THEORY OF ELECTRON DRAGGING BY PHONONS IN A MAGNETIC FIELD

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The behavior of the kinetic coefficients on variation of the magnetic field is studied under conditions when simple and mutual dragging of electrons by phonons must be taken into account. The degree of Fermi degeneracy of the electrons is arbitrary. It is pointed out that an exact solution of the set of kinetic equations can be obtained in the case of a strong magnetic field. A solution of the system of equations for weak and intermediate magnetic field strengths is derived by iteration with respect to the mutual dragging parameter δ_m. The laws of variation of the kinetic coefficients in a magnetic field thus derived can be used to separate the contributions of simple and mutual phonon dragging of electrons to the charge flow.

STARTING with the work of L. É. Gurevich[1], who was the first to point out the importance of taking into account the influence of the non-equilibrium character of the phonon subsystem on the electron subsystem in thermoelectric phenomena, the effect of dragging of electrons and phonons continues to remain an object of both theoretical and experimental research. The purpose of such research is to obtain the kinetic coefficients (the electric conductivity, the thermal conductivity, the thermoelectric power, the Hall and Nernst coefficients, etc.) and to separate the diffusion contribution from the contribution due to the electron-phonon dragging effect. Each of these contributions is determined by electron and phonon times that are characteristic of the considered concrete situation, and consequently can serve as a source of information on the effective relaxation mechanisms.

The development of the theory of the dragging effect in thermoelectric phenomena was continued by Herring[2] and by ter Haar and Neaves[3]. In these studies[1-3], the influence of the non-equilibrium character of the distribution function of the phonon on the electron subsystem was taken into account. Parrot[4], Appel[5] and ter Haar and Neaves[6] were the first to formulate the problem symmetrically with respect to the electron and phonon subsystems, and treated the kinetic equations for the electron and phonon distribution functions as a system of two integral equations. Ter Haar and Neaves[6] replaced the lower integration limit $q/2$ ($\mathbf{q}$ is the wave vector of the phonon) in the integrals with respect to the electron wave vector $\mathbf{k}$ by zero. As noted by Parrot[4], they obtained as a result an incorrect dependence of τ_{pe} (the relaxation time of the phonons on the electrons) on q and overestimated the contribution of the high-energy phonons to the kinetic coefficient. This remark by Parrot applies also to Appel's work[5]. Parrot[4] took into account the so-called mutual dragging, i.e., the non-equilibrium character of the phonon subsystem, which is due to the non-equilibrium character of the electron subsystem and affects the electrons in turn.

The most important means of acting on the electron-phonon coupling itself is to vary the temperature. When the temperature is varied, one dominant scattering mechanism is replaced by others, and the contributions of the diffusion and dragging to the kinetic coefficients are altered. However, an analysis of the temperature dependences of the kinetic coefficients with the object of separating this contribution is quite complicated precisely by virtue of the indicated temperature-dependent dynamics of the process. This raised the question of the possible use for this purpose of variation of the charge and heat transport processes with changing magnetic field in a stationary temperature regime. The first results were obtained by V. Gurevich and Obraztsov[7], who took into account the influence of electron dragging by phonons on the thermomagnetic effects in semiconductors. Natadze and Éfros[8] solved this problem for degenerate semiconductors in a strong magnetic field. A theory of electron dragging by phonons in semimetals was constructed by L. Gurevich and Korenblit[9,10], who solved the system of kinetic equations for phonons and of Fermi-degenerate electrons exactly for an arbitrary value of the magnetic field[9].

In this paper we investigate the system of kinetic equations for phonons and electrons in the case of an arbitrary degree of Fermi degeneracy, and in magnetic fields of arbitrary strength. It is assumed here that it is possible to introduce relaxation times for both the electrons and the phonons. Quantum effects are disregarded. It is our aim to determine the behavior of the kinetic coefficients in a varying magnetic field under conditions when it is necessary to take into account simple and mutual dragging of the electrons by phonons.

1. KINETIC EQUATIONS FOR THE DISTRIBUTION FUNCTIONS OF THE ELECTRONS AND PHONONS

We consider an isotropic conductor placed in a constant homogeneous magnetic field $\mathbf{H}$ (the direction of which is taken to be the z axis) and a constant electric field $\mathbf{E}$ directed perpendicular to $\mathbf{H}$ (along the x axis). In addition, a certain temperature gradient ∇T is maintained along the x axis. The system of kinetic equations for the distribution functions of the electrons $F = F_{0\mathbf{k}}$

+ f_k and phonons $N = N_{0q} + g_q$, which consist of equilibrium Fermi and Planck parts (F_{0k} and N_{0q}) and non-equilibrium increments to them (f_k and g_q), takes the form

$$\left(e\mathbf{vE}-\frac{\varepsilon-\zeta}{T}\mathbf{v}\nabla T\right)\frac{\partial F_{0k}}{\partial\varepsilon}+\frac{e}{c\hbar}[\mathbf{vH}]\frac{\partial f_k}{\partial\mathbf{k}}+\frac{f_k}{\tau_e}+\frac{V}{(2\pi)^2\hbar}\int d\mathbf{q}|C_q|^2 g_q \quad (1)^*$$

$$\times\{(F_{0k}-F_{0k+q})\delta(\varepsilon_k-\varepsilon_{k+q}+\hbar\omega_q)+(F_{0k}-F_{0k-q})\delta(\varepsilon_k-\varepsilon_{k-q}-\hbar\omega_q)\}=0,$$

$$\frac{w}{q}\mathbf{q}\nabla T\frac{\partial N_0}{\partial T}+\frac{g_q}{\tau_p}-\frac{|C_q|^2V}{2\pi^2\hbar}\int d\mathbf{k}\{(N_{0q}+1-F_{0k-q})f_k \quad (2)$$

$$-(N_{0q}+F_{0k})f_{k-q}\}\delta(\varepsilon_k-\varepsilon_{k-q}-\hbar\omega_q)=0.$$

Here e, ϵ, and ζ are the charge, energy, and chemical potential of the electron (hole); $\mathbf{v} = \hbar^{-1}\partial\epsilon/\partial\mathbf{k}$; $\omega_q = wq$ is the frequency of a phonon with wave vector $\mathbf{q}$; w is the speed of sound in the conductor; $|C_q|^2$ is a function characterizing the electron-phonon interaction, T is the temperature in energy units, V is the volume of the crystal, τ_e and τ_p are the relaxation times of the electrons and phonons.

The sum of the last two terms in the left-hand side of (1) is the collision term, and the term in integral form takes into account the non-equilibrium character of the phonon distribution function. Herring[2] separated in similar manner the collision term due to the non-equilibrium character of the electron distribution function.

We seek the non-equilibrium parts of the distribution functions of the electrons and phonons in the form

$$f_k=\hbar\mathbf{k}(A\nabla T-B[\mathbf{h}\nabla T]-A_E\mathbf{E}+B_E[\mathbf{hE}]), \quad (3)$$

$$g_q=\mathbf{q}(a\nabla T+b[\mathbf{h}\nabla T]+a_E\mathbf{E}+b_E[\mathbf{hE}]), \quad (4)$$

The vector $\mathbf{h} = \mathbf{H}/H$ indicates the direction of the magnetic field. After performing the required calculations, we obtain two pairs of integral equations for the coefficients A, B, A_E, and B_E from[3][1)]

$$A_E=(1+\omega_H^2\tau_e^2)^{-1}\left(\frac{e\tau_e}{m}+\frac{\tau_e}{\tau_{ep}}\left\langle\frac{\tau_p}{\tau_{pe}}\langle A_E\rangle_1\right\rangle_2-\right.$$

$$\left.-\omega_H\frac{\tau_e^2}{\tau_{ep}}\left\langle\frac{\tau_p}{\tau_{pe}}\langle B_E\rangle_1\right\rangle_2\right); \quad (5)$$

$$B_E=(1+\omega_H^2\tau_e^2)^{-1}\left(\omega_H\frac{e\tau_e^2}{m}+\frac{\tau_e}{\tau_{ep}}\left\langle\frac{\tau_p}{\tau_{pe}}\langle B_E\rangle_1\right\rangle_2+\right.$$

$$\left.+\omega_H\frac{\tau_e^2}{\tau_{ep}}\left\langle\frac{\tau_p}{\tau_{pe}}\langle A_E\rangle_1\right\rangle_2\right); \quad (6)$$

$$A=(1+\omega_H^2\tau_e^2)^{-1}\left(\frac{\tau_e}{m}S+\frac{\tau_e}{\tau_{ep}}\left\langle\frac{\tau_p}{\tau_{pe}}\langle A\rangle_1\right\rangle_2-\omega_H\frac{\tau_e^2}{\tau_{ep}}\left\langle\frac{\tau_p}{\tau_{pe}}\langle B\rangle_1\right\rangle_2\right), \quad (7)$$

$$B=(1+\omega_H^2\tau_e^2)^{-1}\left(\omega_H\frac{\tau_e^2}{m}S+\frac{\tau_e}{\tau_{ep}}\left\langle\frac{\tau_p}{\tau_{pe}}\langle B\rangle_1\right\rangle_2+\omega_H\frac{\tau_e^2}{\tau_{ep}}\left\langle\frac{\tau_p}{\tau_{pe}}\langle A\rangle_1\right\rangle_2\right) \quad (8)$$

$$S=T^{-1}\left(\varepsilon-\zeta+mw^2\frac{\langle\tau_p\rangle_2}{\tau_{ep}}\right), \quad (9)$$

$$\langle X\rangle_1=\int_{q/2}^{\infty}X\frac{\partial F_0}{\partial\varepsilon}k\,dk\Big/\int_{q/2}^{\infty}\frac{\partial F_0}{\partial\varepsilon}k\,dk, \quad (10)$$

$$\langle Y\rangle_2=\int_0^{2k}Yq^3\,dq\Big/\int_0^{2k}q^3\,dq, \quad (11)$$

$\omega_H = |e|H/mc$ is the cyclotron frequency. The finite lower integration limits (in (10)) and upper limits (in (11)) are the consequence of the energy and momentum conservation laws in electron-phonon scattering. We have introduced here the relaxation times τ_{ep} and τ_{pe} for the scattering of electrons by phonons and of phonons by electrons, respectively,

$$\tau_{pe}^{-1}=-\frac{mwV|C_q|^2}{\pi\hbar^2}\int_{q/2}^{\infty}\frac{\partial F_0}{\partial\varepsilon}k\,dk, \quad (12)$$

$$\tau_{ep}^{-1}=\frac{mV}{2\pi\hbar^3k^3}\int_0^{2k}q^3|C_q|^2N_{0q}\,dq. \quad (13)$$

From expression (3) for f_k, with allowance for the fact that the coefficients in it are determined in (5)—(8), we can calculate in principle all the kinetic coefficients of interest to us. If the expression for the current density is written in the form

$$j_i=\sigma_{ii'}E_{i'}-\beta_{ii'}\nabla_{i'}T, \quad (14)$$

then the components of the electric conductivity tensor $\sigma_{ii'}$ and of the thermal magnetic tensor $\beta_{ii'}$ are determined by the following expressions (we recall that the driving forces act in the xy plane and consequently all fluxes flow in this plane)

$$\begin{aligned}\sigma_{xx}&=n_0e\langle A_E\rangle_3, & \beta_{xx}&=n_0e\langle A\rangle_3,\\ \sigma_{xy}&=n_0e\langle B_E\rangle_3, & \beta_{xy}&=n_0e\langle B\rangle_3,\end{aligned} \quad (15)$$

where the averaging symbol $\langle\ldots\rangle_3$ denotes

$$\langle Z\rangle_3=-z_0^{-3/2}\int_0^{\infty}dt\,t^{3/2}Z\frac{\partial F_0}{\partial t}, \qquad t=\frac{\varepsilon}{T}, \quad (16)$$

$z_0 = \zeta_0/T$ is the dimensionless chemical potential, ζ_0 is the chemical potential at T = 0, and n_0 is the electron (hole) concentration.

The very form of Eqs. (5)—(8) and relation (9) is such that it is possible to determine in natural fashion the characteristic parameters of the "simple" and "mutual" dragging. We define as simple dragging the dragging of electrons by phonons that are taken out of equilibrium by the temperature gradient. In (7) and (8), the simple dragging is described by the term of (9) in the form

$$\frac{mw^2}{T}\langle\tau_\phi\rangle_2\tau_{ep}^{-1}=\frac{mw^2}{T}\langle\tau_{pe}\rangle_2\tau_{ep}^{-1}\delta_s, \quad (17)$$

$$\delta_s=\langle\tau_p\rangle_2/\langle\tau_{pe}\rangle_2. \quad (18)$$

We have introduced the simple-dragging parameter δ_s (18), by multiplying and dividing the left-hand side of (17) by $\langle\tau_{pe}\rangle_2$. It follows from (17) and (18) that simple dragging reaches a maximum if the phonons are scattered predominantly by electrons, i.e., $\delta p \approx 1$. The coefficients of δ_p in (17) is given by

$$\frac{mw^2}{T}\langle\tau_{pe}\rangle_2\tau_{ep}^{-1}=(2k^3)^{-1}\int_0^{2k}q^2F_0^{-1}\left(\frac{\hbar^2q^2}{8m}\right)dq. \quad (19)$$

*[vH] ≡ v × H.

[1)]Since the end purpose of the present paper is the calculation of the charge fluxes, do not present the expressions for the coefficients a, b, a_E and b_E, which we use only during the intermediate stages of the calculations.

In the case of strong Fermi degeneracy of the electrons we have

$$\frac{mw^2}{T}\langle\tau_{pe}\rangle_2\tau_{ep}^{-1}=\frac{4}{3}, \tag{20}$$

and for nondegenerate electrons this quantity is $\sim n_M/n_0$ ($n_M = 2mT/\hbar^2)^{3/2}$), i.e., it can be much larger than unity.

In just as obvious a manner we can introduce the mutual dragging parameter

$$\delta_m=\frac{\tau_e}{\tau_{ep}}\left\langle\frac{\tau_p}{\tau_{pe}}\right\rangle_2. \tag{21}$$

Mutual dragging is dragging of electrons by phonons that are taken out of equilibrium by the electrons themselves. Thus, the mutual dragging exists also in the absence of a gradient ∇T acting directly on the phonons and, as seen from (21), is realized in the case of predominant scattering of electrons by phonons and of phonons by electrons.

2. SOLUTION OF THE SYSTEM OF EQUATIONS (5)—(8)

In this section we present solutions of the system (5)—(8) for the cases of a) strong Fermi degeneracy, b) strong magnetic field, c) weak mutual dragging, $\delta_m \ll 1$.

a) In the case of strong Fermi degeneracy of the electrons, the averaging (10) can be easily carried out, owing to the singularity of the Fermi distribution ($\partial F_0/\partial\epsilon \sim \delta(\epsilon-\zeta)$), as a result of which the system of integral equations reduces to a system of algebraic equations and can be solved exactly[9]. Unlike[9], we first apply the averaging (16) to the left-hand and right-hand sides of (5)—(8), and obtain immediately equations for the quantities that enter directly by the expressions for the components of the electric conductivity tensor σ and the thermomagnetic tensor β (see (15)). We present the final result

$$\begin{aligned}
\langle A\rangle_3&=\frac{4}{3}\frac{\tau_{eF}^2}{m}\frac{1-\delta_{mF}}{(\omega_H\tau_{eF})^2+(1-\delta_{mF})^2},\\
\langle B\rangle_3&=\frac{4}{3}\frac{\tau_{eF}^3\omega_H}{m}\frac{1}{\omega_H^2\tau_{eF}^2+(1-\delta_{mF})^2},\\
\langle A_E\rangle_3&=\frac{e\tau_{eF}^2}{m}\frac{1-\delta_{mF}}{\omega_H^2\tau_{eF}^2+(1-\delta_{mF})^2}\\
\langle B_E\rangle_3&=\frac{e\tau_{eF}^3\omega_H}{m}\frac{1}{\omega_H^2\tau_{eF}^2+(1-\delta_{mF})^2}.
\end{aligned} \tag{22}$$

The subscript F denotes that in the corresponding quantity, which depends on the electron wave vector $\mathbf{k}$, it is necessary to replace $\mathbf{k}$ by $\mathbf{k}_F$. For example,

$$\delta_{mF}=\frac{\tau_{eF}}{\tau_{epF}}\left\langle\frac{\tau_p}{\tau_{pe}}\right\rangle_{2F},\qquad \left\langle\frac{\tau_p}{\tau_{pe}}\right\rangle_{2F}=\int_0^{2k_F}q^3\frac{\tau_p}{\tau_{pe}}dq\Big/\int_0^{2k_F}q^3dq.$$

b) Another important case when the system (5)—(8) can be solved exactly is that of a strong magnetic field, $\omega_H\tau_e \gg 1$. We assume that the coefficients A_E, B_E, A, and B depend on H in the same manner as without mutual dragging, i.e.,

$$A_E\sim H^{-2},\quad B_E\sim H^{-1},\quad A\sim H^{-2},\quad B\sim H^{-1}.$$

Discarding terms that are small relative to the parameter $(\omega_H\tau_e)^{-2}$, we find that the system (5)—(8) reduces to the system

$$\begin{aligned}
A_E&=\omega_H^{-2}\tau_e^{-1}\left(\frac{e}{m}-\frac{\tau_e}{\tau_{ep}}\left\langle\frac{\tau_p}{\tau_{pe}}\langle B_E\rangle_1\right\rangle_2\omega_H\right)\\
B_E&=e/m\omega_H,\qquad B=S/m\omega_H,\\
A&=\omega_H^{-2}\tau_e^{-1}\left(\frac{S}{m}-\omega_H\frac{\tau_e}{\tau_{ep}}\left\langle\frac{\tau_p}{\tau_{pe}}\langle B\rangle_1\right\rangle_2\right),
\end{aligned} \tag{23}$$

in which B and B_E have already been determined. From this we obtain ultimately

$$\begin{aligned}
A_E&=\frac{e}{m\omega_H^2}\left(\tau_e^{-1}-\tau_{ep}^{-1}\left\langle\frac{\tau_p}{\tau_{pe}}\right\rangle_2\right),\\
A&=\frac{1}{m\omega_H}\left(\tau_e^{-1}S-\tau_{ep}^{-1}\left\langle\frac{\tau_e}{\tau_{pe}}\langle S\rangle_1\right\rangle_2\right),\\
B&=S/m\omega_H,\qquad B_E=e/m\omega_H.
\end{aligned} \tag{24}$$

The solutions of (24) satisfy the initial assumption concerning the character of the decrease of A_E, B_E, A, and B with increasing magnetic field.

We take special notice of the fact that the solution (3) with the coefficients (24) is valid for arbitrary Fermi degeneracy of the electrons (in particular, for nondegenerate electrons) and for arbitrary values of the dragging parameters.

c) The mutual-dragging parameter δ_m is always less than unity by virtue of its very definition (21). This circumstance permits an iteration solution of the system (5)—(8) when an exact solution is impossible. A solution by iteration with respect to δ_m is obtained in obvious fashion and yields in the lowest-order approximation in δ_m

$$\begin{aligned}
A&=\frac{\tau_e}{m}(1+\omega_H^2\tau_e^2)^{-1}\left(S+\frac{1}{\tau_{ep}}\left\langle\frac{\tau_p}{\tau_{pe}}\left\langle\frac{\tau_eS}{1+\omega_H^2\tau_e^2}\right\rangle_1\right\rangle_2\right.\\
&\left.+\omega_H^2\frac{\tau_e}{\tau_{ep}}\left\langle\frac{\tau_p}{\tau_{pe}}\left\langle\frac{\tau_e^2S}{1+\omega_H^2\tau_e^2}\right\rangle_1\right\rangle_2\right),\\
B&=\frac{\omega_H\tau_e}{m}(1+\omega_H^2\tau_e^2)^{-1}\left(\tau_eS+\frac{1}{\tau_{ep}}\left\langle\frac{\tau_p}{\tau_{pe}}\left\langle\frac{\tau_e^2S}{1+\omega_H^2\tau_e^2}\right\rangle_1\right\rangle_2\right.\\
&\left.+\frac{\tau_e}{\tau_{ep}}\left\langle\frac{\tau_p}{\tau_{pe}}\left\langle\frac{\tau_eS}{1+\omega_H^2\tau_e^2}\right\rangle_1\right\rangle_2\right),\\
A_E&=\frac{e\tau_e}{m}(1+\omega_H^2\tau_e^2)^{-1}\left(1+\frac{1}{\tau_{ep}}\left\langle\frac{\tau_p}{\tau_{pe}}\left\langle\frac{\tau_e}{1+\omega_H^2\tau_e^2}\right\rangle_1\right\rangle_2\right.\\
&\left.-\omega_H^2\frac{\tau_e}{\tau_{ep}}\left\langle\frac{\tau_p}{\tau_{pe}}\left\langle\frac{\tau_e^2}{1+\omega_H^2\tau_e^2}\right\rangle_1\right\rangle_2\right),\\
B_E&=\frac{e\omega_H\tau_e}{m}(1+\omega_H^2\tau_e^2)^{-1}\left(\tau_e+\frac{1}{\tau_{ep}}\left\langle\frac{\tau_p}{\tau_{pe}}\left\langle\frac{\tau_e^2}{1+\omega_H^2\tau_e^2}\right\rangle_1\right\rangle_2\right.\\
&\left.+\frac{\tau_e}{\tau_{ep}}\left\langle\frac{\tau_p}{\tau_{pe}}\left\langle\frac{\tau_e}{1+\omega_H^2\tau_e^2}\right\rangle_1\right\rangle_2\right).
\end{aligned} \tag{25}$$

3. GALVANOMAGNETIC AND THERMOMAGNETIC KINETIC COEFFICIENTS

In this section we present expressions for the magnetoresistance ρ_{xx}, the Hall coefficient R, the transverse magnetothermoelectric power α_{xx}, and the Nernst coefficient Q, obtained by solving the kinetic equation of the preceding section.

a) In the case of strong Fermi degeneracy of the electrons we have

$$\begin{aligned}
\rho_{xx}&=\sigma_0^{-1}\left[1-\frac{\tau_{eF}}{\tau_{epF}}\left\langle\frac{\tau_p}{\tau_{pe}}\right\rangle_{2F}\right],\qquad \sigma_0=\frac{n_0e^2\tau_{eF}}{m},\\
R&=(n_0ec)^{-1},\\
\alpha_{xx}&=e^{-1}\langle S\rangle_3=\frac{mw^2}{eT}\left\langle\frac{\langle\tau_p\rangle_2}{\tau_{ep}}\right\rangle_3.
\end{aligned} \tag{26}$$

The Nernst coefficient Q vanishes accurate to order

T/ζ. In the case of predominant scattering of phonons by electrons we have $\tau_p \to \tau_{pe}$ and $\alpha_{xx} = 4/3e$[8,9] as a result of simple dragging.

b) In a strong magnetic field, for an arbitrary degree of Fermi degeneracy of the electrons, we have

$$\rho_{xx} = \sigma_0^{-1}\left\langle \tau_e^{-1} - \tau_{ep}^{-1}\left\langle \frac{\tau_p}{\tau_{pe}}\right\rangle_2\right\rangle_3 \langle\tau_e\rangle_3, \quad \sigma_0 = \frac{n_0 e^2\langle\tau_e\rangle_3}{m},$$
$$R = (n_0 ec)^{-1}, \quad \alpha_{xx}^{\infty} = e^{-1}\langle S\rangle_3,$$
$$Q = (eH\omega_H)^{-1}\left\{\langle S\rangle_3\left\langle \tau_e^{-1} - \tau_{ep}^{-1}\left\langle \frac{\tau_p}{\tau_{pe}}\right\rangle_2\right\rangle_3 - \left\langle \tau_e^{-1}S - \tau_{ep}^{-1}\left\langle \frac{\tau_p}{\tau_{pe}}\langle S\rangle_1\right\rangle_2\right\rangle_3\right\}. \quad (27)$$

c) Since an exact solution of the system (5)—(8) exists for a strong magnetic field, it is meaningful to use the iteration solution only in weak magnetic fields. In particular, as $H \to 0$, we have

$$\rho_{xx}^{(0)} = \sigma_0^{-1}\left(1 - \langle\tau_e\rangle_3\left\langle \frac{\tau_e}{\tau_{ep}}\left\langle \frac{\tau_p}{\tau_{pe}}\langle\tau_e\rangle_1\right\rangle_2\right\rangle_3\right), \quad \sigma_0 = \frac{n_0 e^2\langle\tau_e\rangle_3}{m},$$
$$R^{(0)} = (n_0 ec)^{-1}\langle\tau_e\rangle_3^{-2}\left\{\langle\tau_e^2\rangle_3 + \left\langle \frac{\tau_e}{\tau_{ep}}\left\langle \frac{\tau_p}{\tau_{pe}}\langle\tau_e^2\rangle_1\right\rangle_2 + \frac{\tau_e^2}{\tau_{ep}}\left\langle \frac{\tau_p}{\tau_{pe}}\langle\tau_e\rangle_1\right\rangle_2\right\rangle_3 - 2\frac{\langle\tau_e^2\rangle_3}{\langle\tau_e\rangle_3}\left\langle \frac{\tau_e}{\tau_{ep}}\left\langle \frac{\tau_p}{\tau_{pe}}\langle\tau_e\rangle_1\right\rangle_2\right\rangle_3\right\},$$
$$\alpha_{xx}^{(0)} = \frac{\langle\tau_e S\rangle_3}{e\langle\tau_e\rangle_3}\left\{1 + \langle\tau_e S\rangle_3^{-1}\left\langle \frac{\tau_e}{\tau_{ep}}\left\langle \frac{\tau_p}{\tau_{pe}}\langle\tau_e S\rangle_1\right\rangle_2\right\rangle_3 - \langle\tau_e\rangle_3^{-1}\left\langle \frac{\tau_e}{\tau_{ep}}\left\langle \frac{\tau_p}{\tau_{pe}}\langle\tau_e\rangle_1\right\rangle_2\right\rangle_3\right\}, \quad (28)$$
$$Q^{(0)} = n_0 e\{\rho_{xx}^{(0)} H^{-1}\langle B\rangle_3 - R^{(0)}\alpha_{xx}^{(0)}\langle A_E\rangle_3\},$$

where B and A_E are defined in (25).

In addition, there exist for all the kinetic coefficients corrections that depend on the dragging parameter and are proportional to H^2. They can be easily obtained with the aid of (25). For example, the correction to the resistance is given by

$$\rho_{xx}^{(2)} = \frac{\omega_H^2}{\sigma_0\langle\tau_e\rangle_e^2}\left\{\langle\tau_e\rangle_3\langle\tau_e^3\rangle_3 - \langle\tau_e^2\rangle_3^2 - \left(2\langle\tau_e^3\rangle_3 - 3\frac{\langle\tau_e^2\rangle_3^2}{\langle\tau_e\rangle_3}\right)\left\langle \frac{\tau_e}{\tau_{ep}}\left\langle \frac{\tau_p}{\tau_{pe}}\langle\tau_e\rangle_1\right\rangle_2\right\rangle_3 + \langle\tau_e\rangle_3\left\langle 2\frac{\tau_e^3}{\tau_{ep}}\left\langle \frac{\tau_p}{\tau_{pe}}\langle\tau_e\rangle_1\right\rangle_2 + \tau_e\left\langle \frac{\tau_p}{\tau_{pe}}\langle\tau_e^2\rangle_1\right\rangle_2\right\rangle_3 - 2\langle\tau_e^2\rangle_3\left\langle \frac{\tau_e}{\tau_{ep}}\left\langle \frac{\tau_p}{\tau_{pe}}\langle\tau_e^2\rangle_1\right\rangle_2 + \tau_e\left\langle \frac{\tau_p}{\tau_{pe}}\langle\tau_e\rangle_1\right\rangle_2\right\rangle_3\right\}. \quad (29)$$

When comparison is made with experiments, expressions of this type are made more concrete by making definite assumptions concerning the character of the dependence of the relaxation times on the energies (wave numbers) of the electrons and phonons. For example, if the electron relaxation time τ_e does not depend on the electron energy, then for nondegenerate carriers we have $R^{(0)} = R^{(\infty)}$, whereas $\alpha_{xx}^{(0)} \neq \alpha_{xx}^{(\infty)}$ (owing to the presence of dragging). It is interesting to note that for nondegenerate electrons all the terms proportional to ζ/T cancel out in the difference $\alpha_{xx}^{(\infty)} - \alpha_{xx}^{(0)}$ (see expression (9) for S).

4. DISCUSSION OF RESULTS

The expressions presented in the preceding section for the kinetic coefficients enable us to draw certain general conclusions concerning the study of the effective dragging with the aid of an investigation of kinetic phenomena in a magnetic field. As follows from (22), in the case of strong Fermi degeneracy and one type of carrier, the kinetic coefficients are generally independent of the magnetic field. The electric conductivity contains a contribution from the mutual dragging, and the thermoelectric power, neglecting the corrections for incomplete degeneracy, is governed entirely by simple dragging.

In the case of nondegenerate carriers, and also in the case of intermediate degeneracy in a weak magnetic field, the dragging effect makes a contribution to all the discussed kinetic coefficients (mutual dragging contributes to ρ_{xx} and R, and mutual and simple dragging contribute to α_{xx} and Q), with the dragging contributing both to the values of the kinetic coefficients at H = 0 and to the coefficient of H^2, which describes the change in the magnetic field.

On going to strong magnetic fields, the Hall coefficient ceases to depend on the dragging, and only simple dragging contributes to $\alpha_{xx}^{(\infty)}$. Thus, in these cases the magnetic field can serve, as it were, as a separator of diffusion effects from dragging effects. On the other hand, in the case when the dragging effects make an appreciable contribution, one can hope they separate the contributions of the simple and mutual dragging to the thermomagnetic effects.

Naturally, the necessary condition remains a sufficiently reliable separation of the diffusion contribution to the charge fluxes, for which purpose it is necessary to treat simultaneously the aggregate of all the kinetic phenomena, in a manner similar, for example, to that used earlier[11,12]

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