BOSE CONDENSATION OF EXCITONS IN A STRONG MAGNETIC FIELD

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A model is considered of a semiconductor in which the edges of the conduction and valence bands in momentum space coincide, placed in the field of a coherent source of light, the frequency Ω of which is in the range below the threshold of the interband transitions but above the level of the exciton transition. When $\hbar\Omega$ only slightly exceeds the excition transition energy, the equation for the function of the Bose condensate of excitons has the form of an inhomogeneous Schrödinger equation with a coherent source in the right-hand side. The latter thus plays the role of a source of the exciton Bose condensate. The equation for the density of the Bose condensate is found to be cubic. When the intensity of the source exceeds a certain critical value, there exists a range of values of Ω in which all three roots of this equation are real and in which there is a discontinuous change in the density of the Bose condensate on variation of either the frequency Ω or the intensity. Correspondingly, either frequency or amplitude hysteresis occurs. Bose condensation of excitons, like the magnetization of a ferromagnet in the presence of an external field, occurs at any temperature. However, the system possesses a first-order phase transition, related to the discontinuous change in the density of the Bose condensate on variation of the temperature.

THE collective properties of a system of nonequilibrium excitons created by powerful optical sources have been widely investigated recently[1]. The experiments are usually performed under conditions when the photon energy $\hbar\Omega$ is much greater than the gap width E_g. In this case, free electrons and holes are generated, and these thermalize and combine into excitons in times much shorter than the exciton lifetime.

The existing light sources make it easily possible to attain exciton concentrations sufficient for Bose condensation at temperatures of the order of 10–100°K. At the same time, Bose condensation of excitons has, apparently, not been observed, and this may be connected with the competing process of formation of electron-hole drops[2] in the case when the "vaporization" temperature of the drops is higher than the Bose-condensation temperature.

In this paper, we consider the situation when excitons are generated directly by a strong electromagnetic field of frequency Ω close to the exciton level ($\hbar\Omega \gtrsim E_g - E_{ex}^0$, E_{ex}^0 is the binding energy of an isolated exciton). After a unitary transformation removing the explicit time dependence in the Hamiltonian[3], the gap width E_g is replaced by $E_g' = E_g - \hbar\Omega$, so that the effective gap width E_g' is less than the exciton binding energy E_{ex}^0. As is well known[4], such a system turns out to be unstable with respect to the formation of an exciton Bose-condensate. The effect of a finite exciton concentration on the position of the exciton energy level under Bose-condensation conditions was taken into account in the work of Keldysh and Kozlov[5], who showed that the position of the exciton level is shifted linearly with the concentration (which is assumed given) towards the continuous spectrum. In the present work, the position of the exciton level is fixed by the frequency of the electromagnetic field, and the detuning $\hbar\Omega - (E_g - E_{ex}^0)$ determines the concentration. Here it is assumed that the electromagnetic field intensity is sufficient for the exciton level E_{ex} to adjust to the frequency of the electromagnetic field.

It is interesting to note that the dependence of the density of the exciton condensate on the electromagnetic field intensity has a non-monotonic character. With increasing field, the density first grows monotonically, and then, at a certain critical value, increases discontinuously to the value determined by the detuning.

Another interesting result, obtained below, is that, since a coherent electromagnetic field is a source of Bose-condensed excitons, the Bose condensation occurs at any temperature. In this sense, there is a definite analogy with a ferromagnet in the presence of an external magnetic field, when the magnetization exists at any temperature. Thus, for temperatures above the vaporization temperature of the electron-hole drops, Bose condensation of excitons is possible in a strong electromagnetic field.

We shall consider a semiconductor in which the edges of the conduction and valence bands coincide in momentum space and in which the electron and hole masses are equal, in the field of a strong electromagnetic wave with frequency in the range $E_g > \hbar\Omega > E_g - E_{ex}^0$, $E(\mathbf{r}, t) = E_0 \cos(\Omega t - \mathbf{k}\cdot\mathbf{r})$. In the effective-mass approximation, the Hamiltonian of the system has the form

$$H(t) = \sum_{\mathbf{p}} \left\{ \left(\frac{E_g}{2} + \frac{\mathbf{p}^2}{2m} \right) (a_{c\mathbf{p}}^+ a_{c\mathbf{p}} - a_{v\mathbf{p}}^+ a_{v\mathbf{p}}) + \lambda_{\mathbf{p}} a_{c\mathbf{p}}^+ a_{v\mathbf{p}} e^{-i\Omega t} + \lambda_{\mathbf{p}}^* a_{v\mathbf{p}}^+ a_{c\mathbf{p}} e^{i\Omega t} \right\} + \sum_{\mathbf{qpp}'} V_{\mathbf{q}} a_{c\mathbf{p}}^+ a_{v\mathbf{p}'}^+ a_{v\mathbf{p}'-\mathbf{q}} a_{c\mathbf{p}+\mathbf{q}}. \quad (1)$$

Here $a_{c\mathbf{p}}^+$ and $a_{v\mathbf{p}}^+$ are creation operators for electrons in the conduction and valence bands; the third and fourth terms describe the resonance interaction with the electromagnetic field, and the last term describes the Coulomb interaction between the electrons and holes;

$$V_{\mathbf{q}} = \frac{e^2}{\varkappa q^2}, \qquad \lambda_{\mathbf{p}} = \frac{e}{2\Omega} \mathbf{v}_{cv}(\mathbf{p}) \mathbf{E}_0,$$

where κ is the dielectric permittivity. The matrix element $v_{cv}(\mathbf{p})$ of the allowed interband transition depends weakly on the momentum and, for $p/\sqrt{2m} \lesssim \sqrt{E^0_{ex}} \ll \hbar/a\sqrt{2m}$, can be assumed constant (a is the lattice constant).

After the unitary transformation[3]

$$U(t) = \exp\left\{-i\frac{\Omega t}{2}\sum_{\mathbf{p}}(a_{c\mathbf{p}}^{+}a_{c\mathbf{p}} - a_{v\mathbf{p}}^{+}a_{v\mathbf{p}})\right\} \tag{2}$$

the Hamiltonian (1) is brought to the form

$$H = \sum_{\mathbf{p}}\{\xi_{\mathbf{p}}(a_{c\mathbf{p}}^{+}a_{c\mathbf{p}} - a_{v\mathbf{p}}^{+}a_{v\mathbf{p}}) + \lambda a_{c\mathbf{p}}^{+}a_{v\mathbf{p}} + \lambda^{*}a_{v\mathbf{p}}^{+}a_{c\mathbf{p}}\} + \sum_{\mathbf{p}\mathbf{p}'\mathbf{q}} V_{\mathbf{q}}a_{c\mathbf{p}}^{+}a_{v\mathbf{p}'}^{+}a_{v\mathbf{p}'-\mathbf{q}}a_{c\mathbf{p}+\mathbf{q}}, \tag{3}$$

where $\xi_{\mathbf{p}} = E'_g/2 + p^2/2m$. We shall assume that $0 < E'_g < E^0_{ex}$, i.e., that the photon energy lies below the threshold of the interband transitions, but above the level of the exciton transition. Then the system turns out to be unstable with respect to the formation of a Bose condensate of excitons[4]. To describe such a state, we introduce the Green functions

$$G_c(\mathbf{p}, t-t') = -i\langle Ta_{c\mathbf{p}}(t)a^{+}_{c\mathbf{p}}(t')\rangle,$$
$$G_{vc}(\mathbf{p}, t-t') = -i\langle Ta_{v\mathbf{p}}(t)a^{+}_{c\mathbf{p}}(t')\rangle,$$

which satisfy the following equations:

$$\begin{aligned}(\omega-\xi)G_c(\mathbf{p},\omega) &= (\lambda+\Sigma_{cv})G_{vc}(\mathbf{p}\omega) + 1,\\ (\omega+\xi)G_{vc}(\mathbf{p},\omega) &= (\lambda+\Sigma_{cv})G_c(\mathbf{p}\omega),\end{aligned} \tag{4}$$

where the quantity

$$\Sigma_{vc}(\mathbf{p}) = \frac{1}{(2\pi)^4}\int V_{\mathbf{p}-\mathbf{p}'}G_{vc}(\mathbf{p}',\omega')\,d^3p'\,d\omega' \tag{5}$$

is chosen to be real.

The difference between the system (4) and (5) and the corresponding system of equations from the paper[5] consists in the fact that, in our case, the position of the exciton chemical potential is specified by the expression E'_g in ξ, and the exciton concentration is a parameter to be determined. But in the conditions of paper[5], conversely, the exciton concentration is given. The second difference is that the role of the Bose-condensate function Σ_{vc} is played by the sum $\Sigma_{vc} + \lambda$, i.e., the external coherent field is a source of the Bose condensate.

It should be noted that the system (4) was obtained by neglecting the photon momentum. This approximation is justified by the ratio of the exciton radius to the wavelength of the light.

The system of equations (4) corresponds to the self-consistent field approximation. As was shown in[5], allowance for the correlation effects for the case $Na_0^3 \ll 1$ leads to a shift of the chemical potential in the same linear approximation in the concentration, and, consequently, to a renormalization of the numerical coefficient of the concentration. With this in mind, we do not consider correlation effects explicitly in the present work.

From the system (4), (5), we obtain the following expression for G_c and G_{vc}:

$$G_c = \frac{\omega+\xi}{\omega^2-\xi^2-\Delta^2(\mathbf{p})}, \quad G_{vc} = \frac{\Delta(\mathbf{p})}{\omega^2-\xi^2-\Delta^2(\mathbf{p})}, \tag{6}$$
$$\Delta(\mathbf{p}) = \lambda + \Sigma_{vc}(\mathbf{p}).$$

Substituting the expression for G_{vc} from (6) into (5) and integrating over ω', we obtain

$$\Delta(\mathbf{p}) = \lambda + \int\frac{d^3p'}{(2\pi)^3}V_{\mathbf{p}-\mathbf{p}'}\frac{\Delta(\mathbf{p}')}{2[\xi_{\mathbf{p}'}^2+\Delta^2(\mathbf{p}')]^{1/2}}. \tag{7}$$

Equations of this type were obtained earlier for the case $Na_0^3 \gg 1$ in[6,7]. In[8], a system of equations for the exciton Bose condensate and the electromagnetic field was found.

Denoting $\Delta(\mathbf{p})/2[\xi^2+\Delta^2]^{1/2} = \psi(\mathbf{p})$ and using the expansion $[\xi^2+\Delta^2]^{1/2} \approx \xi + \Delta^2(0)/E'_g$, we find an equation for $\psi(\mathbf{p})$:

$$\left(\frac{p^2}{m} + E_g' + \frac{2\Delta^2(0)}{E_g'}\right)\psi(\mathbf{p}) - \int\frac{V(\mathbf{p}-\mathbf{p}')}{(2\pi)^3}\psi(\mathbf{p}')\,d^3p' = \lambda \tag{8}$$

and for the Fourier component $\psi(\mathbf{r})$:

$$(-m^{-1}\hbar^2\nabla_r^2 + k^2 - V(\mathbf{r}))\psi_k(\mathbf{r}) = \lambda(\mathbf{r}) \equiv \lambda\delta(\mathbf{r}), \tag{9}$$

where

$$k^2 = E_g' + 2\Delta^2(0)/E_g'. \tag{10}$$

Equation (9) has the form of the Schrödinger equation for the Coulomb problem, with the right-hand side playing the role of the source. The general solution of Eq. (9) has the form

$$\psi_k(\mathbf{r}) = \chi_k(\mathbf{r}) + \varphi_k(\mathbf{r}), \tag{11}$$

where $\varphi_k(\mathbf{r})$ is the particular integral of the inhomogeneous equation (9) and $\chi_k(\mathbf{r})$ is the solution of Eq. (9) without the right-hand side.

The number of excitons in the Bose condensate is expressed in terms of the function $G_c(\mathbf{p}, \omega)$:

$$N = 2\int\frac{G_c(\mathbf{p},\omega)\,d^3p\,d\omega}{(2\pi)^4}. \tag{12}$$

Hence, taking (6) into account, we have

$$N = 2\int\frac{d^3p}{(2\pi)^3}\psi^2(\mathbf{p}). \tag{13}$$

It is known[9] that, in the case of the inhomogeneous Schrödinger equation, there is always, for any value of k^2, a finite particular integral of the inhomogeneous equation. However, when k^2 is equal to an eigenvalue k_n^2 of the homogeneous equation, a solution $\varphi_k(\mathbf{r})$ does not exist if $\lambda(\mathbf{r})$ is not orthogonal to $\chi_k(\mathbf{r})$, as is the case for the ground state. We are interested in solving the inhomogeneous equation, since the excitons are generated as a result of the action of the source. Therefore, k^2 should not be equal to k_n^2 and, consequently, the solution of Eq. (9) can be represented by the particular integral of the inhomogeneous equation that was found in[10]:

$$\psi_k(\mathbf{r}) = \frac{\lambda m}{2\pi r\hbar^2}\Gamma(1-\eta)W_{\eta,1/2}(2kr), \tag{14}$$

where $\Gamma(\eta)$ is the gamma-function, $W_{\eta,1/2}$ is the Whittaker function, $\eta = k_n/k$, and $k_n^2 = e^4m/\kappa^2\hbar^2$. Since the quantity k^2 depends on $\Delta^2(0)$ and, consequently, on $\psi_k(p=0)$, the following relation should be fulfilled:

$$\frac{\Delta(0)}{E_g'} = \psi_k(p=0) = \int\psi_k(\mathbf{r})\,d^3r = \lambda\Gamma(1-\eta)B(\eta), \tag{15}$$

where

$$B(\eta) = \frac{m}{2\pi\hbar^2}\int\frac{d^3r}{r}W_{\eta,1/2}(2kr).$$

The equations (14) and (15) determine the value of k

and, consequently, $\Delta(0)$ and the exciton concentration N (12) for given values of λ and $\hbar\Omega$.

Substituting the expression (14) for $\psi_k(\mathbf{r})$ into (13), we obtain

$$N = \lambda^2\Gamma^2(1-\eta)A(\eta), \tag{16}$$

where

$$A(\eta) = \frac{m^2}{4\pi^2\hbar^4}\int \frac{d^3r}{r^2} W^2_{\eta,1/2}(2kr) \approx \frac{m^2}{\pi\hbar^4 k}. \tag{17}$$

Note that η is determined by the condition (15).

In the case of interest when $E^0_{ex} - (E_g - \hbar\Omega) \ll E^0_{ex}$ and, consequently, $\eta - 1 \ll 1$, the functions $B(\eta)$ and $A(\eta)$ can be assumed to be independent of η. In this case, for the function $\Gamma(1-\eta)$ the expansion

$$\Gamma(1-\eta) = \frac{\pi\eta}{\Gamma(1+\eta)\sin\pi\eta} \approx \frac{1}{\Gamma(2)(\eta-1)}. \tag{18}$$

is valid. Then the equation for the exciton concentration N takes the form

$$N = \lambda^2 4A/(\varepsilon - \alpha N)^2, \tag{19}$$

where

$$\varepsilon = 1 - E_g'/E_{ex}^0,\ \alpha = B^2E_g'/AE_{ex}^0 \approx 16\sqrt{2}\pi a_0^3.$$

Up to this point, we have not considered recombination processes. If we take them into account phenomenologically, by adding an imaginary part i/τ to the energy k^2 in Eq. (13), we obtain in place of (19)

$$N = \lambda^2 \frac{4A}{(\varepsilon - \alpha N)^2 + \gamma^2},\quad \gamma = \frac{\hbar}{\tau E_{ex}^0}. \tag{20}$$

The expression (20) for the exciton concentration coincides with the expression for the amplitude of forced oscillations of a nonlinear oscillator under the action of a periodic external resonance force[11] [1)]. This enables us to use the analysis of the solution of Eq. (20) performed in[11].

We shall consider the dependence of N on ϵ for given λ. For sufficiently small values of λ, the concentration N is also small, so that we can neglect the term αN in the denominator of (20), and obtain for the dependence of N on ϵ a curve with a maximum at the point $\epsilon = 0$. With increasing λ, the maximum of N is displaced to positive values of ϵ. Then, of the three roots of Eq. (20) only one is real. Starting from a value $\lambda^2 > \lambda^2_{cr}$,

$$\lambda_{cr}^2 = 2\gamma^3/4A\alpha, \tag{21}$$

there exists a range of frequencies in which Eq. (10) admits two different values for the density of the Bose condensate (see Fig. 1).

On increase of the frequency $\hbar\Omega$, at the point C the density N decreases by a jump to the value at the point E. If we now decrease the frequency, the discontinuous increase of N occurs at another point K, i.e., frequency hysteresis occurs.

In an analogous manner, we can analyze the dependence of N on λ for given ϵ (see Fig. 2). With increase

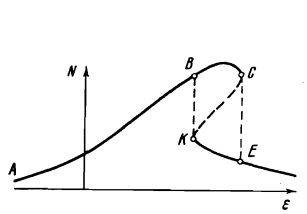

FIG. 1

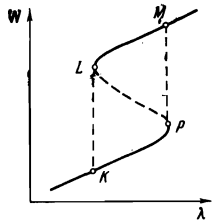

FIG. 2

of λ, the condensate density N, when it reaches the magnitude

$$N_1 = \frac{2\varepsilon - \sqrt{\varepsilon^2 - 3\gamma^2}}{3\alpha} \tag{22}$$

jumps to the higher value at the point M.[2)] The corresponding value of λ is

$$\lambda_{cr}^2(\varepsilon) = \varepsilon^2/9A\alpha. \tag{23}$$

If we now decrease the field amplitude λ, a discontinuous decrease of N occurs at the point L, at which $N = N_2$,

$$N_2 = (2\varepsilon + \sqrt{\varepsilon^2 - 3\gamma^2})/3\alpha.$$

It should be noted that the amplitude hysteresis occurs only when the condition $\epsilon^2 > 3\gamma^2$, which follows from (22), is fulfilled[3)]. If $\epsilon^2 \gg 3\gamma^2$, the expression for N_2 has the form $N_2 \approx \epsilon/\alpha$.

The adjustment, described by Eq. (19), of the level of the exciton Bose condensate to the frequency Ω of the coherent source will hinder the process of agglomeration of excitons into drops. Indeed, when excitons agglomerate into drops, their chemical potential will not correspond to resonance with the source. Therefore, in the drop regime, recombination processes will predominate over generation processes. Consequently, drops can be formed only when the recombination times are much greater than the time $(N^{1/3}v_T)^{-1}$ of agglomeration into a drop (v_T is the thermal velocity of the excitons).

To describe the system at finite tmepratures, we make use of temperature Green functions. The equation for Σ_{VC} will differ from (7) by replacement of the integration over ω by summation over $\omega_n = (2n+1)\pi T$, $(n = \pm 1, \pm 2, \ldots)$. In this case, in the inhomogeneous Schrödinger equation, the Coulomb potential V must be replaced by $V \tanh(\sqrt{\xi^2 + \Delta^2}/2T)$:

$$\left(\frac{p^2}{m} + E_g' + \frac{2\Delta^2(0)}{E_g'}\right)\psi(\mathbf{p}) - \int \frac{V(\mathbf{p}-\mathbf{p}')}{(2\pi)^3}\,\mathrm{th}\frac{\sqrt{\xi^2+\Delta^2}}{2T}\psi(\mathbf{p}')\,d^3p' = \lambda. \tag{24}$$

The phase-transition temperature is determined by the condition $\psi(\mathbf{p}) \to 0$ in this equation. But, because of the temperature-independent right-hand side of Eq. (24), the latter has non-zero solutions at arbitrary temperatures. An analogous phenomenon occurs for a ferromagnet in an external magnetic field, which can create a finite magnetization at any temperature.

On the other hand, drops can exist only at temperatures below the "boiling" temperature. For the values of E_g' that we are considering, i.e., values close to E^0_{ex},

[1)] The analogy with the anharmonic oscillator indicates that the density of the exciton Bose condensate can be described phenomenologically as the appearance of an order parameter under the influence of a driving force. The aim of the microscopic description is to calculate the coefficients in the expansion of the free energy in the order parameter.

[2)] We note that, if we take (22) into account, the small parameter $(\Delta/E_g')^2$ used in obtaining Eq. (8) is equal to $\lambda^2/(E^0_{ex} - E_g')^2$.

[3)] The experimental detection of the amplitude and frequency hystereses would be evidence of the presence of Bose condensation of excitons.

Eq. (24) has the form of an inhomogeneous Schrödinger equation in which the product $V\tanh(E_g'/2T)$ plays the role of the effective potential. The solution for $\psi(\mathbf{p})$ will have the form of (14), if we replace e^2 by $e^2\tanh(E_g'/2T)$ in the expression for k_n^2. Then in Eqs. (19) and (20) for the condensate concentration, the detuning ϵ will depend on the temperature in the following manner:

$$\varepsilon(T)=1-E_g'\left(E_{ex}^0\,\mathrm{th}^2\frac{E_g'}{2T}\right)^{-1}.$$

Thus, for fixed frequency Ω, the parameter $\epsilon(T)$ will vary with changing temperature. Consequently, temperature hysteresis, analogous to the frequency hysteresis considered above, will occur. Although Bose condensation will exist at any temperature, the system possesses a first-order phase transition, which is connected with the discontinuous change of the condensate density.

Above, we considered only the case of direct transitions, when the conduction and valence band edges coincide in momentum space.

In the case when the band edges do not coincide, we can consider two possibilities: a) a "direct" transition, when phonon absorption or emission (or impurity scattering) does not occur in the interaction of the electrons with the coherent wave; b) an indirect transition, when these processes do occur.

In case a), for $E_g > \hbar\Omega > E_g - E_{ex}^0$, the system is a semiconductor with an effective indirect gap $E_g' = E_g - \hbar\Omega$, the latter being smaller than the binding energy E_{ex}^0 of the indirect exciton. Such a system is unstable with respect to the formation of an exciton-Bose condensate; the excitons condense into states with momenta $\mathbf{w}$, where $\mathbf{w}$ is equal to the distance between the band edges, and the non-zero averages are $\langle a_{c\mathbf{p}}a_{v\mathbf{p}+\mathbf{w}}^+\rangle$. For such a state, the term $\lambda a_{c\mathbf{p}}^+a_{v\mathbf{p}}$ in the Hamiltonian (3) does not play the role of a source, and the equation for $\Sigma_{vc}(\mathbf{p}, \mathbf{p}+\mathbf{w})$ will have the form of Eq. (7) without the right-hand side. The latter statement also remains valid for case b). In this case, the process of absorption of a photon with simultaneous emission of a phonon with momentum $\mathbf{w}$ plays the role of the exciton source $\langle a_{c\mathbf{p}}a_{v\mathbf{p}+\mathbf{w}}^+\rangle$. But this source is not coherent, since, generally speaking, the phase of the emitted phonon is random.

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