\delta_{sc} \approx 10^{-5}$ cm, can therefore be neglected with practically no increase in the measurement error; the frequency f_0 can therefore be taken as the reference corresponding to $\delta = 0$. The ratio of the sample thickness d to its diameter D did not exceed 0.06, and consequently the error in the determination of the absolute value of δ_i is not worse than 9%.

The frequency was measured with a digital frequency meter accurate to 1 Hz. The oscillator frequency drift in one hour of operation did not exceed 1×10^{-5} f. The magnetic field H was measured with accuracy $\approx 1.5\%$.

II. RESULTS

The measurements were performed for different directions of the field H on samples of different thicknesses. The results obtained were consistent in all cases.

1. **H** $\parallel$ **n**. General plots of $\partial R/\partial H$ and $\Delta f(H)/f_0$ against the magnetic field are illustrated in Figs. 1 and 2. Under the conditions described in the figure caption, the static resistance increases without limit: $\rho(H) \sim H^2$. Small changes of the frequency in the strongest fields and the reversal of the sign of the derivative $\partial R/\partial H$, corresponding to the absorption maximum, indicate that the depth of penetration of the electromagnetic field in the

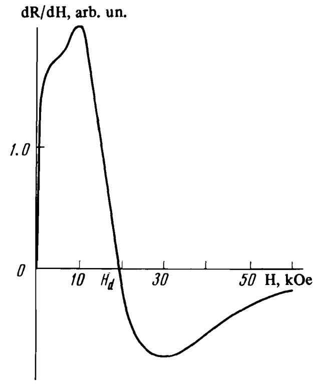

FIG. 1. Dependence of the derivative of the real part of the plate impedance with respect to the magnetic field $\partial R/\partial H$ on the magnetic field intensity H. Sample Sn-1, d = 0.55 mm, **H** ∥ **n** ∥ c_2, **E** ∥ c_2; f = 255 kHz, T = 4.2°K.

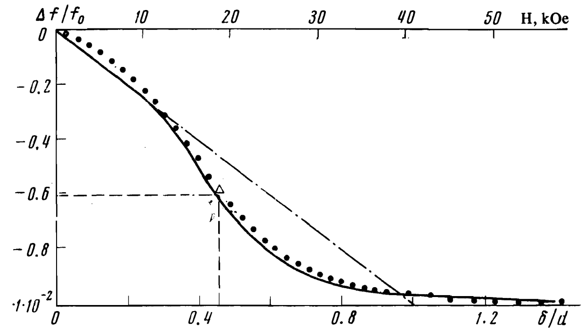

FIG. 2. Relative change $\Delta f(H)/f_0$ of the frequency of the self-oscillator as a function of H, $\Delta f(H) = f(H) - f_0$, where f_0 is the self-oscillator frequency when the sample is superconducting; ●—experimental points, solid line—calculation; the symbol Δ denotes the change of $f(H)/f_0$ when $\partial R/\partial H = 0$. The choice of the scale on the axis in accord with formulas (1) and (2) is illustrated by the dashed lines. The dash-dot line shows the plot of δ/d against H. The sample and the conditions are the same as in Fig. 1.

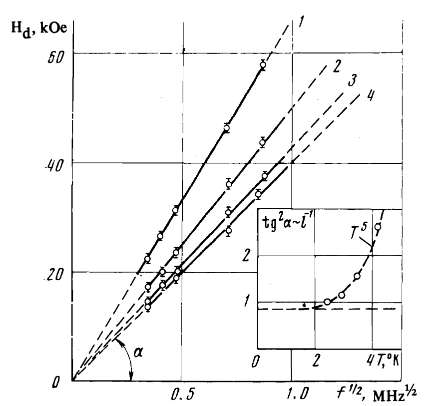

FIG. 3. Values of the magnetic field H_d vs $f^{1/2}$. The insert shows the temperature dependence of the squared slopes of the lines $\tan^2\alpha \sim l^{-1}$. Sample Sn-3, d = 1.0 mm, **H** ∥ **n** ∥ c_2, **E** ∥ c_2; 1—4.2, 2—3.5, 3—3.0, 4—2.5°K.

sample becomes comparable with and exceeds its thickness d (see expressions (1) and (2).

To determine the type of the skin effect, we plotted the magnetic field at which the absorption has a maximum, H_d, against $f^{1/2}$ (Fig. 3). The good agreement between the experimental points and the straight lines drawn through the origin enables us to represent δ(f, H) in the form

$$\delta(f, H) = A(T)\, H/\sqrt{f},$$

which corresponds to the following expression for δ in the normal skin effect:

$$\delta_n(f, H) = c[2\pi\omega\sigma(H)]^{-1/2}, \quad \sigma(H) = \sigma(0)\,(r/l)^2,$$

where σ(0) is the static conductivity in the absence of the magnetic field and r is the Larmor radius.

It follows therefore that $H_d(T_1)/H_d(T_2) = (l_2/l_1)^{1/2}$ and the slope of the line in Fig. 3, at a fixed temperature, is $\tan\alpha = H_d/\delta f^{1/2} \sim l^{-1/2}$. At T = 4.2°K, samples with **n** ∥ c_2 are characterized by the quantities tan α, H_d (in kOe), f (in MHz), and δ = 0.46d. For sample No. 1 we have d = 0.55 mm and tan α = 34; for sample No. 2 we have d = 1 mm and tan α = 32, for sample No. 3, d = 1 mm and tan α = 29; for sample No. 4, d = 0.6 and tan α = 31.

The insert of Fig. 3 shows the temperature dependence of $\tan^2\alpha \sim l^{-1}$ in arbitrary units; it is seen that the electron mean free path increases 2.6 times when the temperature drops from 4.2 to 2.2°K. The experimental points can be reconciled with the $l^{-1} = l_0^{-1} + \beta T^5$ relation corresponding to the law governing the change of the resistance of metals at low temperatures ρ(T) $= \rho_0 + \beta' T^5$, if we put $l_0 \approx 1.1\, l_{2.5°}$. Thus, when the temperature is lowered H_d changes in accordance with the changes of the static conductivity σ(H); taken together with the relation $H_d(f) \sim f^{1/2}$, this allows us to conclude that a normal skin effect takes place in magnetic fields $H \geq H_d$ at **H** ∥ **n**. This fact can be used to set each point of the experimental plot of $\Delta f(H)/f_0$ in correspondence with the curve calculated for the normal skin effect, and they should become identical in fields $H > H_d$.

To this end, using the relation $\delta_n/d = 0.46$ at the absorption maximum, we need experimental plots of $\partial R/\partial H$ and $\Delta f(H)/f_0$ as functions of the ratio δ_n/d. Since the impedance in the normal skin effect depends linearly on H, it suffices for this purpose to put $\delta_n(H_d)/d = 0.46$ and $\Delta f(H_d)/f^{max} = 0.61$. In such a scale, the experimental curve agrees well with the calculated one in fields $H \geq H_d$ (Fig. 2). This enables us to plot $\delta_i(H)$ or $X_\infty(H)$ for a half-space in the employed magnetic field interval (Fig. 4).

With respect to the plots for the normal skin effect $\delta_n(H)$, corresponding to the straight lines passing through the origin, one can indicate three characteristic

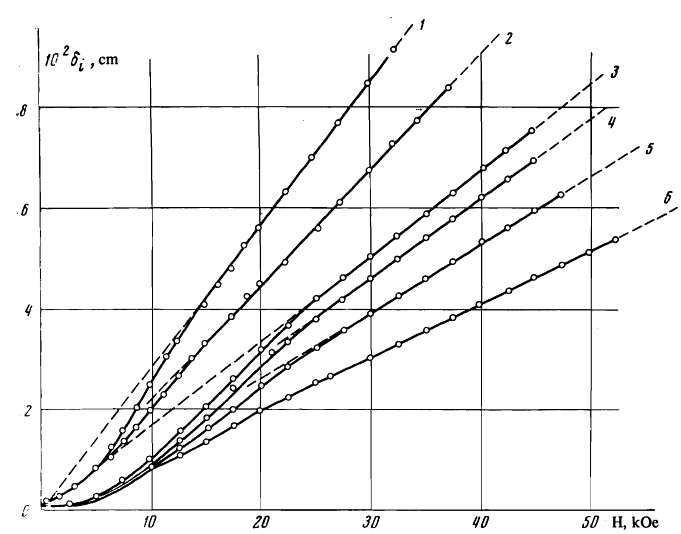

FIG. 4. Plots of $\delta_i(H)$ for a half-space, determined from the experimental plots of $\Delta f(H)/f_0$ for a plate: sample Sn-3, f = 160 kHz: 1—3.5°; 2—4.2°K; f = 506 kHz; 3—2.5; 4—3.0; 5—3.5; 6—4.2°K.

regions on the experimental curves. The first region $\delta_i > \delta_n$ occupies the magnetic-field interval from zero to several hundred Oe and corresponds to the anomalous skin effect. The second, $\delta_i < \delta_n$, occupies the interval from several hundred Oe to fields $H \leq 0.5\, H_d$. In this region, unlike the first, the impedance is sensitive both to changes in the magnetic field and to changes in the temperature; a characteristic feature is here the parabolic dependence of δ_i on H. In the third region, $\delta_i \approx \delta_n$, the field is $H \gtrsim 0.5 H_d$ and the impedance, just as in the second region, varies with changing magnetic field and temperature, but the dependence of δ_i on the magnetic field is linear in this case; this is the region of the normal skin effect.

In the case of tin, the excitation of acoustic waves by an electromagnetic wave in a strong magnetic field is due to the ponderomotive action $\mathbf{j} \times \mathbf{H}/c$ of the current $\mathbf{j}$ flowing in the skin layer. With such excitation, the conductivity $\sigma(H)$ of the surface layer of the metal should exhibit singularities that are revealed by the dependence of H_M on the frequency at which the condition (3) is satisfied. This condition corresponds to a transition from the regime of local excitation of acoustic waves, $\delta < \lambda$, to the nonlocal regime, $\delta > \lambda$. Figure 5 shows plots of the intensities of the standing acoustic waves against the magnetic field. The observed maxima correspond to satisfaction of the condition (3) when $\delta_n \approx \lambda/4.4$. The values of the frequencies f_{res} of the magnetic fields H_M at which the maxima are observed, are

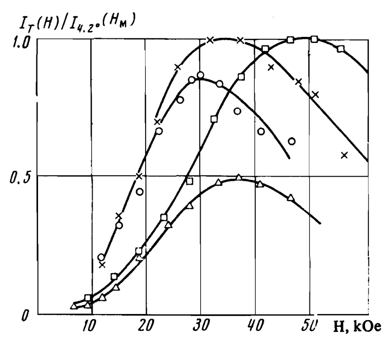

FIG. 5. Relative intensity of acoustic resonances in Sn-1 sample (d = 0.5 mm) vs. the magnetic field at temperatures 4.2 and 2.1°K; E ∥ c_2, H ∥ n ∥ c_2; f_{res} = 1.76 MHz: □—4.2°K, Δ—2.1°K; f_{res} = 5.3 MHz: ×—4.2°K, O—2.1°K.

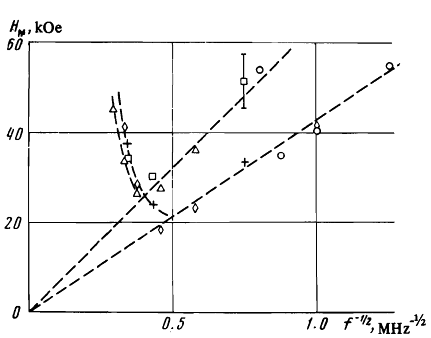

FIG. 6. Magnetic field H_M, at which the intensity of the acoustic resonances is maximal, vs $f^{-1/2}$. H ∥ n ∥ c_2, E ∥ c_2; Sn-1: +—4.2°K, □—2.1°K; Sn-2: ◇—4.2°K, Δ—2.1°K. The calculated values of the magnetic field H'_M at which the acoustic resonances should have maximum intensity under the same conditions are denoted by light circles; $H'_M = H_d s_t/2fd$.

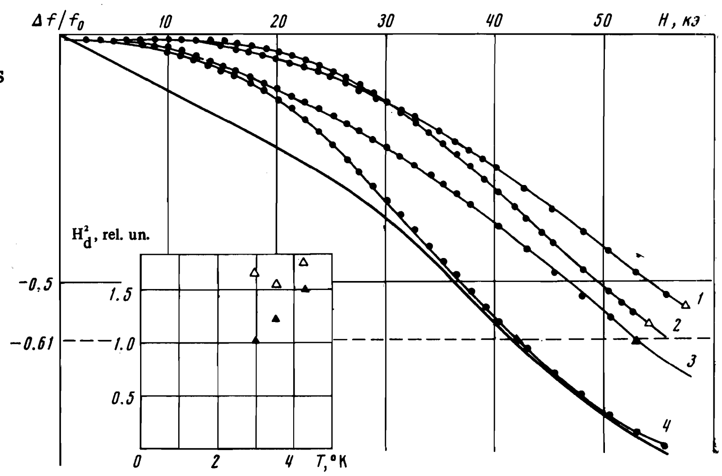

FIG. 7. Relative change of the oscillator frequency as a function of H. The ordinate scale is in units of $\Delta f(H)/f_0 = 0.015$. Sample Sn-5: d = 1.0 mm, E ∥ c_4; T = 4.2°K; 1—H ⊥ n, 3—H ∥ n; T = 3.0°K: 2—H ⊥ n, 4—H ⊥ n. Solid line—calculation for T = 3°K and H ∥ n. The symbols ▲ and Δ denote the value of $\Delta f(H)/f_0$ at which $\partial R/\partial H = 0$ for H ∥ n and H ⊥ n, respectively. The insert shows the temperature dependence of $H_d^2 \sim l^{-1}$, ▲—H ∥ n, Δ—H ∥ n.

plotted in coordinates $f^{-1/2}$ and H (see Fig. 6). In terms of these coordinates, the line determined by condition (3) is straight and passes through the origin if the skin effect is normal. The experimental data at frequencies up to 5 MHz agree quite satisfactorily with this assumption. At higher frequencies, the maxima are observed at larger values of H than expected from calculations using the normal skin effect.

For comparison, the measured values of the impedance obtained from the absorption maximum and from the maximum intensity of the acoustic waves are shown together (Fig. 6). To this end, as follows from (3), the experimental values of H_d must be multiplied by the ratio of half the wavelength of sound at this frequency, $\lambda/2 = s_t/2f$, to the plate thickness d. The values of $H'_M = H_d s_t/2fd$ ($s_t = 1.9 \cdot 10^5$ cm/sec) recalculated in this manner, some of which are shown in Fig. 6, agree well with the acoustic measurements of δ. We note that the increase of H_M at high frequencies agrees qualitatively with the $\delta_i(H)$ dependence in the transition region. Indeed, any specified value of δ_i is attained in this region at a larger value of the magnetic field than in the case of the normal skin effect that takes place at lower frequencies.

2. **H ⊥ n.** At this magnetic-field orientation relative to the surface, we investigated a specially grown sample, the normal to whose plane was in the (001) crystallographic plane and made an angle 22° with the [100] axis, so that by rotation about the c_4 axis magnetic field can be directed along equivalent crystallographic directions both at **H** ∥ **n** and **H** ⊥ **n**.

In comparison with the orientation **H** ∥ **n**, the following new singularities are observed (see Fig. 7). First, a weaker temperature dependence of $\Delta f(H)/f_0$, so that the difference between these dependences at **H** ⊥ **n** and **H** ∥ **n** increases with decreasing temperature. Second, there appears a magnetic-field region in which $\Delta f(H)$ decreases with decreasing temperature. Third, H_d changes with decreasing temperature in an entirely different manner than at $H \parallel n$. Whereas at $H \parallel n$ the values of H_d decrease monotonically with decreasing temperature, at $H \perp n$ the value of H_d does not change within the limits of measurement accuracy.

III. DISCUSSION

It follows from the impedance-measurement results by increasing the intensity of the magnetic field one can change the character of the skin effect in the samples from anomalous to normal. The depth of penetration of the electromagnetic field δ changes by approximately 2 orders of magnitude when H varies up to 60 kOe in the frequency interval from 0.1 to 0.5 MHz. Such changes in the value of δ and in the character of the skin effect are usually observed only when the temperature changes. In the absence of a magnetic field, the transition to the anomalous skin effect is connected with a decrease of the effective conductivity $\sigma_{eff} \approx \sigma(0)\delta/l$, which occurs when the temperature is lowered in sufficiently pure metals. This decrease of σ_{eff} is the reason why δ is larger than the value δ_n corresponding to the static value of $\sigma(0)$ under the same conditions. In a magnetic field, the picture is reversed, namely, $\delta_i < \delta_n$ in the transition region (Figs. 4 and 7), and consequently the effective conductivity is $\sigma_{eff} > \sigma(H)$, in spite of the fact that $l > \delta$. In the strongest magnetic fields in which the normal skin effect is reached at $H \parallel n$, the character of the high-frequency conductivity is apparently altered when the magnetic field is directed along the sample surface. This is evidenced by the fact that the behavior of H_d with decreasing temperature no longer reflects the temperature changes in the static electric conductivity of the sample, as is the case when $H \parallel n$.

A common feature of the noted phenomena is that the electric conductivity increases in a strong magnetic field, $\omega_c \gg \nu$, relative to the static value $\sigma(H)$. According to[12], the high-frequency conductivity of a compensated metal in a strong magnetic field takes the form

$$\sigma(k, H) = \sigma_0\gamma^2(1 - i\,\omega/\nu) + \sigma_0\gamma(kr)^2, \tag{4}$$

where $\gamma = r/l = \nu/\omega_c$, $k \approx 1/\delta$.

For the frequencies used in the present measurements ($\nu \gg \omega$), the first term hardly differs from the static conductivity $\sigma(H)$. The second term is connected with an interaction peculiar to the magnetic field, that of the electrons with the inhomogeneous electric field. In our samples $l \approx 0.5$ mm, so that the second term can be neglected in fields $H > 10$ kOe. Thus, the observed effect must be connected with the first term of expression (4). The increase of the conductivity is due in this case to the increase of γ, which can be caused by the interaction of the conduction electrons with the surface. The additional scattering relative to the volume, in a layer of thickness $\sim r$, increases the value of γ averaged over the entire region in which the high-frequency current of depth δ is concentrated.

Let us explain the quantitative agreement between the experimental results and these concepts. The impedance change due to the surface scattering depends on the relations between δ, l, and r, which determines the type of the skin effect. In a sufficiently strong magnetic field ($\delta \gg r$), these changes should be the same as for the static resistance of the conductors. As shown by calculations[4] and experiment[6], the decrease of the static resistance of plates in a strong magnetic field, due to surface scattering at $H \parallel n$, is much larger than at $H \perp n$, when this decrease is negligibly small. Such a picture agrees well with the relation observed in the present paper between the quantities $\delta_i(H)$ in the region $H \gtrsim H_d$ (Fig. 7). First, at $H \perp n$ the impedance is smaller than at $H \parallel n$; second, the normal skin effect takes place at $H \parallel n$, i.e., the surface scattering is weak. Scattering by the surface, which makes the conductivity in a thin layer of depth of the order of r much larger than the conductivity in the interior of the sample, should influence the magnetic field at which the maximum power is dissipated in the plate, perhaps because of the unusual behavior of H_d at $H \perp n$.

Thus, the results pertaining to the magnetic field region $H \gtrsim H_d$ do not contradict a possible influence of the scattering of electrons by the surface on the impedance.

When the magnetic field is decreased, and with it the ratio δ/l, the dependence of the impedance on H, which is linear in the strongest magnetic field, becomes parabolic. In this magnetic-field region, the deviation from the normal skin effect is maximal and is observed not only at $H \perp n$ (Fig. 7) but also at $H \parallel n$ (Fig. 4). The stronger influence of the surface layer on the impedance, in comparison with the stronger fields, can be naturally explained as being due to the increase of the ratio r/δ, i.e., to the increase of the fraction of the current flowing in the surface layer $\sim r$, relative to the total current in the layer $\sim \delta$. These features of the behavior of the impedance in the transition region, as noted earlier[13], agree in the main with the results of Azbel' and Rakhmanov[12], who have shown that the parabolic dependence of the impedance on H at $\delta < l$ is a manifestation of the properties of the surface layer exclusively.

Thus, the character of the dependence of the imaginary part of the impedance on the magnetic field, both in the region of the normal effect and in the transition region, can be related to the scattering of conduction electrons by the metal surface.

For a quantitative comparison, we used the results of another paper by Azbel' and Rakhmanov[14]. The impedance of a plate placed in the high-frequency coil can be expressed in the form

$$Z_s = \frac{Z_0}{1 + Z_0 S}, \tag{5}$$

where $Z_0 = R_0 + iX_0$ is the impedance of the plate without allowance for the surface scattering, R_0 and X_0 are proportional to expressions (1) and (2), and S is a term that takes into account the surface current. The real and imaginary parts of the expression (5) are equal to

$$R_s = -R_\infty \frac{F(\xi) + 2R_\infty SG(\xi)}{1 + 2R_\infty SF(\xi) + 2(R_\infty S)^2 G(\xi)}, \tag{6}$$

$$X_s = -R_\infty \frac{F'(\xi)}{1 + 2R_\infty SF(\xi) + 2(R_\infty S)^2 G(\xi)}, \tag{7}$$

where

$$R_\infty = \frac{4\pi}{c}\sqrt{\frac{\omega}{2\pi\sigma}}, \qquad F(\xi) = \frac{\operatorname{sh}\xi - \sin\xi}{\operatorname{ch}\xi + \cos\xi},$$

$$F'(\xi) = \frac{\operatorname{sh}\xi + \sin\xi}{\operatorname{ch}\xi + \cos\xi}, \qquad G(\xi) = \frac{\operatorname{sh}^2\xi + \sin^2\xi}{(\operatorname{ch}\xi + \cos\xi)^2}.$$

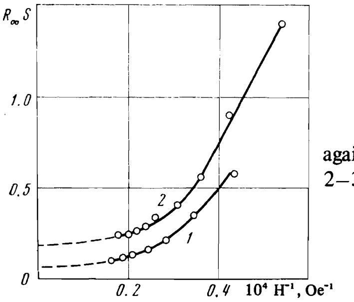

FIG. 8. Plot of $R_\infty S \sim j_s/j_v$ against the magnetic field. 1—4.2°K; 2—3°K.

We have used here also the equality $X_\infty = R_\infty$, which is valid for the normal skin effect. Since $R_\infty F'(\xi) \equiv X_0$, by using the quantity $\Delta f^0(H)/\Delta f_\perp(H)$ ($\Delta f^0(H)$ is the calculated $\Delta f(H)$ dependence at $\mathbf{H} \parallel \mathbf{n}$) for the ratio X_0/X_s at arbitrary H, we obtain from (7) a quadratic equation in $R_\infty S$. Figure 8 shows the positive roots of the equations for different values of the magnetic field at 4.2 and 3.0° K. The quantity $R_\infty S$, as can be easily verified, is the ratio of the current j_s in the surface layer to the current j_v in the skin layer δ_n. The change in the ratio j_s/j_v in the magnetic field is characterized by two features: the first is that this ratio does not tend to zero with increasing H, and the second is that j_s/j_v increases with decreasing H in accordance with the power law H^{-n} with an exponent $n > 1$.

The conductivity of the surface layer in the case of diffuse scattering of the electrons is $S_d = \sigma_0(r/l)r$, and in the case of specular scattering $S_{sp} \approx \sigma_0 r$. Since $r/\delta_n < 10^{-2}$ and $r/l < 10^{-2}$ in fields $H > 20$ kOe, the result $j_s/j_v < 1$ shows that, in the main, the electrons are scattered diffusely from the surface. At the same time, the finite value of j_s/j_v as $H \to \infty$ may be connected with electrons that are reflected specularly. Indeed, let us represent the surface conductivity at $\mathbf{H} \perp \mathbf{n}$ in the form $S \approx \sigma_0(r/l + p)r$, where the first and second terms are connected with the electrons scattered diffusely and specularly, respectively ($p \ll 1$ is the fraction of the specularly reflected electrons). Since the impedance of the half-space in the normal skin effect is $R \sim \sigma_0^{-1/2} l r^{-1}$, it follows that $R_\infty S \sim \sigma_0^{1/2} l(r/l + p)$, yielding a finite value of $R_\infty S$ as $r \to 0$. As seen from Fig. 8, the part of $R_\infty S$ which depends on the magnetic field and is connected with the electrons diffusely scattered at $H \approx 50$ kOe and $T = 4.2$° K is approximately equal to the value connected with the specularly reflected electrons. This leads to the estimate $p \approx \gamma \approx 3 \times 10^{-3}$ under these conditions. With decreasing temperature, the contribution of the diffuse part increases at a rate $\sim l^{1/2}$, which is slower than the rate $\sim l^{3/2}$ for specular scattering. Thus the fact that $R_\infty S$ is three times larger at $T = 3$° K than at $T = 4$° K serves as additional evidence favoring the existence of electrons that are specularly reflected from the surface. We note that the obtained estimate of p does not contradict the estimate of the number of "glancing" electrons $\lambda_e/\Delta \approx 10^{-2}$ (λ_e is the de Broglie wavelength of the electron and Δ is the characteristic dimension of the inhomogeneity of an optically polished surface). The second noted feature, the rapid growth of the surface current relative to the volume current with decreasing H, may be caused by a number of factors: first, by a strong damping of the electromagnetic field in the surface layer than assumed in the calculation, especially if the inequality $r \ll \delta$ becomes weaker, and second, with the dependence of p on the magnetic field, in which case $p \sim H^{-(2-3)}$.

The obtained values of $R_\infty S$ at $\mathbf{H} \perp \mathbf{n}$ enable us to calculate the magnetic field at which a maximum of absorption is observed in the plate, when account is taken of the additional surface currents. Calculation of R_s (Eq. (6)) yields for H_d at $T = 3$° K an approximate value 60 kOe, which is close to the observed 54 kOe.

On going to weaker fields, the quantity $R_\infty S \sim j_s/j_v$ increases rapidly and becomes larger than unity. The impedance and its dependence on the temperature are determined in this case by the corresponding relations for the surface current j_s. This region, in accord with the results of[12], corresponds to the sections of the parabolic dependence of Z on the magnetic field in the experimental curves, and the decrease of the impedance with decreasing temperature at $\mathbf{H} \perp \mathbf{n}$ in fields $H \approx$ $H \approx 20$ kOe agrees with the temperature dependence obtained for the surface current in stronger fields.

In a magnetic field normal to the surface, as shown in[12], the surface scattering of the electrons should also become manifest, but naturally less strongly than at $\mathbf{H} \perp \mathbf{n}$. The necessary condition for this is smallness of the ratio δ/l. The decrease of the impedance in the transition region at $\mathbf{H} \parallel \mathbf{n}$ (Fig. 4) is due to electrons scattered diffusely from the surface. It is of interest in this connection to measure the impedance at δ/l by exciting acoustic waves in the metal. As shown by our measurements, this method gives a correct picture of the variation of the impedance with changing magnetic field, both in the region where the surface scattering of the electron is appreciable, and in the normal skin effect, where good quantitative agreement was obtained with measurements performed by another method. This justifies the use of acoustic measurements as an independent method of determining the impedance.

We note that measurements of the impedance at $\mathbf{H} \parallel \mathbf{n}$ in the region of the normal skin effect of interest from the point of view of investigations of the temperature dependence of the electron mean free path in very pure metals. The results (Fig. 3) show that temperature measurements of the magnetic field at which an absorption maximum is observed in the plate make it possible to trace the variation of l below the temperature of the superconducting transition.

Thus, different features of the behavior of the impedance of tin in a strong magnetic field, both in the region of the normal skin effect and in the region where the transition to the anomalous skin effect takes place, can be explained from a unified point of view as a result of an additional surface current on top of the volume current. This current is a consequence of the inhomogeneous electric conductivity produced in a strong magnetic field by the scattering of the conduction electrons from the metal surface.

In conclusion, I am deeply grateful to Yu. P. Gaĭdukov for fruitful discussion and to S. Ya. Rakhmanov for a discussion of the results.

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Translated by J. G. Adashko
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