HIGH-FREQUENCY INSTABILITY OF AN ELECTROMAGNETIC WAVE IN A NONEQUILIBRIUM MAGNETIZED PLASMA

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It is shown that in a nonequilibrium medium, namely a magnetized plasma which is permeated by an electron beam, an instability of a low-frequency electromagnetic wave should be possible, which results in the excitation of two high-frequency waves and in a decrease of amplitude of the initial pumping wave. The mechanism of such an instability is discussed and its growth rate is determined. The corresponding estimates for a laboratory plasma are presented.

IN a transparent, thermodynamically equilibrated nonlinear medium it is possible to have, as is known, two kinds of instability of a monochromatic wave of finite amplitude—parametric or decaying[1,2], when, at the expense of the energy of frequency ω there are excited only low frequency waves with parameters ω_j and $\mathbf{k}_j$ such that they fulfill the synchronism conditions $\omega = \Sigma n_j\omega_j$ and $\mathbf{k} = \Sigma \mathbf{k}_j n_j$, and an instability of the type $m\omega = \Sigma n_j\omega_j$, $m\mathbf{k} = \Sigma n_j\mathbf{k}_j$, which can be considered as the decay of m quanta that are in a single state. In the simplest case it is the decay of a pair of quanta $\omega + \omega \to \omega_1 + \omega_2$, $\mathbf{k} + \mathbf{k} \to \mathbf{k}_1 + \mathbf{k}_2$, i.e., at the expense of the energy of the monochromatic wave there are simultaneously excited high frequency and low frequency waves. In the particular case when $|\omega_1 - \omega_2| \ll \omega$ this is called "modulation" instability, and brings about a pinching of the wave and a separation into packets[3,4].

The instability of a monochromatic wave with respect to high-frequency perturbations only (excluding the obvious effect of harmonic generation) at $m \leq 2$ in equilibrium mediums is impossible. For a medium with quadratic nonlinearity it means that it is impossible to excite two high-frequency waves with the aid of one low frequency pumping wave. In this work it is shown that for electromagnetic waves such an instability can take place in a nonequilibrium medium, namely in a magnetized plasma which is permeated by an electron beam.

It should be noted that three-wave interactions of the type

$$\omega_1 + \omega_2 = \omega_3, \quad \mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_3 \tag{1}$$

in a nonequilibrium plasma that admits of the existence of waves with negative energy[5] (i.e. waves with whose growth the energy of the medium decreases), have been considered in the literature[6-9]. However, all that work was directed only to the investigation of the so-called "explosive instability," which is characterized by the simultaneous growth of the amplitudes of all interacting waves. For longitudinal waves, the onset of such an instability is possible, for example, in a beam + plasma system, if the energy of the high-frequency or of the two low-frequency waves is negative (the electron current, which causes the nonequilibrium of the medium, serves as the reservoir for the growing waves). For electromagnetic waves the realization of explosive instability is difficult and demands highly specific conditions.

From the point of view of increasing the frequency as the result of conversion, essential interest attaches to the process considered below, interaction of low-frequency and the high-frequency electromagnetic waves with a plasma wave of intermediate frequency, possessing negative energy. Such a process permits the energy to be obtained in the form of electromagnetic radiation from a medium in which it is impossible to obtain directly the instability of electromagnetic waves.

1. In a plasma magnetized along a constant magnetic field, the propagation of waves of several types is possible, including "whistlers" or helicons, longitudinal waves representing oscillations of the charge, and high-frequency electromagnetic waves. Let us study the interaction of these waves in a beam + plasma system with a constant magnetic field. Using the quasihydrodynamic approximation, we write the initial equations in the form[1)]

$$\frac{\partial \mathbf{v}}{\partial t} - \frac{e}{m}\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}\mathbf{H}_0]\right) = \mu\left\{-(\mathbf{v}\nabla)\mathbf{v} + \frac{e}{mc}[\mathbf{v}\mathbf{H}]\right\}, \tag{2}^*$$

$$\frac{\partial \rho}{\partial t} + N\,\mathrm{div}\,\mathbf{v} = -\mu\,\mathrm{div}\,\rho\mathbf{v},$$

$$\frac{\partial \mathbf{v}_s}{\partial t} + (\mathbf{V}_0\nabla)\mathbf{v}_s - \frac{e}{m}\left(\mathbf{E} + \frac{1}{c}[\mathbf{V}_0\mathbf{H}] + \frac{1}{c}[\mathbf{v}_s\mathbf{H}_0]\right) = \mu\left\{-(\mathbf{v}_s\nabla)\mathbf{v}_s + \frac{e}{mc}[\mathbf{v}_s\mathbf{H}]\right\},$$

$$\frac{\partial \rho_s}{\partial t} + N_s\,\mathrm{div}\,\mathbf{v}_s + \mathbf{V}_0\nabla\rho_s = -\mu\,\mathrm{div}\,\rho_s\mathbf{v}_s, \qquad \mathrm{rot}\,\mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} = 0,$$

$$\mathrm{rot}\,\mathbf{H} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} - \frac{4\pi e}{c}(N\mathbf{v} + N_s\mathbf{v}_s) - \frac{4\pi e}{c}\rho_s\mathbf{V}_0 = \mu\left\{\frac{4\pi e}{c}(\rho\mathbf{v} + \rho_s\mathbf{v}_s)\right\},$$

where **E** and **H** are alternating electric and magnetic fields, $\mathbf{H}_0$ is the intensity of the constant magnetic field, ρ, ρ_S, **v** and $\mathbf{v}_S$ are the deviations of the densities and velocities of electrons in the main plasma and in the beam from the equilibrium values N, N_S, 0, and V_0, and the parameter μ is introduced to denote the smallness of the nonlinear terms.

[1)]We consider processes of sufficiently high frequency, for which the motion of the ions can be neglected. Furthermore, the thermal motion is disregarded.

*$[\mathbf{vH}] \equiv \mathbf{v} \times \mathbf{H}$.

Let all the waves propagate along H_0; then the dispersion equation of the medium (with $\mu = 0$) breaks up into two:

$$D(\omega_{1,3}, k_{1,3}) = \omega_{1,3}^2\varepsilon(\omega_{1,3}, k_{1,3}) - k_{1,3}^2c^2 = 0,$$

$$\varepsilon(\omega_{1,3}, k_{1,3}) = 1 - \frac{\omega_0^2}{\omega_{1,3}(\omega_{1,3} - \omega_H)} - \frac{\omega_{0s}^2(\omega_{1,3} + k_{1,3}V_0)}{\omega_{1,3}^2(\omega_{1,3} + k_{1,3}V_0 - \omega_H)}, \quad (3)$$

$$D(\omega_2, k_2) = \varepsilon_{\text{pl}} = 1 - \frac{\omega_0^2}{\omega_2^2} - \frac{\omega_{0s}^2}{(\omega_2 - k_2V_0)^2} = 0, \quad (4)$$

Here $\omega_0 = (4\pi Ne^2/m)^{1/2}$ and $\omega_{0S} = (4\pi N_S e^2/m)^{1/2}$ are the Langmuir frequencies of the plasma and beam electrons; $\omega_H = |eH_0/mc|$ is the gyrofrequency of electrons. Equation (3) corresponds to electromagnetic waves, and (4) to plasma waves. The fact that electromagnetic waves are polarized circularly in the direction of electron rotation ("extraordinary" wave) is taken into consideration in (3).

Assuming that the interacting waves—the helicon (ω_1, k_1), the HF electromagnetic wave (ω_3, k_3), and the plasma wave (ω_2, k_2)—satisfy the synchronism condition (1) (these conditions are verified below), let us write the solution of (2) as

$$E_y = a_1\psi_1 \exp\{i(\omega_1 t - k_1 x)\} + a_3\psi_3 \exp\{i(\omega_3 t - k_3 x)\} + \text{c.c.}, \quad E_z = -iE_y,$$

$$E_x = a_2\psi_2 \exp\{i(\omega_2 t - k_2 x)\} + \text{c.c.}, \quad (5)$$

$$\psi_{1,3} = (D'_{\omega_{1,3}})^{-1}\left\{2 + \frac{\omega_H\omega_0^2}{(\omega_{1,3} - \omega_H)^2\omega_{1,3}} + \frac{\omega_H\omega_{0s}^2}{\omega_{1,3}(\omega_{1,3} + k_{1,3}V_0 - \omega_H)^2}\right\}^{-1},$$

$$\psi_2 = \frac{1}{2\omega_2 D'_{\omega_2}}\left\{\frac{\omega_0^2}{\omega_2^3} + \frac{\omega_{0s}^2}{(\omega_2 - k_2V_0)^3}\right\}^{-1};$$

$$D'_{\omega_{1,3}} = \frac{\partial\omega_{1,3}^2\varepsilon(\omega_{1,3})}{\partial\omega_{1,3}}, \quad D'_{\omega_2} = \frac{\partial\varepsilon_{\text{pl}}}{\partial\omega_2},$$

here a_i are the complex amplitudes of the fields, ψ_i are normalizing factors, and ω_i and k_i are connected by the dispersion equation. For slowly varying amplitudes $a_i(x, t)$ we obtain from (2), (5), and (1) by means of the asymptotic method[10] the following equations:

$$\frac{\partial a_{1,2}}{\partial t} + v_{1,2}\frac{\partial a_{1,2}}{\partial x} = \sigma_{1,2}a_3\dot{a}_{2,1}, \quad \frac{\partial a_3}{\partial t} + v_3\frac{\partial a_3}{\partial x} = -\sigma_3 a_1 a_2, \quad (6)$$

where v_i are the group velocities of the waves and σ_i are the nonlinear-interaction coefficients, equal to

$$\sigma_i = G/D'_{\omega_i},$$

$$G = \frac{e}{m}\omega_2\left\{\frac{\omega_0^2}{\omega_2^2}\left[k_2 - \omega_H\left(\frac{k_3}{\omega_3} - \frac{k_1}{\omega_1}\right)\right]\frac{\omega_3}{(\omega_3 - \omega_H)}\frac{\omega_1}{(\omega_1 - \omega_H)} + \frac{\omega_{0s}^2}{(\omega_2 - k_2V_0)^2}\left[k_2 - \omega_H\left(\frac{k_3}{\omega_3 + k_3V_0} - \frac{k_1}{\omega_1 + k_1V_0}\right)\right] \times \frac{(\omega_3 + k_3V_0)(\omega_1 + k_1V_0)}{(\omega_3 + k_3V_0 - \omega_H)(\omega_1 + k_1V_0 - \omega_H)}\right\} \quad (7)$$

(similar expressions for the coupling coefficients of two transverse and a longitudinal wave in a magnetized plasma with a constant electric field were obtained by the method of coupled waves by other authors[9,11]).

2. It follows from (6) and (7) that the character of the investigated nonlinear processes is determined by the signs of D_{ω_i}, which in the given case coincide with the signs of the energies of the interacting waves. Let us determine the expression for σ_i for concrete wave parameters ω_i, k_i expressed in terms of the characteristic frequencies ω_0 and ω_H.

Let us require that ω_3 considerably exceed ω_1 which is possible if $\omega_0 \gg \omega_H$. In conformance with (3) and (4), the synchronism conditions (1) can be satisfied in this case if $\omega_3 \gtrsim \omega_0 + \omega_H$, $\omega_1 \leq \omega_H$, and $|k_3| \ll |k_1|$, $|k_2|$ (the difference $|\omega_1 - \omega_H|$ is assumed to be not too small to be able to neglect cyclotron absorption of the helicon wave). Depending on the pump frequency ω_1 the traverse waves can be in synchronism either with a wave of frequency

$$\omega_2 = \omega_0 - \delta_1, \quad \frac{\delta_1}{\omega_0} \approx 2^{-4/3}\left(\frac{N_s}{N}\right)^{1/3} \ll 1,$$

or with a wave of frequency

$$\omega_2 = \omega_0 + \delta_2, \quad \frac{\delta_2}{\omega_0} \approx \frac{1}{2}\left(\frac{N_s}{N}\right)\frac{1}{(\beta - 1)^2} \ll 1,$$

$$\left(\beta \approx \frac{V_0}{c}\left(\frac{\omega_1}{|\omega_1 - \omega_H|}\right)^{1/2} > 1\right).$$

In the first case there takes part in the interaction process a plasma wave that grows in the linear approximation (its growth rate is $\gamma/\omega_0 \approx 2^{-4/3} \cdot 3^{1/2} \cdot (N_S/N)^{1/3} \ll 1$), the energy of which is negative at any arbitrarily small concentration ratio N_S/N. In the second case the wave is in the transparency region and its energy is negative when

$$\left(3 + \frac{2}{\beta - 1}\right)\frac{\delta_2}{\omega_0} > 1.$$

It is easy to see that to satisfy the synchronism conditions the plasma and helicon waves must be oppositely directed.

When electromagnetic waves interact with a plasma wave of negative energy we have, unlike in equilibrium quadratic media, where $\sigma_1\sigma_2 > 0$ and $-\sigma_3\sigma_2 < 0$,

$$\sigma_1\sigma_2 < 0, \quad -\sigma_3\sigma_2 > 0, \quad (8)$$

i.e., the high-frequency (ω_3) and the low-frequency (ω_1) waves exchange roles. The most important and principal feature of the considered interaction is the possibility of exciting two HF waves (ω_2, ω_3) by low-frequency pumping (ω_1)[2)]. Indeed, confining ourselves to spatially-homogeneous solutions we obtain from (6) and (8) (assuming $\gamma \approx 0$) the integrals

$$\frac{|a_1|^2}{\sigma_1} + \frac{|a_3|^2}{\sigma_3} = \text{const}, \quad \frac{|a_3|^2}{\sigma_3} - \frac{|a_2|^2}{\sigma_2} = \text{const}, \quad (9)$$

whose existence signifies that in the system under investigation, on one hand, the usual decay instability i.e., an appreciable decrease of the wave amplitude a_3 is impossible if the priming amplitudes a_1 and a_2 are small, and on the other hand, the wave of lower frequency ω_1, which is intense at $t = 0$, can decrease to zero with increasing t, having transferred its energy to the HF waves, whose amplitudes were initially small. The law of conservation of the energy of the waves is in this case, naturally, not fulfilled—the upward frequency transformation requires, besides the expenditure of energy at the pump frequency ω_1 also expenditure of the kinetic energy of the electron beam. To verify this, it is sufficient to determine the sign of the energy, averaged over the period, lost by the electron beam in a unit volume during the excitation of the HF electromagnetic field of frequency ω_3. In our case this quantity is equal to

$$W = \lim_{T\to\infty}\frac{1}{T}\int_0^T \mathbf{jE}\,dt = e\rho_s^0 E_3^0 v_\perp^0 > 0,$$

$$\rho_s^0 = \left|\frac{ek_3Na_2}{m(\omega_2 - k_2V_0)^2}\right|, \quad v_\perp^0 = \left|\frac{ea_1}{m(\omega_1 - \omega_H)}\right|; \quad (10)$$

2) For longitudinal waves in a beam + plasma system, the feasibility of such a process was demonstrated earlier [12].

here ρ_S^0, $\mathbf{E}_3^0$, and $v_\perp^0$ are respectively the amplitudes of the oscillations of the electron density, of the HF perturbation, the velocity of the electron flow transverse to $\mathbf{H}_0$, which is determined by the action of low-frequency pumping.

The growth rate of the HF instability in question, for spatially homogeneous perturbations, is equal to

$$\Gamma = \begin{cases} (|\sigma_2\sigma_3||a_1|^2)^{1/2} = \Gamma_0 \text{ for } \omega_2 \geqslant \omega_0 \\ \gamma/2 + (\Gamma_0^2 + \gamma^2/4)^{1/2} \text{ for } \omega_2 \leqslant \omega_0; \end{cases} \tag{11}$$

$$\gamma \approx 2^{-4/3} \cdot 3^{1/2} \left(\frac{N_s}{N}\right)^{1/3} \omega_0, \quad \sigma_2 \approx \frac{e}{7mV_0} \left| \frac{\omega_1 - \omega_H}{\omega_0} \right| \text{ for } \omega_2, \omega_0 \gg \omega_H,$$

$$\sigma_3 \approx \begin{cases} 7\sigma_2 & \text{for } \omega_2 \leqslant \omega_0 \gg \omega_H \\ \dfrac{4e}{mV_0} \left| \dfrac{\omega_1 - \omega_H}{\omega_0} \right| \left[\dfrac{\delta_2}{\omega_0} \left(3 + \dfrac{2}{\beta - 1} \right) - 1 \right]^{-1} & \text{for } \omega_2 = \omega_0 + \delta_2 \gg \omega_H \end{cases}$$

We note that, independently of the decay of the helicon wave in the system under investigation, two-stream instability there will cause a growth of longitudinal waves with a broad spectrum of frequdncies $0 < \omega < \omega_0$. However, if the growth rate $\Gamma \gg \gamma$, the characteristic times of these processes will differ significantly and we can disregard the slow (over times $t \sim 1/\Gamma$) growth of the longitudinal waves due to the two-stream instability.[3)]

Since Γ depends on $|a_1|$, by proper selection of the pump amplitude it is possible to satisfy the condition $\Gamma \gg \gamma$ and thus use the beam + plasma system, in which oscillations with a broad spectrum $\Delta\omega \sim \omega$ are excited under normal conditions, to amplify and generate quasi-harmonic HF oscillations. Here it should be added that in the presence of a strong helicon wave in the plasma, the motion of the electrons across the constant magnetic field can produce, generally speaking, a Buneman instability[13] that results in damping of the helicon and stops the HF instability. However, if the transverse electron drift velocity is less than their thermal velocity, this instability is absent[4)].

3. We present estimates for a laboratory plasma with parameters $N = 10^{13}$ cm^{-3} ($\omega_0 \approx 10^{11}$ sec^{-1}), $N_s \approx 10^8$ cm^{-3}, $H_0 = 600$ Oe ($\omega_H \approx 10^{10}$ sec^{-1}), $V_0 \approx 5 \times 10^9$ cm/sec, $j = eN_sV_0 \approx 22$ mA/cm^2, $|\omega_1 - \omega_H|/\omega_0 \approx 3 \times 10^{-3}$, and electron temperature $T \approx 5 \times 10^4$°K. This yields an electron thermal velocity $v_T \approx 8 \times 10^7$ cm/sec, i.e., $v_T \ll V_0$. The transverse drift velocity under these conditions is $v_d \approx 5 \times 10^7$ cm/sec, i.e., $v_d < v_T$, which guarantees the absence of the Buneman instability. A helicon of frequency $\omega_1 \leq \omega_H \approx 10^{10}$ sec^{-1} with electric-field amplitude $|a_1| \approx 20$ V/cm will excite a HF electromagnetic wave with a growth rate $\Gamma \approx 10^{10}$ sec^{-1}. The maximum growth rate of beam instability in this case is $\gamma \approx 10^9$ sec^{-1}, i.e., $\gamma/\Gamma \approx 10^{10}$ sec^{-1}. The maximum growth rate of beam instability in this case is $\gamma \approx 10^9$ sec^{-1}, i.e., $\gamma/\Gamma \approx 10^{-1}$, and allowance for the thermal motion lowers γ still more. For the evaluation of the maximum amplitude of HF wave in this case (due to the large growth rate) it is necessary to take into consideration the reaction effect of the generated waves on the electron beam, namely the change in its distribution function.

By this means, in a magnetized beam + plasma system it is possible to effectively excite HF waves with low frequency pumping, with a frequency conversion coefficient $\omega_3/\omega_1 \gtrsim 10$. This effect can also take place in a solid-state plasma in constant electric and magnetic fields. To obtain the corresponding estimates, since $V_0 \approx v_T$ in a solid-state plasma, a kinetic analysis is needed.

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[3)] A similar situation takes place also for increasing helicons, if the conditions of the anomalous doppler effect are satisfied. [13]

[4)] For longitudinal waves, the thermal motion can be neglected if the thermal velocity is much less than the beam velocity V_0.

[1] V. N. Oraevskiĭ and R. Z. Sagdeev, Zh. Tekh. Fiz. **32**, 1291 (1962) [Sov. Phys.-Tech. Phys. **7**, 955 (1963)].

[2] N. Bloembergen, Nonlinear Optics, Benjamin, 1965.

[3] L. A. Ostrovskiĭ, Zh. Eksp. Teor. Fiz. **51**, 1189 (1966) [Sov. Phys.-JETP **24**, 797 (1967)].

[4] B. B. Kadomtsev and V. I. Karpman, Usp. Fiz. Nauk **103**, 193 (1971) [Sov. Phys.-Uspekhi **14**, 40 (1971)].

[5] B. B. Kadomtsev, A. B. Mikhaĭlovskiĭ, and A. V. Timofeev, Zh. Eksp. Teor. Fiz. **47**, 2266 (1964) [Sov. Phys.-JETP **20**, 1517 (1965)].

[6] V. M. Dikasov, L. I. Rudakov, and D. D. Ryutov, ibid. **48**, 913 (1965) [**21**, 608 (1965)].

[7] B. Coppi, M. N. Rosenbluth, and R. N. Sudan, Ann. of Phys. **55**, 207 (1969).

[8] A. Jarmon, L. Steaflo, H. Wilhelmson, and F. Engelmann, Phys. Lett. **28A**, 748 (1969).

[9] H. Wilhelmson, Symposium on Beam—Plasma Inter., Utrecht (1969).

[10] M. I. Rabinovich, Dokl. Akad. Nauk SSSR **191**, 1253 (1970) [Sov. Phys.-Dokl.

[11] H. Wilhelmson, J. Plasma Phys. **3**, 215 (1969).

[12] M. I. Rabinovich and S. M. Faĭnshteĭn, Zh. Eksp. Teor. Fiz. **57**, 1298 (1969) [Sov. Phys.-JETP **30**, 705 (1970)].

[14] A. I. Akhiezer, I. A. Akhiezer, R. V. Polovin, A. G. Sitenko, and K. N. Stepanov, Kollektivnye Kolebaniya v Plazme (Collective Oscillations in Plasma), Atomizdat, (1964).

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