NONLINEAR THEORY OF THE INTERACTION BETWEEN A MONOENERGETIC BEAM AND A DENSE PLASMA

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The interaction between a monoenergetic electron beam and a dense collision plasma is investigated, using the method of partial numerical simulation. The parameters of the plasma-beam system are such that the collision rate in the plasma is greater than the hydrodynamic increment of a collisionless plasma, but smaller than the electron plasma frequency. It is shown that in the linear phase of the interaction there arise in the plasma unstable oscillations leading to a partial capture of the beam particles. The nonlinear stage of the interaction is characterized by the appearance in the initially monoenergetic beam of a thermal spread which impedes the development of an instability, with the result that the beam is able to pass freely through the plasma without any appreciable energy losses.

INTEREST to the study of the behavior of beams in a dense collision plasma has increased recently. This is connected with the problem of the passage of relativistic beams through a plasma and with attempts to use high-power beams for the production of a plasma with thermonuclear parameters[1-5].

In such systems a situation can, in particular, be realized when the rate of collisions of the plasma electrons with the ions, ν_{ei}, or with the neutral gas, ν_{en}, is much greater than the maximum increment of the beam instability in a collisionless plasma, $\gamma_0 = \omega_{pe} \times (n_1/n_0)^{1/3}$ [6], where $\omega_{pe} = (4\pi e^2 n_0/m)^{1/2}$ is the electron plasma frequency, n_0 is the plasma density, and n_1 is the beam density; $n_0 \gg n_1$.

For such parameters of the plasma-beam system, instability is possible only in the case when the thermal spread ΔV of the velocities in the beam is smaller than a certain value, namely;

$$\Delta V / V_0 < \gamma_0 / \omega_{pe}. \quad (1)$$

Here V_0 is the velocity of the beam.

It is not difficult to see that this condition is at the same time the condition of applicability of the hydrodynamic approximation:

$$\gamma_0 > k\Delta V,$$

where $k = \omega_{pe}/V_0$ is the wave number. Such a coincidence is not fortuitous, since in the case of hydrodynamic instability the vibrational energy

$$W = \frac{E^2}{8\pi}\omega\frac{\partial\varepsilon}{\partial\omega},$$

where ϵ is the permittivity of the medium, is negative[7], and the presence of dissipation in the system leads to the growth of the amplitude of the oscillations, in contrast to the case of kinetic instability, when the vibrational energy is positive and dissipation leads to the damping of the oscillations.

Let us consider an electron beam moving with velocity V_0 relative to a dense plasma with an electron collision rate ν. The dispersion relation for such a system will have the form

$$1 = \frac{\omega_{pe}^2}{\omega(\omega + i\nu)} + \frac{\omega_{be}^2}{(\omega - kV_0)^2}, \quad (2)$$

where $\omega_{be} = (4\pi e^2 n_1/m)^{1/2}$. In the case when

$$\gamma_0 < \nu < \omega_{pe}, \quad (3)$$

the oscillations with frequency $\omega \approx \omega_{pe} \approx kV_0$ have the maximum increment $\gamma = (\omega_{be}^2\omega_{pe}/2\nu)^{1/2}$. The ratio of the increment for a collisionless plasma to the increment for a collision plasma is then of the order of

$$\sqrt{\frac{2\nu}{\omega_{pe}}\left(\frac{n_1}{n_0}\right)^{-1/3}},$$

i.e., is a quantity which, according to (3), is greater than unity.

Thus, the linear phase of the interaction between an electron beam moving with velocity V_0 relative to a collision plasma and the plasma is characterized by the development of a beam instability. As the amplitude of the oscillations grows, the beam-plasma interaction process becomes substantially nonlinear, and cannot be described analytically. Therefore, to solve this problem we used the method of partial numerical simulation. This method combines the analytic description of the behavior of the particles of the plasma, considered as a continuous medium, since the plasma density is considerably higher than the beam density, $n_0 \gg n_1$, and it can be assumed that the motion of the plasma electrons in the field of the wave is linear in character, with the simulation of the electron beam by single particles. The beam particles are, at the moment $t = 0$, uniformly distributed over the length of the wave, and an equation of motion is written and the trajectory determined for each particle.

The method of partial numerical simulation has been used to investigate the interaction between a monoenergetic electron beam and a collisionless plasma[8]. It turned out in this case that the important events that occur in the course of this interaction are the capture of the beam particles by the wave field and the cessation of the growth of the wave amplitude when

$$\varphi \sim \frac{m}{e}(V_0 - V_{ph})^2 \approx \frac{mV_0^2}{e}\left(\frac{n_1}{n_0}\right)^{2/3},$$

where φ is the electric potential of the wave and V_{ph} is the phase velocity of the wave. Subsequently, the amplitude executes undamped oscillations about some mean value with a frequency equal to the oscillation frequency of a bunch of beam electrons in the potential well of the wave.

1. Let us consider the interaction of a one-dimensional monoenergetic electron beam with a collisionless plasma in the case when one wave is excited in the plasma. Such a situation is realized when the beam is modulated at the frequency ω_{pe}. The level of the fluctuation noise should in this case be much lower than the maximum value of the electric field of the excitable wave, i.e., the condition

$$\gamma^2 m V_0 / e\omega_{pe} > (4\pi n_0 T / N_D)^{1/2},$$

where N_D ($\sim 10^7 - 10^8$ for the usual plasma parameters) is the number of particles in the Debye sphere. Substituting the value of the increment into the above inequality, we obtain

$$\frac{V_0}{V_{Te}} > \frac{1}{N_D} \frac{n_0}{n_1} \frac{\omega_{pe}}{\nu}.$$

It is not difficult to see that the condition obtained for the excitation in the plasma of one wave is practically always fulfilled.

Let us derive the equations describing the interaction between a monoenergetic electron beam and a collision plasma in the case when one wave is excited in the plasma. Let us for this purpose use the Poisson equation

$$\operatorname{div} \mathbf{E} = -4\pi e (n_e + n_b), \tag{4}$$

where n_e is the perturbation of the plasma-electron density and n_b is the beam density, as well as the equation of motion and the continuity equation for the plasma electrons. The plasma ions can be considered here as being at rest. From the simultaneous solution of the equation of motion and the continuity equation for the plasma particles:

$$\frac{d\mathbf{V}_e}{dt} = -\frac{e}{m}\mathbf{E} - \nu \mathbf{V}_e, \quad \frac{\partial n_e}{\partial t} + \operatorname{div}(n_0 \mathbf{V}_e) = 0,$$

taking into account the condition $\omega_{pe} > \nu$, we obtain for the perturbation of the plasma density in the linear approximation the expression:

$$\frac{\partial^3 n_e}{\partial t^3} = -n_0 \frac{\partial}{\partial x}\left(\frac{e}{m}\frac{\partial E}{\partial t} - \nu \frac{e}{m} E\right) \tag{5}$$

Differentiating the Poisson equation (4) with respect to the time and substituting into it the relation (5), we obtain ($\gamma \lesssim \nu$)

$$\frac{\partial}{\partial x}\frac{\partial^3 E}{\partial t^3} = \frac{\partial}{\partial x}\omega_{pe}^2 \frac{\partial E}{\partial t} - \frac{\partial}{\partial x}\omega_{pe}^2 \nu E - 4\pi e \frac{\partial^3 n_b}{\partial t^3}. \tag{6}$$

Notice that when $\nu \gg \gamma$ the electric field of the excited wave can be determined by equating to zero the two last terms on the right-hand side of Eq. (6). The retention, however, of all the terms in Eq. (6) allows us to consider the more general case when $\nu \gtrsim \gamma$, and to compare the time dependences of the field of the excited wave for different values of the parameter ν/γ within a wide range. When the ratio ν/γ is reduced to unity, the form of the time dependence of the field of the excited wave resembles the result obtained in the paper[8] in which the interaction between a monoenergetic electron beam and a collisionless plasma is considered.

The electric field of the excited wave can be represented in the form[9]

$$E(x, t) = E(t) \exp(ik_0 x - i\omega_{pe} t + i\alpha(t)), \tag{7}$$

where $\alpha(t)$ is the phase of the oscillation: $E(t)$ and $\alpha(t)$ are slowly varying functions of the time (the characteristic variation time for $E(t)$ and $\alpha(t)$ is a quantity of the order of the inverse increment $t_H \sim 1/\omega_{pe}$). Substituting (7) into (6), we obtain

$$-\left[2\frac{\partial E}{\partial t} + 2iE\frac{\partial \alpha}{\partial t}\right]\exp[ik_0 x - i\omega_{pe} t + i\alpha(t)] \tag{8}$$
$$= -\nu E \exp[ik_0 x - i\omega_{pe} t + i\alpha(t)] - i\frac{4\pi e}{k\omega_{pe}^2}\frac{\partial^3 n_b}{\partial t^3}.$$

For the above-described formulation of the problem, the density of the beam particles can be written as follows:

$$n_b(\xi, t') = k\sum_{p=1}^{N} \delta(\xi - \xi_p(t')). \tag{9}$$

We have introduced here the new variables $\xi = k_0 x - \omega_{pe} t$ and $t' = t$; the summation is over all the particles of the beam, $\xi_p(t')$ is the trajectory of the motion in time of the particle with the index p. We shall henceforth drop the prime on the variable t', bearing in mind, however, that $\partial/\partial t = \partial/\partial t' - \omega_{pe}\partial/\partial\xi$.

Substituting (9) into Eq. (8), we obtain after separating the imaginary and real parts and averaging over the time ω_{pe}^{-1}:

$$2\frac{\partial E}{\partial t} = -\nu E + \frac{4\pi e\omega_{pe}}{2\pi}\sum_{p=1}^{N}\cos(\xi_p(t) + \alpha(t)), \tag{10}$$

$$2E\frac{\partial \alpha}{\partial t} = \frac{4\pi e\omega_{pe}}{2\pi}\sum_{p=1}^{N}\sin(\xi_p(t) + \alpha(t)). \tag{11}$$

The trajectories of the motion of the beam particles are determined by integrating the equations of motion

$$\frac{1}{k}\frac{d^2\xi_p}{dt^2} = -\frac{e}{m}E(t)\sin[\xi_p + \alpha(t)]; \quad \frac{d\xi_p}{dt} = kV_p. \tag{12}$$

The system of equations (10)—(12) completely describes the behavior in time of the oscillations under consideration. It is convenient, for the numerical solution of this system, to rewrite it in terms of the dimensionless variables:

$$E = \varepsilon E_{max}, \quad E_{max} = \gamma^2 m / ek, \quad \eta = (kV - \omega_{pe})/\gamma, \quad \tau = \gamma t;$$

$$\frac{d\varepsilon}{d\tau} = -\frac{\nu}{2\gamma}\left\{\varepsilon - \frac{1}{N}\sum_{p=1}^{N}\cos(\xi_p + \alpha)\right\}, \tag{13.1}$$

$$\frac{d\alpha}{d\tau} = \frac{\nu}{2\gamma\varepsilon N}\sum_{p=1}^{N}\sin(\xi_p + \alpha), \tag{13.2}$$

$$d\xi_p / d\tau = \eta_p, \quad d\eta_p / d\tau = -\varepsilon\cos(\xi_p + \alpha). \tag{13.3}$$

The presence in the first equation of the system (13) of the large parameter $\nu/2\gamma$ allows us to estimate the maximum energy of the wave. If we assume that the energy of the wave attains its maximum, $\partial\epsilon/\partial\tau = 0$, when all the beam particles agglomerate at the single point with $\sin(\xi_p + \alpha) = 1$, then it follows from (13.1) that $\epsilon_{max} = 1$.

Let us now show that the motion of the plasma electrons can indeed be considered in the linear approximation. The velocity of the plasma electrons in the field of the wave can be estimated as $V_e \sim eE_{max}/m\omega_{pe}$, where E_{max} is the maximum electric field of the wave

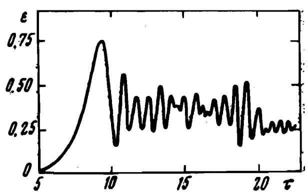

FIG. 1

at the moment of capture of the beam particles. The dominant nonlinear term in the equations describing the motion of the plasma electrons is the term $V_e \partial V_e/\partial x$. Comparing it with the least linear term, γV_e, we find that even at the maximum amplitude of the wave field $\gamma V_e/kV_e^2 \approx \omega_{pe}/\gamma \gg 1$, i.e., our approximation is valid.

The system of equations (13) was integrated on an electronic computer, using the Runge-Kutta method. The electron beam was simulated by single particles distributed in the interval $-\pi \le \xi \le \pi$. The number of particles was varied in the range $25 \le N \le 500$. The program used in the computations allowed us to carry out the integration with a variable spacing, achieved by prescribing the relative error at each step, and with a constant spacing, in which case a spacing was chosen which ensured, in accordance with the requirements of correctness of the problem, the stability of the system.

Figure 1 shows the dependence of the energy of the wave on time for 50 particles. The computations were conducted with a spacing $h = 10^{-2}$, the initial energy was assumed to be equal to 10^{-4}, and, $\nu/2\gamma = 30$. The result did not change when the number of particles was further increased and the integration spacing was decreased. The result also did not depend upon the magnitude of the initial energy. In accordance with the linear theory, $\epsilon \sim e^{\tau}$ when $\epsilon \ll 1$. As τ increases, the rate of growth of the energy decreases, and at $\tau \sim 10$ the energy attains its maximum value $\epsilon \approx 0.75$. This value is $2\gamma/\nu$ times smaller than the level attainable by the energy of the wave when the electron beam interacts with a collisionless plasma[8].

The variation of the energy with time (Fig. 1) in our case differs essentially from the case of the collisionless plasma, when the beam particles gather into one bunch before their capture by the wave and the energy of the wave oscillates about some mean value with a frequency equal to the frequency of oscillation of this bunch in the potential well of the wave. In our case the oscillations of the energy are random, the degree of randomness increasing as τ increases.

Figure 2 shows the dependence of the velocity η of the particles relative to the wave on the coordinate ξ at different moments of time τ. As follows from Fig. 2c, which corresponds to the maximum energy value, the beam particles do not have time to completely agglomerate before their capture and possess appreciable velocity, as well as coordinate straggling. It is natural, therefore, that the capture of a portion of the particles by the wave cannot lead to orderly oscillations of the energy. The capture does not also lead to damping of the oscillations, since the violation of the condition for the buildup of the oscillations with the maximum increment is, on account of the decrease of the velocity of the beam particles, compensated by the phase change α. It is as if the beam is "tuned up," and the energy, having fallen to a level considerably lower than in experiments with a collisionless plasma, begins to increase again. The beats observable in the graph of the function $\epsilon(\tau)$ are connected with the presence in the system of two characteristic times—the oscillation time of the particle bunch in the potential well of the wave and the variation time of the potential well itself.

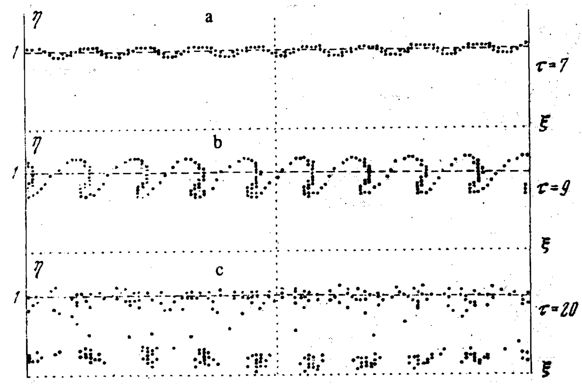

FIG. 2

It is clear from the foregoing that a numerical experiment on the excitation of a single vibrational mode cannot yield the correct picture of the instability under consideration. In fact, in the interaction between an electron beam and a plasma, the increment of the resulting instability is, depending on the detuning (we call the quantity $\overline{\omega} = kv_0 - \omega_{pe}$ detuning), not a δ-function, and there is a buildup in the plasma of not one wave, but a wave packet. This circumstance was taken into account in subsequent experiments.

2. The system of equations describing the behavior of the plasma-beam system when a wave packet is excited in the plasma is obtained by the method described in Sec. 1, with the only difference that the condition $\beta kV = \omega_{pe}$, where β is the number of the harmonic which builds up with the maximum increment, must now be fulfilled. In terms of the dimensionless variables

$$\tau = \gamma t, \quad E = \varepsilon E_{max}, \quad E_{max} = \frac{\gamma_m}{\beta e k}, \quad \eta = \frac{\beta k V - \omega_{pe}}{\gamma},$$

$$\xi = k_0 x - \frac{\omega_{pe}}{\beta} t$$

the system of equations have the following form:

$$\frac{d\varepsilon_j}{d\tau} = -\frac{\nu}{2\gamma}\left\{\varepsilon_j - \frac{1}{N}\frac{\zeta}{\beta^2}\sum_{p=1}^{N}\sin(\zeta\xi_p + \alpha_j)\right\}, \tag{14}$$

$$\frac{d\alpha_j}{d\tau} = \frac{\omega_{pe}}{2\gamma}\frac{[\zeta^2 - \beta^2]\zeta}{\beta^3} + \frac{\nu}{2\gamma}\frac{1}{N\varepsilon_j}\frac{\zeta^2}{\beta^2}\sum_{p=1}^{N}\cos(\zeta\xi_p + \alpha_j),$$

$$d\xi_p/d\tau = \eta_p, \quad \frac{d\eta_p}{d\tau} = -\frac{4}{\beta}\sum_j \varepsilon_j \sin[\zeta\xi_p + \alpha_j].$$

Here, $\xi = kV - \omega_{pe}t/\beta$, β is the number of the resonance mode, and $\zeta = \beta + j$, where $j = \pm 1, \pm 2, \pm 3 \ldots$ is the number of the nonresonance mode. When $\eta = \beta = 1$ the system (14) goes over into the system (13).

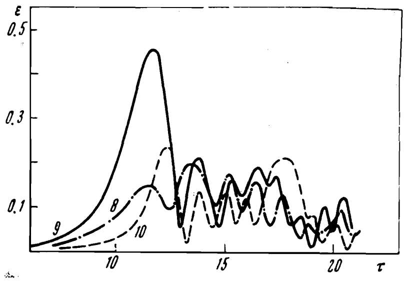

FIG. 3

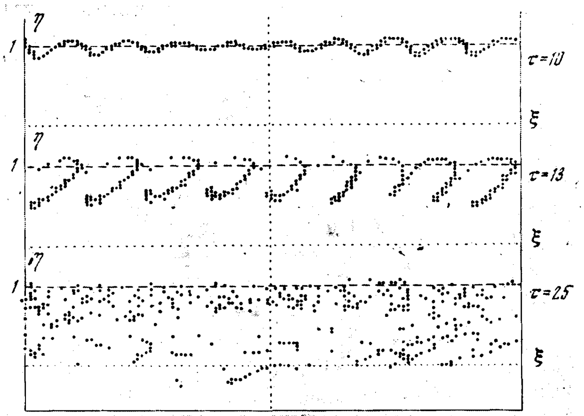

FIG. 4

The system (14) was integrated by the Runge-Kutta method with the following values of the parameters: N = 500, $\omega_{pe}/2\gamma = 50$, and $\nu/2\gamma = 10$. The velocity η of the beam relative to the wave was set equal to unity. The time variation of the amplitude and phase of the 8th, 9th, 10th, 11th, and 12th harmonics was investigated.

The plot of the function $\epsilon(\tau)$ for five harmonics is shown in Fig. 3. It can be seen that at the initial stage the waves behave in accordance with the linear theory. The 9th harmonic has the maximum increment, followed by the 10th and the 8th; there is no buildup of the 11th and 12th harmonics. After the first maximum the oscillations become randomized and attenuate. Figure 4 shows the dependence on the coordinate of the velocity of the beam particles relative to the wave at different moments of time. It is easy to see that there is, even at the moment of capture, a more considerable spread in the velocities and coordinates than in the case of one wave. A complete thermalization of the beam occurs at $\tau = 25$, and the oscillations attenuate.

It follows from the foregoing that for the indicated beam and plasma parameters, $\gamma_0 < \nu < \omega_{pe}$; in the case when a packet of oscillations is excited in the plasma, the beam becomes thermalized. This leads to the damping of the plasma oscillations, in accordance with the relation (1), and allows the beam to freely pass through the plasma without any appreciable change in its energy. The mean free path will not, in the present case, be related to the instability, but will be determined by the dissipation of the reverse current, and is in order of magnitude equal to $x_0 \approx v_0 n_0 / v n_1$[10].

The results of the present paper can also be applied to the relativistic beam, since the introduction of the relativistic factor will affect only the normalization of the systems (13) and (14).

It should be noted that the results obtained above in the case of a steady injection of a beam into a bounded plasma are valid so long as the plasma temperature and, consequently, the collision time do not, owing to the dissipation of the reverse current, increase so much so that the condition (3) is violated. In that case the slowing-down length will be determined by the quasi-linear theory[3-5] and may be substantially smaller than the value x_0 given above.

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