ABSORPTION OF FOURTH SOUND IN He II *NEAR THE* λ *POINT*

J. G. SANIKIDZE

Institute of Cybernetics, Georgian Academy of Sciences

Submitted June 16, 1972

Zh. Eksp. Teor. Fiz. **63**, 1919–1921 (November, 1972)

The various absorption mechanisms of fourth sound near the λ point are investigated. It is shown that the major mechanism of energy dissipation is the decay of fourth sound into second-sound quanta with wavelengths of the order of the correlation length. The sound absorption coefficient is calculated and its temperature dependence near the λ point is determined.

In sufficiently narrow capillaries or in a porous medium saturated with superfluid helium, unusual oscillations can be propagated that are a modification of first sound and are known as fourth sound.[1] The velocity of fourth sound is equal to

$$u_4=\left(\frac{\rho_s}{\rho}u_1^2+\frac{\rho_n}{\rho}u_2^2\right)^{1/2}, \tag{1}$$

where u_1 and u_2 are the velocities of first and second sound, and ρ_s and ρ_n are the densities of the superfluid and normal components of the liquid.

The width of the capillaries d, according to the conditions of propagation of fourth sound, should be much less than the penetration depth of the viscous wave $\lambda_v = (2\eta/\omega\rho_n)^{1/2}$ (η is the viscosity of helium and ω is the sound frequency). Under the same conditions, second sound is transformed into a strongly damped thermal wave (fifth sound), the velocity of propagation of which is equal to[2]

$$u_5=\left(\frac{\rho}{2\rho_s}\right)^{1/2}\frac{d}{\lambda_v}u_2. \tag{2}$$

Upon approach to the λ point, the velocity of fourth sound tends to zero and the sound dies out. Since sound absorption near the λ point is basically due to relaxation of ρ_s, it is necessary for the consideration of this problem to make use of a set of equations of hydrodynamics which also contains an equation describing the approach of ρ_s to its equilibrium value.[3,4] In the case of propagation of fourth sound, one can set the velocity of the normal component of the liquid equal to zero, which simplifies the set of equations[5]

$$\dot{\mathbf{v}}_s+\nabla(\mu_0+\mu_s)=0,\quad \frac{\partial\rho}{\partial t}+\operatorname{div}\rho_s\mathbf{v}_s=0,\quad \frac{\partial(\rho\sigma)}{\partial t}=0,$$
$$\frac{\partial\rho_s}{\partial t}+\operatorname{div}\rho_s\mathbf{v}_s=-\frac{2\Lambda m}{\hbar}\left(\frac{\partial E}{\partial\rho_s}\right)_{\sigma,\rho}\rho_s;$$
$$\mu_0=\left(\frac{\partial E}{\partial\rho}\right)_{\sigma,\rho_s},\quad \mu_s=\left(\frac{\partial E}{\partial\rho_s}\right)_{\sigma,\rho}, \tag{3}$$

where E is the internal energy per unit mass of the liquid, and σ is the entropy per unit mass. The dimensionless kinetic coefficient Λ, which determines the approach of ρ_s to its equilibrium value, is determined by the mechanism of dissipation of the sound energy near the λ point.

Solving the set of equations (3), we find the sound absorption coefficient

$$\alpha_4=\omega^2\rho_s\zeta/2u_4^3, \tag{4}$$

where the coefficient of second viscosity ζ is determined in the following manner:

$$\zeta=\frac{\hbar}{2\Lambda m\rho_s}\left\{\frac{1}{\rho}\left(\frac{\partial\rho_s}{\partial T}\right)_p\left[\left(\frac{\partial\sigma}{\partial T}\right)_p\frac{\partial T_\lambda}{\partial p}+\sigma\left(\frac{\partial\rho}{\partial p}\right)_T+\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_p\right.\right.$$
$$\left.\left.+\sigma\left(\frac{\partial\rho}{\partial T}\right)_p\frac{\partial T_\lambda}{\partial p}\right]\left[\left(\frac{\partial\rho}{\partial p}\right)_T\left(\frac{\partial\sigma}{\partial T}\right)_p-\frac{1}{\rho^2}\left(\frac{\partial\rho}{\partial T}\right)_p^2\right]^{-1}+1\right\}. \tag{5}$$

Near the λ point, the system can be characterized by some correlation length ξ. Fourth sound can propagate only when $k_4\xi \ll 1$ (k_4 is the wave number of fourth sound). The applicability of the equations of hydrodynamics (3) is determined by this same condition. The quanta of fourth sound, with a wavelength of the order of ξ, will be entirely dissipated and a characteristic time $\tau_4 = \xi/u_4$ can be introduced which determines the rate of energy dissipation of the waves of fourth sound. Taking into account the dependence (see[6]) of ξ and u_4 on $\epsilon = (T_\lambda - T)/T_\lambda$, we obtain the result that

$$\tau_4\sim\varepsilon^{-1+\alpha/2},$$

where α is the critical index of the heat capacity. The kinetic coefficient Λ can be defined as

$$\Lambda_4=mu_4\xi/\hbar\sim\varepsilon^{-1/3+\alpha/6}.$$

(m is the mass of the He^4 atom). Such a definition of Λ is in accord with the introduction of the characteristic relaxation time

$$\frac{1}{\tau}=\frac{2\Lambda m}{\hbar}\left(\frac{\partial\mu_s}{\partial\rho_s}\right)\rho_s,$$

which follows from Eqs. (3). Taking into account the dependence of Λ on the temperature, and introducing in (5) the singular parts of the derivatives of the thermodynamic quantities, we obtain the temperature dependence of the absorption coefficient

$$\alpha_4^{(4)}\sim\varepsilon^{-4/3+5\alpha/3},$$

which agrees with the result obtained directly from the theory of dynamic similarity[7] (for helium, we can take $\alpha \approx 0$).

However, for channels whose transverse dimensions satisfy the condition $\xi \ll d \ll \lambda_v$, another dissipation mechanism is possible, connected with the decay of fourth sound into a wave of second sound. (For $d \ll \lambda_v$, although hydrodynamically the second sound is not propagated, fluctuations of second sound are possible, since $\xi \ll d$.) Such a mechanism is similar to the mechanism of first sound absorption, considered by Pokrovskiĭ and Khalatnikov.[8] The relaxation time is

$$\tau_2 = \xi / u_2 \sim \varepsilon^{-1}, \quad \Lambda_2 = mu_2\xi / \hbar \sim \varepsilon^{-1/3+5\alpha/6}.$$

In view of the fact that $\tau_2 \gg \tau_4$, this mechanism of absorption should be dominant. In this case, we get

$$\alpha_4^{(2)} \sim \varepsilon^{-4/3+7\alpha/6}.$$

Thus, in spite of the fact that fourth sound is a modification of first sound, in relation to absorption, it behaves in the same way as second sound[4] ($\alpha_1 \sim \epsilon^{-1+\alpha}$, $\alpha_2 \sim \epsilon^{4/3-\alpha/3}$). The absorption over one wavelength is the same for all sounds:

$$\operatorname{Im} k / \operatorname{Re} k \sim \varepsilon^{-1}.$$

(We have set $\alpha = 0$.)

Under conditions when $d \leq \xi$, a shift occurs in the λ point[9] (a new critical temperature T_0 appears); the temperature dependences of the thermodynamic characteristics change. For temperatures $T < T_0$, under the assumption that the temperature dependence of the coherence length does not change,[9-11] the picture of the absorption of fourth sound considered above remains valid. In the range of temperatures $T_0 < T < T_\lambda$ ($\xi \geq d$) the superfluid state is unstable. If we introduce a new correlation length $\tilde{\xi}$ in this region, then, inasmuch as we have an essentially one-dimensional situation, we can obtain from the scaling laws[6] the result that $\rho_s \sim \tilde{\xi}^2/d^2$. In this region, the relaxation can be determined by the processes of decay into quanta of fourth sound with the critical wavelength $\tilde{\xi}$. The decay into quanta of fifth sound can be more important, since $\tau_5 \sim \tilde{\xi}/u_5 \gg \tilde{\xi}/u_4$. For the estimate of the temperature dependence of the absorption in this region, it is necessary to know the dependence of $\tilde{\xi}$ on the temperature.

The author is grateful to V. L. Pokrovskiĭ and I. M. Khalatnikov for useful discussions.

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Translated by R. T. Beyer
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