CYCLOTRON RESONANCE IN SPECULAR REFLECTION OF ELECTRONS FROM THE SURFACE OF A METAL

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A theory of cyclotron resonance in metals in the specular reflection of electrons from a surface is developed. An asymptotically exact solution of the Maxwell equations is derived, and it is shown that, under certain conditions, the resonance contribution to the impedance can be small in comparison with the major nonresonance term. In this way, a microscopic and quantitative explanation is obtained of the phenomenological resonance line shape proposed by Chambers.[4] Inversion of the resonance curve in comparison with the case of diffuse scattering occurs during specular reflection. The dependence of the resonance amplitude, width, and shape, on the magnetic field strength, the anisotropy of the dispersion law, and the external wave polarization is analyzed.

1. INTRODUCTION

AFTER the prediction in 1956 of cyclotron resonance in metals,[1] this phenomenon was discovered experimentally in many metals and became one of the most effective methods for the study of their electron properties. At the time, when the theory of cyclotron resonance was developed,[2] it seemed indubitable that the scattering of electrons from the surface of a metal was diffuse, or close to diffuse. This point of view obtained wide acceptance and most investigators assumed that analysis of the case of specular reflection of electrons from the surface was only of academic interest. The effect of the character of the reflection of the electrons on cyclotron resonance was considered by one of us in 1957,[3] and it was pointed out that for nonspecular scattering the surface impedance of the metal in the neighborhood of resonances was practically independent of the character of reflection of the electrons. It was then pointed out that "the case of specular reflection is a special one, since the principal contribution to the current density in this case is made by electrons that undergo multiple collisions with the surface of the metal." In this special case, the principal term of the asymptotic expression for the current density does not have resonance singularities and depends smoothly on the magnetic field, while the cyclotron resonance arises only in the next terms of the asymptotic expansion. For specular reflection, the resonance in the surface impedance of the metal was not studied in[2], although the asymptotic current density corresponding to this limiting case was obtained.

Later on, an experimental investigation of the line shape of cyclotron resonance showed that significant deviations from the theory are observed.[2] The phenomenological calculations of Chambers,[4] based on the assumption that the resonance contribution to the current density is small in comparison with the nonresonant part, was in significantly better correspondence with the results of many experiments. The Chambers hypothesis finds a natural microscopic explanation if one assumes the scattering of the electrons to be close to specular. In this case, the screening current and the skin layer are created by electrons that are accelerated along the surface of the metal and do not take part in the resonance. The latter is due to the so-called "volume" electrons, which do not collide with the surface of the metal.

The oscillations of the surface impedance of many metals, discovered by Khaikin,[5] also favor specular reflection. These occur in weak magnetic fields ($H \sim 1-10$ Oe) and represent an effect similar to cyclotron resonance; they are due to transitions of the electrons between magnetic surface levels.[6] The very existence of such levels and the observation of oscillations in weak fields are connected with the specular reflection of the electrons from the surface. The physical reason for the specular reflection is the weak diffraction of the electrons by the random inhomogeneities of the surface and the sample under conditions when the height of the roughnesses is less than the reciprocal of the normal component of the wave vector of the electron.[7] The electrical conductivity of thin monocrystalline filaments (whiskers) is well explained by specular reflection of the electrons. At any rate it is impossible at the present time to state that the scattering of the electrons by the boundaries of a metal is diffuse. At the present time it is evidently possible to prepare samples of metals with such smooth and uniform surfaces that the reflection of the electrons will be sufficiently close to specular. Therefore, the theoretical study of cyclotron resonance for specular reflection is of definite interest.

In addition to the already mentioned work of one of the authors,[3] the effect of the character of the reflection on cyclotron resonance in metals was studied by Meĭerovich,[8] who considered another limiting case, and did not analyze the situation in which the reflection of the electrons is sufficiently close to specular.

In the present work, an asymptotically exact solution of the problem of cyclotron resonance for specular reflection has been obtained on the basis of further development of the method proposed by Hartmann and Luttinger. It is shown that the resonance component of the surface impedance constitutes a small increment to its nonresonant part. The dependence of the line shape and of the amplitude of the resonances on the magnetic field, the dispersion law of the electrons, the frequency, and other parameters are analyzed.

2. STATEMENT OF THE PROBLEM

We consider first an alkali metal with a spherical Fermi surface. A constant and uniform magnetic field **H** is parallel to the metal-vacuum interface and is directed along the z axis. The x axis coincides with the direction of the inward normal to the boundary. The electric vector **E** of the external electromagnetic wave is parallel to the z axis (the ordinary wave). In order to find the high-frequency field in the metal, it is necessary to solve the Maxwell equations. We write them down for the spatial Fourier component:

$$\mathscr{E}(k)=2\int_0^\infty dx\,E(x)\cos kx, \tag{2.1}$$

($E(x)$ is continued in even fashion to the region $x<0$ outside the metal). Neglecting the displacement current, we have

$$k^2\mathscr{E}(k)+2E'(0)=4\pi i\omega c^{-2}j(k), \tag{2.2}$$

where $j(k)$ is the Fourier component of the z component of the current density, ω is the frequency of the wave, and c is the velocity of light; the prime denotes the derivative with respect to x.

To find the connection between $j(k)$ and $\mathscr{E}(k)$, we must solve the kinetic equation for the electron distribution function with account of the reflection of the electrons from the interface. Such calculations for an arbitrary reflection coefficient ρ have been carried out previously.[3] Using the results of these calculations, we write down the final formula for the current:

$$j(k)=K(k)\mathscr{E}(k)-\int_0^\infty dk'\,Q(k,k')\mathscr{E}(k'). \tag{2.3}$$

Here $K(k)$ is the Fourier component of the conductivity of the unbounded metal and does not depend on the conditions of reflection of the electrons from the boundary. According to[2]

$$K(k)=\frac{3\sigma\gamma}{\pi}\int_0^{\pi/2}d\theta\sin\theta\cos^2\theta\int_0^\infty dx\,e^{-\gamma x}\int_0^\pi d\tau\cos[kR_\perp(\cos\tau-\cos(\tau-x))]$$

$$=3\sigma\gamma\int_0^{\pi/2}d\theta\sin\theta\cos^2\theta\sum_{n=-\infty}^{\infty}\frac{J_n^2(kR_\perp)}{\gamma+in}; \tag{2.4}$$

$$\sigma=\frac{Ne^2}{m(\nu-i\omega)},\qquad \gamma=\frac{\nu-i\omega}{\Omega},\qquad R_\perp=\frac{v}{\Omega}\sin\theta, \tag{2.5}$$

where N is the concentration, m the effective mass, v the Fermi velocity, e the absolute value of the charge on the electron, $\Omega=eH/mc$ the cyclotron frequency, $R_\perp$ the radius of the orbit of the electron in the magnetic field, θ the polar angle with the polar axis z, τ the azimuthal angle ($v_x=v_\perp\sin\tau$, $v_y=v_\perp\cos\tau$), ν the electron-scatterer collision frequency, $R=v/\Omega$ the maximal radius of the orbit, and $J_n(x)$ a Bessel function.

The nonlocal kernel of the conductivity operator $Q(k,k')$ takes into account the presence of the interface and depends on the character of the reflection of the electrons from the surface. Makarov and one of the authors[10] have shown that $Q(k,k')$ represents the sum of two components:

$$Q(k,k')=Q_{\mathrm{vol}}(k,k')+Q_{\mathrm{sur}}(k,k'). \tag{2.6}$$

The first term is the contribution to the conductivity from the so-called "volume" electrons. Their centers of orbit are found in the interior of the metal at a distance greater than the radius of revolution $R_\perp$. They are not scattered by the surface of the metal and therefore $Q_{\mathrm{vol}}(k,k')$ does not depend on the reflection coefficient. According to[10]

$$Q_{\mathrm{vol}}(k,k')=\frac{3\sigma\gamma}{\pi}\int_0^{\pi/2}d\theta\sin\theta\cos^2\theta \tag{2.7}$$

$$\times\sum_{n=-\infty}^{\infty}\frac{J_n(kR_\perp)J_n(k'R_\perp)}{\gamma+in}\left\{\frac{\sin[(k-k')R_\perp]}{k-k'}+(-1)^n\frac{\sin[(k+k')R_\perp]}{k+k'}\right\}.$$

The "surface" electrons, which collide with the interface in each turn, determine the second component in (2.6). The centers of the orbits of these electrons are outside the metal, or inside it at a depth less than $R_\perp$. For specular reflection, the Q_{sur} have the following form:

$$Q_{\mathrm{sur}}(k,k')=-\frac{6\sigma\gamma R}{\pi^2}\int_0^{\pi/2}d\theta\sin^2\theta\cos^2\theta\sum_{n=-\infty}^{\infty}\int_0^\pi\frac{d\varphi\sin\varphi}{\gamma\varphi+in\pi} \tag{2.8}$$

$$\times\int_0^\varphi d\lambda\cos\frac{\pi n\lambda}{\varphi}\cos[kR_\perp(\cos\lambda-\cos\varphi)]\int_0^\varphi d\mu\cos\frac{\pi n\mu}{\varphi}\cos[k'R_\perp(\cos\mu-\cos\varphi)].$$

In the general case, it is not possible to solve Eq. (2.2). We shall therefore find its asymptotic solution.

3. ASYMPTOTIC VALUE OF THE CURRENT

In the region of the anomalous skin effect, the formulas for $K(k)$ and $Q(k,k')$ can be simplified by substituting for them the asymptotic expressions for large values of kR and $k'R$. The asymptotic forms of $K(k)$ and $Q_{\mathrm{vol}}(k,k')$ are simply obtained by replacing the Bessel functions by the approximate expressions for large arguments. As a result of simple calculations, we obtain

$$K(k)=\frac{3\pi}{4}\frac{\sigma\gamma}{kR}\operatorname{cth}\pi\gamma, \tag{3.1}$$

$$Q_{\mathrm{vol}}(k,k')=\frac{3}{8}\frac{\sigma\gamma}{(kk')^{1/2}R}\operatorname{cth}\pi\gamma\left[\pi\delta(k-k')+\frac{1}{k+k'}\right] \tag{3.2}$$

This asymptote is valid for satisfaction of the inequality

$$kR\gg 1+|\gamma|. \tag{3.3}$$

It is somewhat more complicated to find the asymptotic expression for $Q_{\mathrm{sur}}(k,k')$. For large values of k and k″, and also of $|k-k'|$, the principal contribution to the integral over φ is made by the vicinity of points of the saddle points $\varphi=0$ and $\varphi=\pi$. We first consider the saddle point $\varphi=0$. Of all the terms of the sum over n, the largest is the component with $n=0$. The remaining terms of the sum are small in comparison with unity. Keeping only the term with $n=0$, we get the following asymptotic expression for the contribution to Q_{sur} from the point $\varphi=0$:

$$Q_{\mathrm{sur}}^{(0)}(k,k')=-\frac{6\sigma R}{\pi^2}\int_0^{\pi/2}d\theta\sin^2\theta\cos^2\theta\int_0^\infty d\varphi\int_0^\varphi d\lambda\cos\left[\frac{kR_\perp}{2}(\varphi^2-\lambda^2)\right]$$

$$\times\int_0^{\varphi} d\mu\cos\left[\frac{k'R_\perp}{2}(\mu^2-\varphi^2)\right]. \tag{3.4}$$

Direct calculation of the integrals leads to the result:

$$Q_{\rm sur}^{(0)}(k,k')=-\frac{3}{4}\frac{\sigma\alpha}{(kk'R)^{1/2}}[|k-k'|^{-1/2}-(k+k')^{-1/2}],$$
$$\alpha=\pi^{-1/2}\int_0^{\pi/2}d\theta\sin^{1/2}\theta\cos^2\theta=\frac{4\pi\sqrt{2}}{5\Gamma^2(0{,}25)}\approx 0{,}27. \tag{3.5}$$

Analysis shows that this formula is valid upon satisfaction of the following conditions:¹⁾

$$1-\rho\ll 2|\gamma|/(kR)^{1/2}\ll 1. \tag{3.6}$$

We now investigate the asymptotic contribution to $Q_{\rm sur}$ from the saddle point $\varphi=\pi$. Here the principal role is played by their relation between the resonance parameter $|1-e^{-2\pi\gamma}|$ and the quantity $2|\gamma|(kR)^{-1/2}$. If the first of them is much less than the second, then the "volume" electrons make the principal contribution to the current density, $|Q_{\rm vol}|\gg|Q_{\rm sur}|$, and the current density near resonance generally does not depend on the character of the reflection of the electrons from the boundary.

In the opposite limiting case,

$$|1-e^{-2\pi\gamma}|\gg 2|\gamma|/(kR)^{1/2} \tag{3.7}$$

In the calculation of the contribution from the point $\varphi=\pi$ the function $(\gamma\varphi+in\pi)^{-1}$ can be replaced by its value for $\varphi=\pi$. Calculation gives the following asymptotic expression:

$$Q_{\rm sur}^{(\pi)}(k,k')=-\frac{3}{8}\frac{\sigma\gamma}{R}\operatorname{cth}\pi\gamma\left\{\frac{1}{(kk')^{1/2}}\left[\pi\delta(k-k')+\frac{1}{k+k'}\right]-\frac{2}{\pi}\frac{\ln(k/k')}{k^2-k'^2}\right\}. \tag{3.8}$$

The component with the δ function is due to the contribution of the saddle points $\lambda=0$ and $\mu=0$, while the two other terms are connected with integration over λ and μ near the upper limits in the general formula (2.8).

It is evident from a comparison of (3.2) and (3.8) that the kernel $Q_{\rm vol}(k,k')$ is exactly compensated by the first two components in (3.8), in analogy with the case of large fields, when $|\gamma|\ll 1$.[10] Thus, the asymptotic current density in the region of large values of kR (3.3), upon satisfaction of the inequalities (3.6) and (3.7), is of the form

$$j(k)=\frac{3}{4}\frac{\sigma\alpha}{R^{1/2}}\int_0^\infty dk'\frac{\mathscr{E}(k')}{(kk')^{1/2}}[|k-k'|^{-1/2}-(k+k')^{-1/2}] \tag{3.9}$$
$$+\frac{3\pi}{4}\frac{\sigma\gamma}{R}\operatorname{cth}\pi\gamma\left[\frac{\mathscr{E}(k)}{k}-\frac{1}{\pi^2}\int_0 \frac{dk'}{k^2-k'^2}\mathscr{E}(k')\ln\frac{k}{k'}\right].$$

The relative value of the resonance components is of the order of $\gamma\coth\pi\gamma(kR)^{-1/2}$ and is small because of the condition (3.7). We note that the asymptotic y component of the current density differs from (3.9) only in the value of α, that is, $\alpha_y=1.5\,\alpha_z$.

4. SOLUTION OF THE MAXWELL EQUATIONS

We proceed to the solution of Eq. (2.2), in which the current density is given by the expression (3.9). We introduce the dimensionless variables:

$$k=k_0\xi,\quad \mathscr{E}(k)=-2E'(0)F(\xi)k_0^{-2}, \tag{4.1}$$

where k_0 is the characteristic value of the inverse of the skin depth and is determined by the formula ($\tau=1/\nu$)

$$k_0=\left(\frac{3\pi\alpha\omega\sigma}{c^2R^{1/2}}\right)^{2/5}\equiv|k_0|\exp\left(\frac{2i}{5}\operatorname{arc\,tg}\omega\tau\right), \tag{4.2}$$

Then we get the following expression for $F(\xi)$;

$$\xi^2F(\xi)-i\int_0^\infty\frac{d\xi'F(\xi')}{(\xi\xi')^{1/2}}[|\xi-\xi'|^{-1/2}-(\xi+\xi')^{-1/2}]$$
$$=1+iA\left[\frac{F(\xi)}{\xi}-\frac{1}{\pi^2}\int_0^\infty\frac{d\xi'}{\xi^2-\xi'^2}F(\xi')\ln\frac{\xi}{\xi'}\right], \tag{4.3}$$

in which the parameter

$$A=\frac{\pi\gamma\operatorname{cth}\pi\gamma}{\alpha(k_0R)^{1/2}} \tag{4.4}$$

is small because of the inequality (3.7).

Equation (4.3) can be solved by perturbation theory if we expand the solution in powers of iA. We seek the solution in the form

$$F(\xi)=F_0(\xi)+iAF_1(\xi),$$

where $F_0(\xi)$ and $F_1(\xi)$ satisfy the following equations:

$$\xi^2F_0(\xi)-i\xi^{-1/2}\int_0^\infty\frac{d\xi'}{(\xi')^{1/2}}[|\xi-\xi'|^{-1/2}-(\xi+\xi')^{-1/2}]F_0(\xi')=1, \tag{4.5}$$

$$\xi^2F_1(\xi)-i\xi^{-1/2}\int_0^\infty\frac{d\xi'}{(\xi')^{1/2}}[|\xi-\xi'|^{-1/2}-(\xi+\xi')^{-1/2}]F_1(\xi')$$
$$=\frac{F_0(\xi)}{\xi}-\frac{1}{\pi^2}\int_0^\infty\frac{d\xi'}{\xi^2-\xi'^2}F_0(\xi')\ln\frac{\xi}{\xi'}. \tag{4.6}$$

The solution of the zeroth-approximation equation (4.5) is easily found by a method first applied by Hartmann and Luttinger[9] to equations of a similar type. We find $F_0(\xi)$ in the form of a contour integral:

$$F_0(\xi)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}dz\,\xi^zM_0(z), \tag{4.7}$$

$$M_0(z)=\int_0^\infty d\xi\,\xi^{-z-1}F_0(\xi); \tag{4.8}$$

$M_0(z)$ is the Mellin transform of the function $F_0(\xi)$. It follows from Eq. (4.5) that as $\xi\to\infty$ the function $F_0(\xi)\to\xi^{-2}$, while as $\xi\to 0$, the function $F_0(\xi)\to i\xi^{1/2}$. Consequently, $M_0(z)$ should be regular in the strip $-2<\operatorname{Re}z<+\frac{1}{2}$, and the constant c in (4.7) is chosen inside this strip. If we substitute (4.7) in (4.5), then the following equation is obtained for $M_0(z)$:

$$\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}dz[M_0(z)\xi^{z+5/2}-i\Phi_0(z)M_0(z)\xi^z]=\xi^{1/2}, \tag{4.9}$$

$$\Phi_0(z)=\int_0^\infty du[(u-1)^{-1/2}-(u+1)^{-1/2}]u^{-z-1}=\frac{\sqrt{\pi}\,\Gamma(z+1/2)}{\Gamma(z+1)}\left(1+\operatorname{tg}\frac{\pi z}{2}\right), \tag{4.10}$$

where $\Gamma(z)$ is the Euler gamma function.

We require that $M_0(z)$ have a simple pole with a residue equal to unity at the point $z=-2$. Then, displacing the contour of integral in the first component in (4.9) by $\frac{5}{2}$ to the left, and going around the point $z=-2$, we obtain the component $\xi^{1/2}$, which preserves the same form on the right hand side of (4.9).

After a change of variables, the homogeneous function equation

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¹⁾A limiting case that is the opposite of (3.6) has been considered by Meĭerovich. [8]

$$M_0(z-{}^5/_2)=i\Phi_0(z)M_0(z) \tag{4.11}$$

is obtained for $M_0(z)$. Its general solution evidently contains as a factor an arbitrary periodic function with period $5/2$. A unique choice of this function is made by regularization of the general solution in the strip $-2 \leq \mathrm{Re}\, z < 1/2$ for the condition that the function $M_0(z)$ should have a simple pole at the point $z = -2$ with unity residue. Such a regular solution was obtained earlier[10] for Eq. (4.11) in the form of a rather complicated integral. However, it can be demonstrated directly that the function

$$M_0(z)=\left(\frac{4}{25\sqrt{2\pi}}\right)^{2(z+2)/5}\Gamma^{-1}\left(\frac{7}{5}\right)e^{\pi i(z+2)/5} \times \cos\frac{\pi z}{2}\Gamma(z+1)\Gamma\left(\frac{1-2z}{5}\right)\Gamma\left(\frac{3-2z}{5}\right) \tag{4.12}$$

satisfies the equation and is regular in the strip $-4 < \mathrm{Re}\, z < 1/2$, with the exception of a single point $z = -2$, where its residue is equal to unity.[2)] Thus the formula (4.12), together with (4.7), gives an asymptotically exact solution of the problem in zeroth approximation in the parameter A.

In order to obtain a solution of the next-approximation equation (4.6), we make use of the following example. We represent the right side of (4.6) in the form of the contour integral

$$\frac{1}{2\pi i}\int_{c_1-i\infty}^{c_1+i\infty}\psi(\zeta)\xi^{\zeta-1}\,d\zeta. \tag{4.13}$$

Then it is sufficient to construct for the functional equation (4.11) a solution that is regular in the strip

$$\mathrm{Re}\,\zeta-3\leqslant \mathrm{Re}\,z<\mathrm{Re}\,\zeta-{}^1/_2, \tag{4.14}$$

which has a simple pole at the point $z = \zeta - 3$ with a residue equal to $\psi(\zeta)$. Then calculating the integral over ζ, we find $M_1(z)$, and consequently $F_1(\xi)$.

Direct calculation gives

$$\psi(\zeta)=M_0(\zeta)\left[1-\left(2\cos\frac{\pi\zeta}{2}\right)^{-2}\right]. \tag{4.15}$$

The solution of Eq. (4.11), which is regular in the strip (4.14), with residue $\psi(\zeta)$ at the point $z = \zeta - 3$, is

$$M_1^{(\zeta)}(z)=\frac{{}^2/_5\pi M_0(z)M_0(\zeta)\left[1-(2\cos{}^1/_2\pi\zeta)^{-2}\right]\sin\left[{}^2/_5\pi(z+2)\right]}{M_0(\zeta-3)\sin\left[{}^2/_5\pi(z-\zeta+3)\right]\sin\left[{}^2/_5\pi(\zeta-1)\right]}. \tag{4.16}$$

Consequently, the Mellin transform of $F_1(\xi)$ is

$$M_1(z)=\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}d\zeta\, M_1^{(\zeta)}(z). \tag{4.17}$$

5. THE SURFACE IMPEDANCE

In order to find the surface impedance, there is no necessity of calculating the spatial distribution of the high-frequency field in a metal; it is necessary only to know the function $M(z)$ at the point $z = -1$. Actually, the impedance is expressed in terms of $M(z)$ in the following way:

$$Z=-\frac{8i\omega}{k_0c^2}\int_0^\infty F(\xi)\,d\xi=-\frac{8i\omega}{k_0c^2}M(-1)=Z_1(1+iA\mu), \tag{5.1}$$

$$Z_1=-\frac{8i\omega}{k_0c^2}M_0(-1)=\left[\frac{2\pi}{\Gamma(0.4)}\right]^2\left(\frac{5}{4\pi}\right)^{1/5}\frac{1}{\sin(2\pi/5)}\frac{\omega}{k_0c^2}e^{-3\pi i/10} \approx 7.5\frac{\omega}{k_0c^2}e^{-3\pi i/10}=-7.5i\frac{\omega}{c^2}\left(\frac{c^2R^{1/2}}{3\pi\alpha\omega|\sigma|}\right)^{2/5}\exp\left(\frac{2i}{5}\,\mathrm{arc\,tg}\frac{1}{\omega\tau}\right) \tag{5.2}$$

The quantity Z_1 represents the nonresonant part of the impedance, and depends smoothly on the magnetic field ($Z_1 \propto H^{-1/5}$). At high frequencies ($\omega\tau \gg 1$) the impedance Z_1 is in the main imaginary. The complex constant μ is given by the formula

$$\mu=\frac{M_1(-1)}{M_0(-1)}=\frac{25}{64}\left(\frac{5}{\pi}\right)^{3/5}2^{4/5}\sin\frac{2\pi}{5}e^{3\pi i/5}J, \tag{5.3}$$

$$J=\int_0^\infty dx\frac{x}{\mathrm{th}\,x}\left(1-\frac{1}{4\,\mathrm{ch}^2 x}\right)\frac{|\Gamma(1{,}2+i\cdot 4x/5\pi)\Gamma(1{,}6+i\cdot 4x/5\pi)|^2}{(x^2+\pi^2/16)(x^2+9\pi^2/16)}.$$

The integral J in (5.3), found by numerical integration, is equal to 0.166 and

$$\mu\simeq 0.142\exp(3\pi i/5).$$

Substituting (5.2) and (5.3) in formula (5.1), and using the expression (4.4) for A, we obtain

$$Z=\frac{7.5\omega}{qc^2\alpha^{2/5}}\exp\left[-\frac{3\pi i}{10}-\frac{2i}{5}\mathrm{arctg}(\omega\tau)\right] +\frac{3.35\omega}{\alpha^{8/5}q^{3/2}c^2R^{1/2}}i\gamma\,\mathrm{cth}\,\pi\gamma\exp\left(\frac{3i}{5}\mathrm{arctg}\frac{1}{\omega\tau}\right). \tag{5.4}$$

Here

$$q=\left(\frac{3\pi\omega|\sigma|}{c^2R^{1/2}}\right)^{2/5}\propto H^{1/5},\qquad |\sigma|=\frac{Ne^2\tau}{m(1+\omega^2\tau^2)^{1/2}}, \tag{5.4'}$$

while the parameter $\alpha = 0.270$ for the polarization $\mathbf{j} \parallel \mathbf{H}$ and $\alpha = 0.405$ for $\mathbf{j} \perp \mathbf{H}$.

In the resonance region, the first component in Eq. (5.4) describes the slow and gradual decrease of the nonresonant part of the surface impedance with increase in the magnetic field ($Z_1 \propto H^{-1/5}$, cf.[3]). The cyclotron resonance is described in the case considered by the second component in (5.4). In the neighborhood of resonance, when $\Omega\tau \gg 1$ and $|\omega - n\Omega| \ll \Omega$, the resonance contribution can be represented in the form

$$\Delta Z_n\equiv Z-Z_1=\Delta R_{1\,max}\frac{n^{-1/2}}{1-i(\omega-n\Omega)\tau}, \tag{5.5}$$

where

$$\Delta R_{1\,max}=\frac{1.07}{\alpha^{8/5}c^2}\left(\frac{\omega^3}{q_0^3v}\right)^{1/2}\omega\tau\propto\omega^{2.2}\tau$$

represents the maximum value of the real part of the impedance near the fundamental resonance $\omega = \Omega$ and q_0 differs from q by the replacement of R by v/ω. Thus, in the case considered of an isotropic dispersion law and specular reflection of the electrons, the resonance curve for the impedance has a Lorentzian form. The amplitude of the resonance decreases slowly with increase in the number (as $n^{-1/2} \sim H^{1/2}$) and depends on the frequency and time of free path as $\omega^{2.2}\tau$.

It is interesting to note that for specular reflection the surface impedance increases at resonance, while it decreases for diffuse reflection.

The anisotropy of the resonance relative to the mutual orientation of the high-frequency current and the magnetic field is characterized by the factor $\alpha^{-3/5}$. For $\mathbf{j} \parallel \mathbf{H}$, the impedance is $(1.5)^{8/5} \approx 1.9$ times that for $\mathbf{j} \perp \mathbf{H}$. It is curious that this anisotropy should exist in an alkali metal with an isotropic dispersion law for the electrons. It is connected with the fact that the value of the screening current, which is due to the glancing electrons, is greater for $\mathbf{j} \perp \mathbf{H}$ than for $\mathbf{j} \parallel \mathbf{H}$.

2)It can be shown that the solution (4.12) is identical to that obtained in [10] in integral form.

Thus, in specular reflection of electrons from the surface of a metal (the inequalities (3.6) and (3.7)), we arrive at the result postulated by Chambers[4] for the surface impedance in the resonance region. In the same way, the phenomenological supposition[4] as the smallness of the resonance contribution to the impedance obtains its physical and rigorous quantitative foundation. We shall assume that some of these discrepancies between theory[1,2] and experiment (see, for example,[11]), which were spoken of in the Introduction, can be explained by specular reflection of the electrons from the surface.

We now consider the effect of a nonquadratic electron dispersion on the cyclotron resonance in the case of specular reflection. We limit ourselves to a closed Fermi surface. In order to obtain the corresponding generalization of Eq. (5.1) to an arbitrary dispersion law, it is necessary to find expressions for the quantities $\sigma\alpha/R^{1/2}$ and $\sigma\gamma \coth \pi\gamma/R$, which enter in (3.9). It is evident that these quantities will be tensors of second rank. Using the results of the earlier work,[2] it is not difficult to establish the fact that the coefficient in the resonance part of the current (3.9) must be replaced as follows:

$$\frac{\sigma\gamma}{R}\operatorname{cth}\pi\gamma \to \frac{e^2}{3(\pi\hbar)^3}\oint \frac{n_\alpha n_\beta}{K}\operatorname{cth}\left[\pi\frac{\nu - i\omega}{\Omega}\right]d\chi. \tag{5.6}$$

Here $\alpha, \beta = y, z$ are certain indices, $n_\alpha = v_\alpha/v$ is the unit vector of the velocity of the electron, $n_y = \sin\chi$, $n_x = \cos\chi$, and K is the absolute value of the Gaussian curvature; integration in (5.6) is carried out along the line $v_x = v\cos\vartheta = 0$ on the Fermi surface $\epsilon(p) = \epsilon_F$. The quantities χ and ϑ represent the azimuthal and polar angles in the velocity space with polar axis v_x. The components of the tensor (5.6), which contain the index x, are obviously equal to zero because $v_x = 0$.

Equation (5.6) can also be rewritten in invariant form:

$$\frac{e^2}{3(\pi\hbar)^3}\int d^3p\,\delta(\varepsilon - \varepsilon_F)\,\delta(v_x)\,v_\alpha v_\beta \operatorname{cth}\left[\pi\frac{\nu - i\omega}{\Omega}\right]. \tag{5.6$'$}$$

If we take it into account that $d^3p = d\epsilon do_v/Kv$, then $do_v = d\chi dn_x$, and carry out the integration over ϵ and n_x with the aid of δ functions, we then obtain (5.6).

In a similar way we can generalize the coefficient for the nonresonance component in (3.9):

$$\frac{\sigma\alpha}{R^{1/2}} \to \frac{\pi^{1/2}e^2}{12(\pi\hbar)^3(\nu - i\omega)}\oint \frac{d\chi}{K} n_\alpha n_\beta v_\perp^{1/2}\left|\frac{eH}{c\mu_{xx}}\right|^{1/2}, \tag{5.7}$$

where $v_\perp = (v_x^2 + v_y^2)^{1/2} \approx |v_y|$ is the component of the electron velocity perpendicular to the magnetic field, and the mass $\mu_{xx} = (\partial^2\epsilon/dp_x^2)^{-1/2}$ is the x component of the effective mass tensor. The integral in (5.7) can also be easily transformed to an invariant form of the type (5.6$'$).

Now, instead of the single equation (2.2), it is generally necessary to solve the set of two equations for $\mathscr{E}_y(k)$ and $\mathscr{E}_z(k)$. However, if we recognize that the components of the tensor (5.7) are small by virtue of the condition (3.7)[3)], then the solution can be obtained by reducing the tensor (5.7) to the principal axes. Essentially, this does not differ from the solution which was obtained in the previous section. We therefore write down immediately the final form of the impedance tensor in the case $\omega\tau \gg 1$:

$$\Delta Z_\alpha = Z_\alpha - Z_{\alpha 1} = \frac{4.28\sqrt{\pi}\omega^2}{c^2|k_\alpha|^{3/2}}\frac{\langle v_\alpha^2 \operatorname{cth}[\pi(\nu - i\omega)/\Omega]\rangle}{\langle v_\alpha^2(v_\perp\Omega_{xx})^{1/2}\rangle}. \tag{5.8}$$

Here the angular brackets denote the integral over d^3p of the type (5.6):

$$\langle U\rangle = \int d^3p\,\delta(\varepsilon - \varepsilon_F)\,\delta(v_x)\,U, \qquad \Omega_{xx} = \frac{eH}{c\mu_{xx}},$$

and k_α is expressed in terms of the principal value of the tensor (5.7), in correspondence with Eq. (4.2). The nonresonance impedance Z_α is almost imaginary and is given by Eq. (5.2) with the replacement of k_0 by k_α.

The difference of Eq. (5.8) from (5.4) is that, for an arbitrary dispersion law, the resonance factor $\coth[\pi(\nu - i\omega)/\Omega]$ depends on p_z (or on χ), thanks to the dependence of the cyclotron frequency Ω on p_z. For this reason, the cyclotron resonance becomes smeared out, its amplitude decreases and the shape of the resonance curve changes. As usual,[1,2] the resonance occurs at the extremal frequencies Ω_{ext} and their harmonics $n\Omega_{ext}$. In order to determine the behavior of the impedance near resonance, the cyclotron frequencies is expanded in powers of p_z (or in χ) near Ω_{ext} and the integration is carried out. Simple calculation leads to the following result:

$$\begin{aligned}\Delta Z_\alpha^{(n)} &= \Delta R_{\alpha 1}\frac{n^{-1/5}}{(1 - i\Delta_n)^{1/2}}\exp\left(\frac{\pi i s}{4}\right) \\ &= \Delta R_{\alpha 1}\frac{n^{-1/5}}{(1+\Delta_n^2)^{1/4}}\exp\left[\frac{\pi i s}{4} + \frac{1}{2}\operatorname{arctg}\Delta_n\right],\end{aligned} \tag{5.9}$$

where

$$\Delta_n = (\omega - n\Omega_{ext})\tau, \tag{5.10}$$

$$\Delta R_{\alpha 1} = \frac{4.28\pi^{1/2}\omega^2\tau^{1/2}}{c^2|k_{\alpha 1}|^{3/2}}\left(\sum \frac{n_{\alpha\,ext}^2}{K_{ext}}\right)\Big/\oint\frac{d\chi n_\alpha^2}{K}\left|v_\perp\frac{m''_{ext}}{2\mu_{xx}}\right|^{1/2} \propto \omega^{1,7}\tau^{1/2}, \tag{5.11}$$

and s denotes the sign of the second derivative $m_{ext}'' = \partial^2 m/\partial\chi^2$ at the point of the Fermi surface $\epsilon(p) = \epsilon_F$, $v_x = 0$, where $m(\chi)$ reaches an extremum. Summation in (5.10) is carried out over all χ_{ext}, where $m(\chi) = m''_{ext}$. In this case, if the anisotropy of the Fermi surface is of the order of unity ($|m''_{ext}| \approx m_{ext}$), the quantity ΔR_α is smaller by a factor of about $(\omega\tau)^{1/2}$ than for an isotropic dispersion law for the electrons. It is evident that in this case the surface impedance increases at resonance, and does not decrease.

Figure 1 shows the dependence of the resonance contribution to the real and imaginary parts of the surface impedance on the parameter $\Delta = \omega\tau(1 - H/H_n)$. The ordinate scale is arbitrary. The index 1 denotes that the curves describe the Lorentzian shape of the resonance line (5.5). The index 2 indicates that they represent the resonance curve (5.9) for s = +1, i.e., for local minimum of the cyclotron mass, $m = m_{min}$. For the maximum mass (s = −1), the dependence of the real part of the impedance on Δ is obtained by mirror reflection of the curve of ΔR_2 relative to the ordinate axis, and the dependence of the imaginary part by reflection relative to the ordinate axis and change of sign of the curve ΔX_2, shown in Fig. 1b. The change in the shape and width of the resonance curves is due to

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[3)]Strictly speaking, it suffices for this that $kR|1 - e^{-2\pi\gamma_e}| \gg 1$, where $\gamma_e = (\nu - i\omega)/\Omega_{ext}$, and Ω_{ext} is the extremal value of the cyclotron frequency.

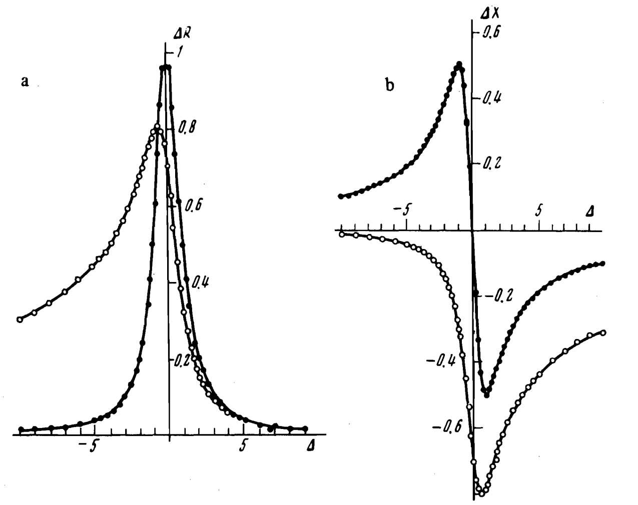

FIG. 1. Dependence of the resonance contribution to the impedance on the reciprocal of the magnetic field, $\Delta = \omega\tau(H_n/H-1)$: a–$\Delta R$(●–$\Delta R_1$, ○–$\Delta R_2$) and b–$\Delta X$(●–$\Delta X_1$, ○–$\Delta X_2$). The index 1 denotes the quadratic dispersion law (m = const), 2 denotes the nonquadratic dispersion law ($m = m_{min}$, s = +1). The scale on the ordinates is arbitrary.

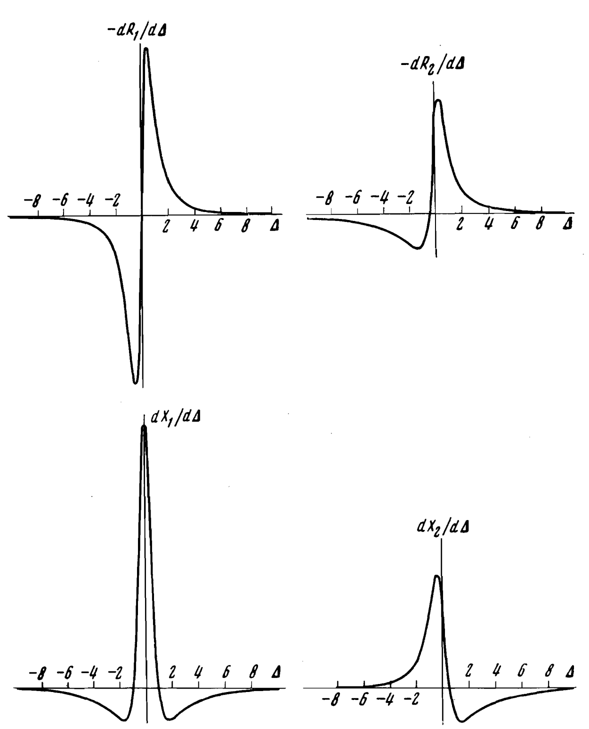

FIG. 2. Dependence of the derivative of the resonance part of the surface impedance on the reciprocal of the magnetic field. The notation is the same as in Fig. 1.

the effect of the electron dispersion law. Figure 2 gives the derivatives of the real and imaginary parts of the impedance with respect to the magnetic field, inasmuch as the derivative $\partial Z/\partial H$ is usually the quantity measured experimentally.

For comparison, plots of the resonance contribution to the surface impedance are given in Fig. 3. They are constructed from the data of Meĭerovich[8] and refer to the limiting case opposite to (3.6):

$$2|\gamma| / (kR)^{1/2} \ll 1-\rho \ll 1.$$

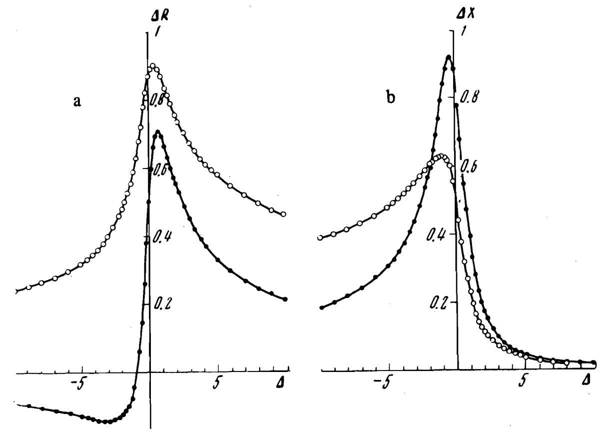

FIG. 3. Dependence of the resonance contribution to the impedance on the reciprocal of the magnetic field for a finite value $1-\rho$ (the inequality (5.12′), [8] $\Delta = \omega\tau(H_n/H-1)$: a–$\Delta R$ (●–R_3. ○–R_4), b–ΔX (●–ΔX_3, ○–ΔX_4). The index 3 denotes the quadratic dispersion law (m = const); 4–the nonquadratic dispersion law ($m = m_{min}$, s = +1). The scale on the ordinates is arbitrary.

The curves with the index 3 refer to the isotropic quadratic dispersion law for the conduction electrons ($\Omega(p_z)$ = const), and the curves with the index 4 to the nonquadratic dependence of $\epsilon(p)$ and $\Omega(p_z) = \Omega_{max} \simeq \omega/n$. It is seen from a comparison of Figs. 1 and 3 that the shape of the resonance curves depends essentially on the relation between the parameters $1-\rho$ and $2|\gamma|(kR)^{-1/2}$. Inasmuch as the value of the latter is determined by the magnetic field, the shape of the curves for finite values of $1-\rho$ will change upon change in H.

We shall now discuss the dependence of the shape, width, and amplitude of the resonance on the magnetic field in the case (3.6) considered by us. For a quadratic dispersion law, in the region of strong fields, when

$$1/\Omega\tau \approx n/\omega\tau \ll 1, \tag{5.12'}$$

the resonance curves have a Lorentzian shape. Their relative width $\Delta H/H_n \approx 1/\omega\tau$ does not depend on the magnetic field, i.e., the resonance curves become narrower with increase in the number (with decreasing the magnetic field). We emphasize that this conclusion is valid upon satisfaction of the conditions (3.7) and (5.12′). The inequality (3.7) can be rewritten in the form

$$n^{2/5} \gg \frac{\omega\tau}{\pi(1+\Delta_n^2)^{1/2}}\left(\frac{\omega}{q_0 v}\right)^{1/2}. \tag{5.13}$$

In the range of very weak fields, when the inequality (5.12) is replaced by the inverse, the resonance oscillations take on a harmonic form:

$$\Delta Z_{osc} = \Delta R_{1\,max} \cdot 2\pi\left(\frac{\omega}{\Omega}\right)^{4/5} \exp\left(-\frac{1}{\Omega\tau} + \frac{8i}{5}\operatorname{arctg}\frac{1}{\omega\tau} - 2\pi i\frac{\omega}{\Omega}\right). \tag{5.14}$$

The amplitude of these oscillations decreases essentially exponentially with decreasing H.

For an analysis of the shape of the resonance curves, and also of the dependence of the resonance amplitude on the number n, it is necessary to have in mind the following circumstance. The inequality (3.7) (or the equivalent condition (5.13)) can be satisfied on the wings of the resonance line (when $\Delta_n^2 \gg 1$) and violated near its center. The latter takes place if the parameter $\pi(qR)^{1/2}/\omega\tau$ is small:

$$\pi(qR)^{1/2}/\omega\tau \ll 1. \tag{5.15}$$

In this case, the surface electrons give a much smaller contribution to the current density than the volume electrons. As a result, it is shown that the skin layer in the metal and the cyclotron resonance are determined by the volume electrons (i.e., nonresonance electrons). In other words, one can neglect the quantity Q_{sur} in comparison with Q_{vol} in (2.7) and the current density and the surface impedance cease to depend on the character of the reflection of the electrons from the surface, and have the same asymptotic form as for diffuse scattering ($\sigma_0 = Ne^2\tau/m$):

$$Z_0 = \frac{8\pi^{2/3}\omega}{3^{1/2}c^2}\left(\frac{c^2 v\tau}{3\pi\omega\sigma_0}\right)^{1/3}(1-e^{-2\pi\gamma})^{1/3}e^{-\pi i/3}. \tag{5.16}$$

At resonance, the real and imaginary parts of the surface impedance are minimal:

$$R_n = \frac{4\pi^{2/3}\omega}{3^{1/2}c^2}\left(\frac{c^2 v\tau}{3\pi\omega\sigma_0}\right)^{1/3}\left(\frac{2\pi n}{\omega\tau}\right)^{1/3}, \quad \Delta = \left(\frac{\omega\tau}{2\pi n}\right)^{1/3};$$
$$X_n = \frac{8\pi^{2/3}\omega}{3^{1/2}c^2}\left(\frac{c^2 v\tau}{3\pi\omega\sigma_0}\right)^{1/3}\left(\frac{\pi n}{\omega\tau}\right)^{1/3}, \quad \Delta = -1. \tag{5.17}$$

The formulas (5.4) and (5.16) are matched together under the condition $|\Delta| \sim \omega\tau(qR)^{-1/2}/\pi$, where q is determined from (5.4′). Consequently, for specular reflection, the shape of the resonance curve and also the amplitude of the resonance depend significantly on the value of the parameter $\omega\tau(qR)^{-1/2}/\pi$ and change with the magnetic field.

Not long ago, we obtained a preprint of the work of Kamgar et al.[12], in which experimental data on the behavior of the amplitude and shape of resonance curves in gallium upon a change in the number of the resonance are analyzed. An exponential decrease in the amplitude with number and a frequency dependence of the exponent were discovered. The authors explain the results of their experiments by retardation effects, when the path lengths of the electron in the period of the high-frequency field v/ω become smaller than the characteristic distance $(R\delta)^{1/2}$ which the electron travels in the skin layer. Here the right side of the fundamental inequality (2.6) is violated, i.e., the quantity $|\gamma|(kR)^{-1/2} \approx (R\delta)^{1/2}\omega/v$ becomes larger than unity. A qualitative theory for this case, based on an exponential law of decay of the electromagnetic field in the skin layer, was recently proposed by Drew.[13] The results of the present work cannot be applied to the experiment of[12], inasmuch as they refer to another limiting case. Now there exists the important problem of adequately explaining the dependence of the amplitude of the resonance and the shape of the resonance curves on the number (magnetic field) observed in many metals.

It is possible that neither one of the limiting cases (diffuse or specular) reflection of the electrons is capable of completely explaining the real experimental situation, and that it is necessary to take into consideration the dependence of the reflection coefficient of the electrons on the momentum.

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