SOME PROPERTIES OF ELECTRONS IN GASEOUS HELIUM IN THE PRESENCE OF A STRONG MAGNETIC FIELD

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Possible localized free electron states in dense gaseous helium are studied in the self-consistent field approximation. It is shown that when the magnetic field is switched on, along with the usual ion states (small radius ions) encountered in a sufficiently dense gas with $n \gtrsim n_{cr} \sim 10^{21}\ \mathrm{cm^{-3}}$, new, weakly channeled states (large radius ions) arise in the system, their existence being possible at $n < n_{cr}$. The main parameters of large radius ions and their mobility along the magnetic field are estimated for gaseous helium concentrations only slightly smaller than the critical value and in magnetic fields $H < 10^6$ g. Under these conditions the ion mobility is 2 to 3 orders of magnitude lower than that of free electrons in a gas of the same density.

IT is well known that when electrons are introduced into dense helium (solid, liquid, or gas), complexes consisting of regions with deformed helium density and an electron localized inside this region turn out to be energywise favored. In the case of solid and liquid helium, the electron crowds out the helium atoms from the region of its localization almost completely. The resultant empty sphere of radius $R_0 \approx 20$ Å is called a "negative ion" or "anion." For gaseous helium, depending on the density, electron states with different degrees of localization are possible. The critical helium density that separates the regions of the existence of strongly and weakly localized states lies, according to the experimental data of Levin and Sanders[1], in the vicinity of $n_{cr} \sim 10^{21}\ \mathrm{cm^{-3}}$. In this region, which corresponds to only a threefold change in the gas density, the carrier mobility decreases by approximately five orders of magnitude, starting with values that agree well with calculations in accordance with the usual gas-kinetic theory for free electrons, to a value which is almost equal to the mobility of the anions in liquid helium. Levin and Sanders[1] attribute, in natural fashion, such an abrupt decrease of the mobility to the onset of anion formation in the dense helium gas. This qualitative explanation of the transition region is subject to no doubt, but it should be noted that there is still no sufficiently satisfactory quantitative description of the abrupt decrease, in spite of a number of attempts[2].

In the present paper we study the behavior of electrons situated in gaseous helium at gas densities immediately preceding the transition region. According to the cited experimental data[1] and the simple calculations presented below, localized states are energywise not favored in this concentration region without external fields, and are therefore not realized. The picture changes if the electron plus gas system is placed in a strong magnetic field. This makes it possible for localized states to be produced; the structure of these states is analogous to the structure of the so-called "magnetic condensons," the existence of which in homopolar semiconductors was predicted by one of the authors[3]. It was also noted that such states are possible in gaseous helium[4]. We present below a detailed description of the local electronic states brought about in gaseous helium by a strong magnetic field, and calculate their longitudinal mobility.

1. LOCALIZED STATES IN GASEOUS HELIUM

1. The equations that determine the self-consistent states of an electron in gaseous helium are obtained, as usual, by minimizing the free energy of the system. We assume that the de Broglie wavelength of the electron greatly exceeds the average distance between the helium atoms; the electron is located here in the average field produced by the helium atoms and determined by their concentration $n(\mathbf{r})$. The contribution of the interaction between the electron and the gas to the free-energy density takes, following[1], the form

$$\tilde{F}_{int} = \frac{2\pi\hbar^2 a_0}{m} n(\mathbf{r}) |\varphi(\mathbf{r})|^2, \tag{1}$$

where a_0 is the length of the scattering of the electron by the helium atom in the pseudopotential approximation ($a_0 = 0.62$A); $\varphi(\mathbf{r})$ is the wave function of the electron and m is its mass. Thus, if we neglect the interaction between the helium atoms and regard the gas as classical, then the free-energy density of the system in an external field with vector potential $\mathbf{A}(\mathbf{r})$ is

$$\tilde{F} = \frac{1}{2m} \left| \left(\hat{\mathbf{p}} + \frac{e}{c}\mathbf{A} \right) \varphi \right|^2 + \tilde{F}_{int} + nT\ln(nB), \tag{2}$$

where B(T) is a known function of the temperature.

By varying the free energy $F = \int_V \tilde{F} d\mathbf{r}$ with respect to $n(\mathbf{r})$ and $\varphi(\mathbf{r})$ at a constant volume V of the system, under the condition

$$\int_V |\varphi(\mathbf{r})|^2 d\mathbf{r} = 1,$$

we obtain as $V \to \infty$ ($N/V = n_0$)

$$n(\mathbf{r}) = n_0 e^{-\psi/T}, \tag{3}$$

$$\psi(\mathbf{r}) = 2\pi\hbar^2 a_0 |\varphi(\mathbf{r})|^2 / m, \tag{4}$$

$$F = \int \left[\frac{1}{2m} \left| \left(\hat{\mathbf{p}} + \frac{e}{c}\mathbf{A} \right) \varphi \right|^2 + n_0 T (1 - e^{-\psi/T}) \right] d\mathbf{r} + n_0 TV \ln(n_0 B). \tag{5}$$

The second term in (5) is the free energy of an ideal gas.

The normalized extremal φ of the functional (5) satisfies the equation

$$\frac{1}{2m}\left(\hat{\mathbf{p}}+\frac{e}{c}\mathbf{A}\right)^2\varphi+\frac{2\pi\hbar^2 a_0}{m}n(\mathbf{r})\varphi=E\varphi. \qquad (6)$$

2. Let us investigate the possibility of electron localization in the absence of a magnetic field. This problem was considered earlier by two methods[1,2]. We write down the dependence of the free energy (5) on the reciprocal radius of the local state k; for simplicity, we choose in this state an electron wave function in the form $\varphi(\mathbf{r}) = \pi^{-1/2}k^{3/2}e^{-kr}$. The change of the free energy $\delta F_0(k)$, due to the electron localization

$$\delta F_0(k)=F(k)-n_0TV\ln(n_0B)-2\pi\hbar^2a_0n_0/m, \qquad (7)$$

and expressed in the dimensionless variables

$$\varkappa=\frac{k}{k_0},\quad \tilde{n}=\frac{n}{n^*},\quad \delta F=\frac{\delta F}{\varepsilon_0};\quad k_0^3=\frac{mT}{2\hbar^2a_0}, \qquad (8)$$
$$n^*=\frac{k_0^2}{2\pi a_0},\quad \varepsilon_0=\frac{\hbar^2k_0^2}{2m},$$

turns out to be

$$\delta F_0(\varkappa)=\varkappa^2+\tilde{n}\left[\frac{1}{3}\int_0^\infty s^3e^{-s}\exp(-\varkappa^3e^{-s})\,ds-2\right].$$

For δF_0 there are the asymptotic formulas

$$\delta F_0(\varkappa)=\begin{cases}\varkappa^2-{}^1/_6\tilde{n}\varkappa^3, & \varkappa\ll 1\\ \varkappa^2+\tilde{n}\{{}^1/_3\varkappa^{-3}[\ln^3(\varkappa^3\gamma_0)+\psi'({}^1/_2)\ln(\varkappa^3\gamma_0)-\psi''(1)]-2\}, & \varkappa\gg 1\end{cases} \qquad (9)$$

where $\psi(x)$ is the Euler function and $\gamma_0 = 1.78$ is the Euler constant. A satisfactory joining of the asymptotic forms (9) occurs at $\kappa \approx 1.8$.

Plots of $\delta F_0(\kappa)$ obtained with the aid of these asymptotic formulas for different n are shown in the figure (curves I–V). The bound electronic states corresponding to the negative minimum of $\delta F_0(\kappa)$ appear here starting with $\tilde{n} \geq \tilde{n}_{cr} \approx 7$. In dimensional units, at $T \approx 4°$K, we have $n_{cr} \approx 2\times 10^{21}$ cm^{-3} and $k_{cr}^{-1} \approx 1.6\times 10^{-7}$ cm. These critical parameters satisfy the inequality

$$\gamma\equiv\psi(0)/T\gg 1, \qquad (10)$$

i.e., according to (3), $n(\mathbf{r}) \ll n_0$ in the region of localization of the electron; this can serve as the justification of the empty-sphere model.

3. In the presence of a strong magnetic field directed along the z axis, the infinite motion of the electron is preserved only along z. But in one-dimensional problems, any interaction that plays the role of a potential well for the electron should lead to the onset of bound electronic states (in three-dimensional problems, this well should have a finite depth). It is precisely for this reason that the interaction (1), which stimulates the deformation of the gas density in the vicinity of the electron (the deformed gas density plays the role of an effective potential well for the electron), turns out to be sufficient to localize the electron motion also along the z axis when the magnetic field is turned on. The mechanism of this localization, unlike that considered above, has no threshold with respect to the gas density, so that ions of magnetic origin exist also in the region $n < n_{cr}$.

To verify this, it is convenient to obtain first, as above, simple variational estimates. Choosing the trial function in the form

$$\varphi(\mathbf{r})=\frac{k^{1/2}}{(2\pi)^{1/2}\rho_0}\exp\left(-\frac{\rho^2}{4\rho_0^2}\right)e^{-k|z|},\qquad \rho_0=\left(\frac{c\hbar}{eH}\right)^{1/2}$$

(ρ is the polar coordinate; ρ_0 is the characteristic magnetic length), determining the change of the free energy $\delta F_H(k)$ connected with the localization of the electron in the magnetic field, in analogy with (7), and calculating the asymptotic form of $\delta F_H(k)$ in the limit of small k, we get in the dimensionless units (8)

$$\delta F_H(\varkappa)\approx\varkappa^2-\tilde{n}\varkappa/8(k_0\rho_0)^2,\ \varkappa<1. \qquad (11)$$

Expression (11) for arbitrary $\tilde{n} < \tilde{n}_{cr}$ has a negative minimum, which was to be proved. Plots of (11) for n = 7 and 8 and $H = 5\times 10^5$ G are shown in the figure (curves III″ and IV″). Using (11) and (8), we find that in this case $\gamma \ll 1$ for $n < n_{cr}$ and $H < 10^6$ G. Thus, energywise favored self-consistent states generated by a strong magnetic field actually exist. Subsequently, such states will be called large-radius ions, since their characteristic dimensions can greatly exceed the radius of the ion in the absence of a field.

Owing to the inequality $\gamma \ll 1$, it becomes possible to obtain the $\varphi(\mathbf{r})$ and $n(\mathbf{r})$ corresponding to large-radius ions. To this end, we expand the exponential in (5) up to terms of order γ^2 inclusive, and obtain

$$\delta F_H=\int\left[\frac{1}{2m}\left|\left(\hat{\mathbf{p}}+\frac{e}{c}\mathbf{A}\right)\varphi\right|^2-\left(\frac{2\pi\hbar^2a_0}{m}\right)^2\frac{n_0}{2T}\varphi^4\right]d\mathbf{r}. \qquad (12)$$

Expression (12) differs from the corresponding expression obtained by one of the authors[3] only in the meaning of the constants that enter in it, and we therefore use the results obtained there. In the approximation of the lowest Landau band, which is valid if the magnetic field H is strong enough, the wave function of the ground state is

$$\varphi(\mathbf{r})=\frac{1}{(2\pi)^{1/2}\rho_0}\exp\left(-\frac{1}{4}\bar{\rho}^2\right)\chi(z),\qquad \bar{\rho}=\frac{\rho}{\rho_0}, \qquad (13)$$

and the free energy is given by

$$\delta F_H=\int_{-\infty}^{+\infty}\left[\frac{\hbar^2}{2m}\left(\frac{d\chi}{dz}\right)^2-\left(\frac{2\pi\hbar^2a_0}{m\rho_0}\right)^2\frac{n_0}{8\pi T}\chi^4\right]dz.$$

We write the Euler-Lagrange equation for $\chi(z)$:

$$-\frac{\hbar^2}{2m}\frac{d^2\chi}{dz^2}-\left(\frac{2\pi\hbar^2a_0}{m\rho_0}\right)^2\frac{n_0}{4\pi T}\chi^3=E_0^*\chi, \qquad (14)$$

where $E_0^* = E_0 - \mu H$. The solutions of (14) that decrease at infinity are given by

$$\chi(z)=\pm(2r_z)^{-1/2}\,\mathrm{ch}^{-1}\frac{z-z_0}{r_z},$$

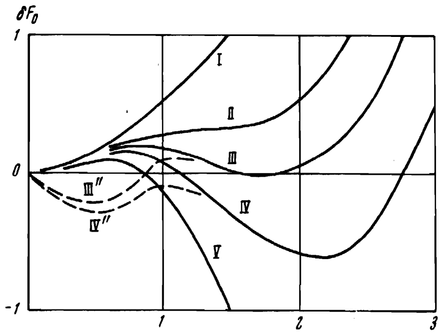
where z_0 is an arbitrary constant and r_z and the energy E_0^* of the local state are equal to

$$r_z=\frac{1}{\pi}\frac{2mT}{\hbar^2n_0}\left(\frac{\rho_0}{a_0}\right)^2,\qquad E_0^*=-\frac{\hbar^2}{2mr_z^2}.$$

The self-energy of the large-radius ion, which constitutes the pure gain in the system free energy due to the localization, is equal to $(1/3)|E_0^*|$. We present the numerical values of the large-radius ion characteristics for $n_0 = 2\times 10^{21}$ cm^{-3}, T = 4°K, and $H = 5\times 10^5$ G:

$$r_z\cong 6\cdot 10^{-7}\ \text{cm},\ \rho_0\cong 3\cdot 10^{-7}\text{cm},\ |E_0^*|\cong 20°,\ \gamma\cong 0{,}1 \qquad (15)$$

4. In concluding this section, we make a few remarks. In the construction of the localized phase it was assumed that the potential field produced for the electron by the helium atoms can be described with the aid of the averaged gas density $n(\mathbf{r})$. In the absence of a



Plot of δF_0 against κ for different n. Curves I–V correspond, in increasing order, to n = 4, 6, 7, 8, and 10. The dashed curves are plots of δF_H against κ at small κ. Curves III'' and IV'' pertain to n = 7 and 8, respectively.

magnetic field, at densities close to n_{cr}, the radius of the local states turned out to be of the order of the average distance between the helium ions, so that the use of n(**r**) for the description of the electron-gas interaction is not sufficiently well justified. The characteristic dimensions of large-radius ions at $n \lesssim n_{cr}$ and $H < 10^6$ G noticeably exceed $n_0^{-1/3}$, and this justifies the average-density approximation and ensured stability of these states with respect to density fluctuations.

It should also be noted that the choice of $\varphi(\mathbf{r})$ in the form (13) is a good approximation if the following inequality holds:

$$\alpha^2 \equiv (r_z / \rho_0)^2 \gg 1. \tag{16}$$

Allowance for the next Landau bands increases the self-energy of the ion[5].

Some additional comments are also in order for the initial expressions (2) and (5) for the free energy of the system. We have left out from these expressions the terms connected with the electron entropy S_{el} (per electron). To obtain an expression for the electron entropy in the general case it is necessary to know the electron spectrum for an arbitrary density distribution n(**r**). Such a problem can hardly be solved. However, if the localized states of the electron are separated from the continuous spectrum by a sufficiently large gap, (in comparison with the temperature), then the estimate of the contribution of the electron entropy to the total free energy of the system becomes simpler. Such a situation has already arisen in its time in a discussion of different localized states of electrons interacting with a medium (see, for example,[1,6]). It turned out then that the electron entropy does not influence the structure of the localized state, i.e., does not change the form of (6), if the depth $|E_0^*|$ of the local level greatly exceeds T. In our case, according to (15), $|E_0^*| \approx 20°$K and $T \approx 4°$K, i.e., the inequality $|E_0^*| \gg T$ is satisfied, albeit not very well. This inequality can be improved either by increasing the magnetic field, which is experimentally very difficult, or by lowering the temperature, which is possible by using, for example, He^3 instead of He^4. As to the curve shown in the figure, allowance for S_{el} in the spirit of[1,6], leads to an additional deepening of the minimum by an amount on the order of T.

2. LONGITUDINAL CONDUCTIVITY OF LARGE-RADIUS IONS

1. Since the mean free path of the helium atoms with respect to the interatomic collisions is of the order of 10 Å at $n_0 \approx 10^{21}$ cm^{-3}[1], which is much less than the characteristic dimensions of the large-radius ions, we can use for their description the hydrodynamic approximation. To calculate the mobility we find first the velocity field for uniform motion of the ion. The external electron field, and consequently the ion velocity v_0, will be assumed to be small enough to ensure stability of the ion and an ohmic character of the current-voltage characteristic; the last two apparently are violated even in electric fields that are not too strong.

We consider the system of equations

$$\frac{\partial n}{\partial t} + \operatorname{div} n\mathbf{v} = 0,$$

$$Mn\left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v}\right] = -T\nabla n - n\nabla\psi + \eta\left({}^4/_3\nabla \operatorname{div}\mathbf{v} - \operatorname{rot}\operatorname{rot}\mathbf{v}\right), \tag{17}$$

where M is the mass of the helium atom, n and ψ are the concentration of the helium atoms and the wave function of the electron in the moving ion (see (4)). We seek a solution of (17) corresponding to steady-state motion of the ion with velocity v_0 along the magnetic field; then **v**, n, and ψ are functions of $(x, y, z - v_0 t)$, and their expansions in powers of v_0 are given by

$$n = n^{(0)} + v_0 n^{(1)} + \ldots, \quad \mathbf{v} = v_0 \mathbf{v}^{(0)} + \ldots,$$
$$\psi = \psi^{(0)} + v_0 \psi^{(1)} + \ldots$$

In the zeroth order in v_0, as expected, we obtain from (17)

$$n^{(0)}(\mathbf{r}) = n_0 \exp(-\psi^{(0)}(\mathbf{r}) / T),$$

which agrees with (3). In first order, which we shall need for the subsequent calculations,

$$\operatorname{div}\mathbf{v}^{(0)} = -\frac{1}{T}\frac{\partial\psi^{(0)}}{\partial z} + \mathbf{v}^{(0)}\frac{\nabla\psi^{(0)}}{T}, \tag{18}$$
$$\nabla n^{(1)} + n^{(1)}\nabla\frac{\psi^{(0)}}{T} + n^{(0)}\nabla\frac{\psi^{(1)}}{T} = \frac{\eta}{T}\left(\frac{4}{3}\nabla\operatorname{div}\mathbf{v}^{(0)} - \operatorname{rot}\operatorname{rot}\mathbf{v}^{(0)}\right).$$

We use furthermore the fact that $\gamma \ll 1$. Since it follows from (18) that $\mathbf{v}^{(0)}$, $n^{(1)}$, and $\psi^{(1)}/T$ are of first order in γ, then we can rewrite (18) in first order in γ as follows:

$$\operatorname{div}\mathbf{v}^{(0)} = -\frac{1}{T}\frac{\partial\psi^{(0)}}{\partial z}, \tag{19}$$
$$\frac{\eta}{T}\operatorname{rot}\operatorname{rot}\mathbf{v}^{(0)} = -\nabla\left(\frac{4}{3}\frac{\eta}{T^2}\frac{\partial\psi^{(0)}}{\partial z} + n^{(1)} + n_0\frac{\psi^{(1)}}{T}\right).$$

It is seen from (19) that in this approximation there exists vortex-free solutions of the problem, which are described by the system (the superscript (0) of $\mathbf{v}^{(0)}$ and $\psi^{(0)}$ will henceforth be omitted)

$$\operatorname{div}\mathbf{v} = -\frac{1}{T}\frac{\partial\psi}{\partial z}, \qquad \operatorname{rot}\mathbf{v} = 0 \tag{20}$$

with the condition that **v** decrease at infinity. The solution (20) is usually expressed with the aid of potentials, but their second derivatives, which enter into the expression for the mobility, are quite cumbersome and difficult to investigate. We therefore use another representation for **v**, which takes into account explicitly the actual symmetry of the problem. We seek **v** in the form ($J_0(x)$ and $J_1(x)$ are Bessel functions)

$$v_\rho = \int_0^\infty c_\rho(u, z) J_1(\rho u)\, du, \qquad v_z = \int_0^\infty c_z(u, z) J_0(\rho u)\, du. \tag{21}$$

Substitution of (21) in (20) leads to differential equations for c_ρ and c_z:

$$\frac{\partial^2 c_\rho}{\partial z^2} - u^2 c_\rho = -\frac{2\gamma\rho_0{}^2u^2}{r_z}\frac{\operatorname{th}\xi}{\operatorname{ch}^2\xi}\exp\left(-\frac{\rho_0{}^2u^2}{2}\right) \equiv g_\rho(u,z),$$
$$\frac{\partial^2 c_z}{\partial z^2} - u^2 c_z = -\frac{2\gamma}{\alpha^2}u\frac{2\operatorname{sh}^2\xi - 1}{\operatorname{ch}^4\xi}\exp\left(-\frac{\rho_0{}^2u^2}{2}\right) \equiv g_z(u,z), \tag{22}$$

$\xi \equiv z/r_z$. Solutions of (22), which ensure a decrease of v at infinity, take the form ($i = \rho, z$)

$$c_i(u,z) = -\frac{1}{2u}\left[e^{uz}\int_z^\infty e^{-us}g_i(u,s)\,ds + e^{-uz}\int_{-\infty}^z e^{us}g_i(u,s)\,ds\right],$$

i.e., finally we have ($\tilde{\rho} = \rho/\rho_0$)

$$v_\rho(\tilde{\rho},\xi) = \frac{\alpha\gamma}{2}\int_0^\infty J_1(\tilde{\rho}\sigma)\sigma^2\exp\left(-\frac{\sigma^2}{2}\right)d\sigma, \tag{23}$$

$$v_\rho(\tilde{\rho},\xi) = \frac{\alpha\gamma}{2}\int_0^\infty J_1(\tilde{\rho}\sigma)\sigma^2\exp\left(-\frac{\sigma^2}{2}\right)d\sigma\int_0^\infty e^{-\alpha\sigma s}[\operatorname{ch}^{-2}(\xi-s) - \operatorname{ch}^{-2}(\xi+s)]\,ds,$$

$$v_z(\tilde{\rho},\xi) = \frac{\alpha\gamma}{2}\int_0^\infty J_0(\tilde{\rho}\sigma)\sigma^2\exp\left(-\frac{\sigma^2}{2}\right)d\sigma$$
$$\times\int_0^\infty e^{-\alpha\sigma s}[\operatorname{ch}^{-2}(\xi-s) + \operatorname{ch}^{-2}(\xi+s) - 2\operatorname{ch}^{-2}\xi]\,ds.$$

A qualitative investigation of (23), a determination of their asymptotic forms, and numerical calculations of **v** are simple tasks in view of the good convergence of the integrals in (23).

2. We proceed to calculate the ion mobility. To this end, we write down the rate of change of the kinetic energy of an ion moving in a compressible viscous liquid with a viscosity coefficient η (the coefficient of the second viscosity can be assumed equal to zero)

$$-\frac{\partial E_{\text{kin}}}{dt} = 2\pi\eta v_0{}^2\int_0^\infty \rho\,d\rho\int_{-\infty}^{+\infty}\left[-\frac{1}{3T}\left(\frac{\partial\psi}{\partial z}\right)^2 + \left(\frac{\partial v_z}{\partial z}\right)^2\right.$$
$$\left. + \left(\frac{\partial v_\rho}{\partial\rho}\right)^2 + 2\left(\frac{\partial v_z}{\partial\rho}\right)^2 + \left(\frac{v_\rho}{\rho}\right)^2\right]dz.$$

Equating this quantity to the work eEv_0 performed on the ion by the electric field per unit time, we obtain for the ion mobility the expression

$$\mu_i = \frac{v_0}{E} = -ev_0{}^2\Big/\frac{\partial E_{\text{kin}}}{\partial t}. \tag{24}$$

We now recognize that $\alpha \gg 1$ according to (16); at large α we have from (23)

$$v_\rho = 2\frac{\gamma}{\alpha\tilde{\rho}}\frac{\operatorname{th}\xi}{\operatorname{ch}^2\xi}\left(1 - \exp\left(-\frac{1}{2}\tilde{\rho}^2\right)\right),$$
$$\frac{\partial v_\rho}{\partial\rho} = 2\frac{\gamma}{\alpha\rho_0}\frac{\operatorname{th}\xi}{\operatorname{ch}^2\xi}\left[\exp\left(-\frac{1}{2}\tilde{\rho}^2\right) - \tilde{\rho}^{-2}\left(1 - \exp\left(-\frac{1}{2}\tilde{\rho}^2\right)\right)\right].$$

The expressions $\partial v_\rho/\partial z$ and $\partial v_z/\partial z$ are of higher order in α^{-1}, so that accurate to α^{-2} the mobility can be expressed in the form of the Stokes formula

$$\mu_i = e/6\pi\eta R_0, \tag{25}$$

where the effective radius of the sphere is

$$R_0 \approx 0.12(\gamma/\alpha)^2 r_z.$$

For the values of γ, α, and r_z given in (15) and (16) we have $R_0 = 1.8\times10^{-10}$ cm, and the mobility of the large-radius ions under conditions close to (15) turns out to be smaller by two or three orders of magnitude than the free-electron mobility μ_e calculated with the aid of the gas-kinetic theory, and larger by two or three orders of magnitude than the mobility of a vacuum sphere of radius 15 Å. Thus, in a strong magnetic field, the conditions for the observation of large-radius ions at densities immediately preceding the critical value may turn out to be favorable, all the more since inclusion of the terms $(\partial v_\rho/\partial z)^2$ and $(\partial v_z/\partial z)^2$ in expression (24) and of the next Landau bands in the expansion of $\varphi(\mathbf{r})$, which is necessary for $\alpha \approx 0$ (1), leads to an additional decrease of the mobility.

Concluding the discussion of the mobility of the large-radius ions, we must present at least a qualitative estimate of the influence exerted on the effective mobility μ_i^* of the finite lifetime of the localized electronic states. In order of magnitude we have for μ_i^*:

$$\mu_i^* \approx \left(1 - \frac{N_e}{N_i}\right)\mu_i + \frac{N_e}{N_i}\mu_e. \tag{26}$$

where $\mu_i/\mu_e \sim 10^{-2}$–10^{-3}, and the ratio N_i/N_e, which determines the relative population of the localized states, is obtained from the approximate equality (see[6]):

$$\frac{N_i}{N_e} \approx \left(\frac{M}{m}\right)^{1/2}\exp\left(-\frac{\delta F_H}{T}\right),$$

where δF_H is the total gain in the energy upon localization of the electron, $\delta F_H = -(1/3)|E_0^*|$, with $|E_0^*|$ taken from (15); M and m are the effective masses of the ion and of the electron. By way of estimate, we use for M the quantity M_0, which plays the role of the attached hydrodynamic mass of the large-radius ion:

$$\frac{M_0}{2} = \int\rho\frac{v^2(\mathbf{r})}{2}\,d\mathbf{r}, \quad M \gg M_0, \tag{27}$$

ρ is the density of the helium and $v(\mathbf{r})$ is the velocity field (23) induced by the ion motion. The value of M_0 that follows from (27) under the conditions (15) turns out to be of the order of $\sim m_{He^4}$. As a result we have $N_i/N_e \sim 10^2$ and consequently both terms in (26) are of the same order of magnitude. In other words, the effective mobility μ_i^* coincides in order of magnitude with μ_i from (25).

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