SOVIET PHYSICS JETP VOLUME 36, NUMBER 5 MAY, 1973

ACCELERATION OF ATOMS BY A RESONANCE FIELD

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Submitted June 15, 1972

Zh. Eksp. Teor. Fiz. 63, 1628–1634 (November, 1972)

It is shown that intense laser radiation can be used to accelerate atoms. In a field of a special type (22) the effective atomic potential takes the form of a sinusoid moving with uniform acceleration. The captured atoms have an acceleration proportional to the rate at which the field frequency is varied. It is shown that under the conditions which can be attained in practice the velocity of the atom during its excited state lifetime may reach 10^8 cm/sec.

In this paper we investigate the question as to whether high-intensity laser radiation can be used to accelerate atoms. The force acting on an atom in an external field is expressed in terms of the real and imaginary parts of the atomic polarizability. The force associated with energy dissipation is, in fact, the force of recoil which occurs during the absorption or emission of a photon, and this changes the momentum of the atom by an amount of the order of the photon momentum $\hbar\mathbf{k}$. Some of the effects associated with the force of recoil are considered in[1,2]. The other part is the ponderomotive force connected, as in a continuous medium[3], with the gradient of the electromagnetic energy density. In a strong field this force will be quite large, so that the acceleration of the atoms while they are in the upper working levels can be substantial.

We shall consider some general expressions for the force acting on at atom. We shall then investigate the motion of the atom in the field of a special form [Eq. (22)], and will determine the acceleration effect.

1. THE FORCE OF AN ATOM

Let us begin by considering the general expression for the force acting on an atom in a resonance radiation field

$$\mathbf{E}e^{-i\omega t}+\mathbf{E}^*e^{i\omega t}, \tag{1}$$

where $\mathbf{E}(t, \mathbf{r})$ is the slowly varying amplitude of the electric field which is such that $dE/dt \ll \omega E$. We shall use the Lorentz formula for the force $\mathbf{F}$ acting on the atomic dipole moment

$$\mathbf{p}(t)e^{-i\omega t}+\mathbf{p}^*(t)e^{i\omega t},\quad \mathbf{p}(t)=\mathrm{Sp}(\rho(t)\hat{\mathbf{d}}), \tag{2}$$

which is induced by the external field. In these expressions ρ is the atomic density matrix and $\hat{d}$ is the dipole moment operator for the working transition. Using the Maxwell equation

$$i\omega\mathbf{H}=c\,\mathrm{rot}\,\mathbf{E}, \tag{3}$$

in the resonance approximation, we obtain the following expression for the force:

$$F_i=\sum_j p_j(t)\nabla_i E_j^*+\text{c.c.} \tag{4}$$

To obtain the final answer to our question we must solve the equation of motion for the atomic density matrix ρ with the Hamiltonian

$$H=\hbar\Delta+\hat{\mathbf{d}}\mathbf{E}^*+\hat{\mathbf{d}}^+\mathbf{E}, \tag{5}$$

where $\Delta = \omega_0 - \omega$ is the frequency detuning and ω_0 is the transition frequency.

To establish the orders of magnitude we shall suppose that $\mathbf{E} \sim e^{i\mathbf{k}\cdot\mathbf{r}}$, $\mathbf{k} = \mathbf{k}_1 + i\mathbf{k}_2$, and $\mathbf{k}_1$ and $\mathbf{k}_2$ are the real and imaginary parts of the wave vector:

$$\mathbf{F}=\mathbf{F}_1+\mathbf{F}_2; \tag{6}$$

$$\mathbf{F}_1=-i\mathbf{k}_1(\mathbf{p}^*\mathbf{E}-\text{c.c.}),\quad \mathbf{F}_2=\mathbf{k}_2(\mathbf{p}^*\mathbf{E}+\text{c.c.}). \tag{7}$$

In a progressive wave of constant amplitude the atom experiences only the force $\mathbf{F} = \mathbf{F}_1$ which is proportional to the field energy $\mathrm{Im}(\mathbf{p}^*\mathbf{E})$ dissipated in the atom per unit time. The order of magnitude of this force is $\mathbf{F}_1 \sim \hbar\mathbf{k}_1\gamma$, and γ^{-1} is the lifetime of the excited state. The total momentum transferred to the atom while it is in the working state can be written in the form

$$\int_0^\infty dt\mathbf{F}_1(t)=\pm\hbar\mathbf{k}_1W, \tag{8}$$

where W is the probability of stimulated emission or absorption of a photon of the external field, and the sign is determined by the initial difference between the level populations. Thus, in the simplest model of a two-level atom in which both levels have the same lifetime γ^{-1} we have for the progressive monochromatic wave

$$W=\frac{2d^2|\mathbf{E}|^2}{\hbar^2(\gamma^2+\Delta^2)+4d^2|\mathbf{E}|^2}. \tag{9}$$

In this expression d is the matrix element of the dipole transition. It follows that the force of recoil as a function of frequency is described by the Lorentz curve with a maximum at the line center. In a weak field, W is small, whereas in a strong field $W = \frac{1}{2}$. The momentum transferred to the atom is shown by Eq. (6) never to exceed $\hbar\mathbf{k}_1$.

Let us now consider the ponderomotive force $\mathbf{F}_2$ which is associated with the field gradient. We shall restrict our attention to the case of a strong field, i.e.,

$$d|\mathbf{E}|\gg\hbar\gamma. \tag{10}$$

The force $\mathbf{F}_2$ can be quite large if the condition $\hbar\Delta \sim d|\mathbf{E}|$ is satisfied.

We shall now find the final expression for the force in the following important case of a quasistationary field:

$$dE/dt\ll\Delta E. \tag{11}$$

In this case, we can readily show that the induced dipole moment is

$$\mathbf{p}(t)=\frac{d^2\mathbf{E}}{\hbar\Delta_E\Delta_E{}^0}\Delta, \qquad (12)$$
$$\Delta_E{}^2=\Delta^2+4d^2|\mathbf{E}|^2/\hbar^2.$$

In these expressions we have not taken into account the decay of the atomic states (Δ_E^0 is the value of Δ_E at the initial time of excitation). In the model which we shall consider below, ΔE is much less than Ω_0 (the characteristic frequency of the captured atoms). For this reason we have neglected in (12) the terms which oscillate rapidly with frequency Δ_E. We then have the following effective potential for the atom in the resonance field.

$$F_z=\nabla U, \qquad U=\pm\frac{\Delta}{2\Delta_E{}^0}(\Delta_E-|\Delta|) \qquad (13)$$

The frequency dependence of the potential is shown in Fig. 1. It is important that the maximum value of the potential (U_m) is of the order of dE. For comparison, let us consider the potential of a charged particle placed in the same field. According to[4] we have

$$U=(e/2M\omega)^2|\mathbf{E}|^2. \qquad (14)$$

Comparison of Eqs. (13) and (14) can conveniently be carried in terms of the atomic units. In these units $U_m \sim E$, for the electron $U \sim E^2$, and for an ion $U \sim (E/M)^2$. Thus the atomic potential in the light field which is small in comparison with the atomic Coulomb field ($E \ll E_{at}$) has its maximum value: It is greater by the factor E_{at}/E than the potential of the electron. It is clear that this fact and the uncertainty in the sign of the potential (13) are connected with the resonance character of the interaction. In a weak field $dE \ll \hbar\Delta$, and

$$U=\pm d^2|\mathbf{E}|^2/\hbar\Delta. \qquad (15)$$

In this case, the Stark shift is quadratic in the field.

In a field of the order of 3×10^6 V/cm, the atomic potential U_m is of the order of the kinetic energy of the atoms at room temperature. An atom placed in a field of a standing wave of this amplitude changes its energy by a substantial amount over a very small distance (a quarter of the wavelength). In the next section we shall show that, in the field derived from the potential which is in the form of a uniformly accelerated sinusoid, the energy can increase by an amount of the order

$$(kv_0t)^2U_m, \qquad (16)$$

where v_0 is the characteristic initial velocity of the captured atom.

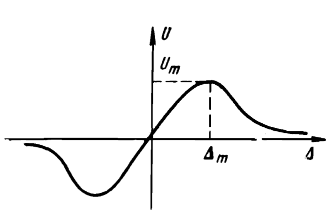

FIG. 1

2. ACCELERATION OF ATOMS BY A NONSTATIONARY FIELD

Consider, to begin with, a field of the form

$$\mathbf{E}=\mathbf{E}_1\exp\{i(k+\Omega/c)x-i\Omega t\}+\mathbf{E}_2\exp\{-i(k-\Omega/c)x+i\Omega t\}, \qquad (17)$$

where $\mathbf{E}_1$ and $\mathbf{E}_2$ lie in the plane perpendicular to the x axis, and $k = \omega/c$. We shall suppose that

$$\Omega\ll\Delta, \qquad (18)$$

and this enables us to use Eq. (13) or, in a weak field, the expression given by Eq. (15). For the sake of simplicity, we shall use the latter case:

$$F_x=F\sin(2kx-2\Omega t+\varphi),\ F=4d^2k|\mathbf{E}_1\mathbf{E}_2{}^*|/\hbar\Delta, \qquad (19)$$

where φ is a phase constant.

If the Mandel'shtam-Brillouin resonance condition

$$\Omega=kv, \qquad (20)$$

is satisfied for an atomic velocity v, the atom experiences a constant force. However, this force modifies the velocity of the atom, and the condition given by Eq. (20) is violated. On the average, therefore, the atoms move with constant velocity: captured atoms move with a velocity Ω/k and uncaptured atoms with a velocity determined by the initial conditions.

To ensure that the resonance is not violated during the acceleration process, Ω must change with time in such a way that Eq. (20) is satisfied at all times. To investigate this, consider the superposition of two fields for which the frequencies are linear functions of time:

$$\Omega(t)=\Omega_0+\dot{\Omega}t, \qquad (21)$$

where $\dot{\Omega}$ is the constant rate of change of the frequency. Instead of Eq. (17), we now have

$$\mathbf{E}=\mathbf{E}_1\exp\{i[kx-\Omega_0(t-x/c)-{}^1\!/_2\dot{\Omega}(t-x/c)^2]\}$$
$$+\mathbf{E}_2\exp\{i[-kx+\Omega_0(t+x/c)+{}^1\!/_2\dot{\Omega}(t+x/c)^2]\}. \qquad (22)$$

This field clearly satisfies the wave equation. In the next section we shall consider how a field of this kind can be used in practice.

The formula for the force now takes the form

$$F_x=F\sin(2kx-2\Omega_0t+\dot{\Omega}(t^2+x^2/c^2)). \qquad (23)$$

The phase of the wave determined by Eq. (23) contains a term which is nonlinear in x, and is multiplied by the small coefficient c^{-2}. So long as $x < c/\sqrt{\dot{\Omega}}$, this term is small and can be ignored. In the numerical example discussed below, $\dot{\Omega} = 10^{20}$ Hz^2, so that the last condition means that $x < 3$ cm. When $x > c/\sqrt{\dot{\Omega}}$, the nonlinear term becomes greater than unity. However, so long as it is less than the linear component, i.e.,

$$x\ll L=\omega c/\dot{\Omega}, \qquad (24)$$

this can be ignored. This estimate will be justified later. For the moment, we shall adopt Eq. (24) as valid, and will neglect the term including x^2 in Eq. (23).

Thus, over the region $x \ll L$, the constant-phase front of the force F_x propagates with uniform acceleration. To analyze the motion of the particles in this case, let us transform to the uniformly accelerated set of coordinates:

$$x = x_0(t) + y,\ x_0(t) = v_0 t + at^2/2; \tag{25}$$

$$kv_0 = \Omega_0,\ ka = \dot{\Omega}. \tag{26}$$

It is clear from Eq. (26) that the resonance condition (20) is now satisfied for the mean velocity $v(t) = v_0 + at$. In the new coordinate system the equation of motion for the particles is

$$\ddot{y} + a = (F/M)\sin(2ky + \varphi), \tag{27}$$

where M is the mass of the atom. This equation is well known and describes the so-called pulling phenomena (for example, the frequency pulling in a rotating laser[5]). The particle now moves in the field of the potential

$$U(y) = May + U_0\cos(2ky + \varphi),\ U_0 = F/2k, \tag{28}$$

shown in Fig. 2. As long as the acceleration does not exceed the maximum value

$$a_m = F/M, \tag{29}$$

there is a bunch of captured particles which lies in this region of the potential (shown shaded in Fig. 2). The number of captured particles depends on the two parameters $\eta = 2U_0/Mv_0^2$ and a/a_m. If the wave propagates at constant velocity then for $\eta \gtrsim 1$ the fraction of captured particles is of the order of unity, and for $\eta \ll 1$. The fraction is of the order of η. Transition to the uniformly accelerated motion is accompanied by the fact that a fraction of the captured particles enters the continuous spectrum. The number of the remaining particles can be estimated very approximately by the ratio of the shaded area to the area bounded by the potential curve and the broken line. The corresponding formulas are quite complicated and will not be written out here.

The condition for the existence of captured particles, $a < a_m$, imposes a restriction on the rate at which the frequency can be varied, i.e., $\Omega < \dot{\Omega}_m = ka_m$ or

$$\dot{\Omega}_m = 2k^2U_0/M. \tag{30}$$

The maximum rate of change of the frequency is thus seen to coincide with the square of the frequency of small oscillations of the captured particles about the minimum potential A.

For further estimates it will be convenient to write the last condition in the form

$$\dot{\Omega}_m = \eta\Omega_0^2. \tag{31}$$

Let us now consider the acceleration of the captured atoms. For the maximum possible acceleration we have

$$v_m(t) = v_0(1 + \eta\Omega_0 t). \tag{32}$$

The case $\eta \sim 1$ is, clearly, of particular interest, since here the fraction of captured particles is of the order of unity. In this case, for large t we have $v_m(t)/v_0 = \Omega_0 t$. It is precisely for this situation that we shall consider the following numerical example.

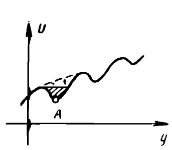

FIG. 2

Let us suppose that $v_0 = 10^5$ cm/sec, $k = 10^5$ cm^{-1}, $\Omega_0 = 10^{10}$ Hz, and $\Delta = 10^{13}$ Hz. The adiabatic condition (11) can be written in the form $\Omega(t) \ll \Delta$ and, during the initial stage of acceleration, this is easily satisfied. Next, from the condition $\eta = 1$ and for $M = 10^{-24}$ g we have $U_0 = 0.03$ eV and $E = 2 \times 10^6$ V/cm. In the last estimate the dipole moment of the transition is assumed to be equal to 1 d. According to Eq. (31), we then have $\dot{\Omega}_m = 10^{20}$ Hz^2. The acceleration parameter is restricted by the lifetime of the atom in the working level $\tau = \gamma^{-1}$. For $\tau = 10^{-7}$ sec, we have $\Omega_0\tau = 10^3$. Therefore, in the present example, the velocity of the captured atoms at the end of the acceleration process is 10^8 cm/sec and the energy is 3 keV. The path length is 5 cm. We note that when $t = \tau$ we have $\Omega = 10^{13}$ Hz, so that, at the end of the process, the adiabatic condition (11) is violated.

3. DISCUSSION

Let us now consider the assumptions adopted in the course of the above calculations.

1. In connection with the field given by Eq. (22), we must consider the following points. A field with a frequency varying linearly with time can be obtained in various ways. We shall consider the simplest case, i.e., that in which the variation is achieved by varying the length of a Fabry-Perot resonator. This turns out to be sufficient to realize the numerical value adopted for $\dot{\Omega}_m$ above.

If one of the mirrors of the resonator travels in accordance with the law $\delta l(t)$, the radiated frequency is given by

$$\omega(t) = \frac{\omega}{1 + \delta l(t)/l}, \tag{33}$$

where ω and l are the initial frequency and length of the resonator. Hence, we find that for small displacements

$$\dot{\Omega} = v\omega/l, \tag{34}$$

where v is the velocity of the mirror. The value $\dot{\Omega} = 10^{20}$ Hz^2 in which we are interested can be achieved by adopting $\omega = 10^{15}$ Hz, $v = 10^5$ cm/sec, and $l = 1$ cm. We must, however, take into account the restriction on this mechanism of frequency "acceleration" due to the finite Q of the resonator. The total change in the frequency, $\Delta\omega$, during the lifetime of the photon in the resonator with a moving wall can be written in the form

$$\Delta\Omega = 2kv/(1 - R), \tag{35}$$

where R is the reflectance of the mirror. This formula has a very simple interpretation: 2kv is the shift in the photon frequency when it collides with the wall, and $(1 - R)^{-1}$ is the number of collisions. To obtain $\Delta\Omega = 10^{13}$ Hz for $kv = 10^{10}$ Hz we must have a transmittance $1 - R = 10^{-3}$.

Thus, the field generator necessary for our problem can be of the form illustrated in Fig. 3. At the initial time we have two identical resonators filled with monochromatic radiation of frequency ω. The mirror of one of the resonators then moves in the outward direction with velocity v, and the mirror of the other resonator moves in the inward direction with the same velocity. As a result of the interference between the two fields

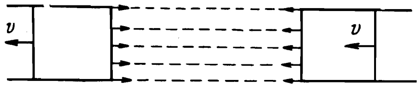

FIG. 3

in the space between the two resonators, we have the effective potential with takes the form of a sinusoid moving with uniform acceleration.

2. Let us now consider the condition given Eq. (24). It will be convenient to substitute

$$x = x_0(t) + \dot{\Omega} x^2 / 2\omega c + y. \tag{36}$$

If we look upon x^2 as a small quantity in this equation, we have instead of (27)

$$(\ddot{y} + a)\left(1 + \frac{x_0 + y}{L}\right) + \frac{1}{2}(\dot{x}_0 + \dot{y})^2 = F \sin(2ky + \varphi). \tag{37}$$

The amplitude y for the captured particles is of the order of the wavelength. We therefore have $y \ll x_0$ and $\dot{y}/\dot{x}_0 \sim 1/\Omega_0\tau \ll 1$. Neglecting y and $\dot{y}$ on the left-hand side of Eq. (37), we obtain the criterion given by Eq. (24), i.e., $x_0 \ll L$ (for the above numerical example $L = 10^5$ cm). It is clear that this condition can be written in an equivalent form as a restriction on the time:

$$t \ll T = \omega / \dot{\Omega}. \tag{38}$$

In the above example $T = 10^{-5}$ sec.

3. Let us now estimate the minimum power necessary to realize the above model. This is determined by the field strength $E = 2 \times 10^6$ V/cm and the minimum beam cross section λx necessary to produce a uniform field over a distance of the order of x. For $x = 5$ cm and $\lambda = 5 \times 10^{-5}$ cm, we have $\lambda x = 2 \times 10^{-4}$ cm^2. Hence, we find the power flux of 10^{10} W/cm^2 and a power output of 2×10^6 W. The total energy consumed is less than 1 J. The total number of particles which can be accelerated by this method for an expenditure of 1 J in energy is estimated to be of the order of 10^{15}.

We are indebted to S. I. Anisimov and V. L. Pokrovskiĭ for valuable discussions, and to S. G. Rautian for discussions and a number of very valuable suggestions.

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Translated by S. Chomet
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