PULSATIONS OF THE RADIATION FROM A LASER WITH FREQUENCY DISPERSION

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The equation of motion of a laser whose cavity possesses frequency dispersion is obtained. It is found that frequency dispersion can lead to pulsations of the emission of solid and gas lasers.

RESONATORS with frequency dispersions are widely used in lasers to control the emission spectrum and mode selection. Many problems solved with the aid of lasers lead to the important particular case of a resonator with dispersion—a three-mirror resonator. At the same time, the influence of frequency dispersion on the kinetics and on the spectral characteristics of laser emission has not yet been investigated sufficiently well.

A theoretical investigation of the spectrum of a laser with dispersion usually reduces in the literature to the determination of the threshold conditions and the natural frequencies of different concrete schemes of linear resonators (without allowance for the nonlinearity of the active medium). Perel' and Rogova[1] evaluated the saturation and the nonlinear mode interaction of a gas laser with a three-mirror resonator. The analysis of[1] is based on the use of Slater's method[2] and on an expansion of the electromagnetic field in terms of the modes of two weakly-coupled resonators with high mirror reflectivities; this imposes a certain limit on the applicability of the theory of[1].

The influence of the spatial dispersion of the resonator on the emission of solid-state lasers was investigated for the most part experimentally[3]. Ratner[4] has indicated that allowance for spatial dispersion of the active medium does not account for the regimes of undamped radiation pulsations. The possibility of explaining such pulsations of solid-state lasers, particularly as due to the presence in the resonator of additional reflecting surfaces and their motion relative to the main mirrors of the resonator has been diligently investigated of late[5,6]. The effect of such a nonstationary dispersion was reduced in the description simply to a time modulation of the Q of the unaltered modes. As we shall show, such interpretations are insufficient in many cases.

In the present paper we develop the theory of a laser whose resonator has frequency dispersion. It is found that under definite conditions the stationary frequency dispersion can be the cause of pulsations of the emission of gas in solid-state lasers. The principal point of our analysis is that we forego in part the mode expansion of the field and analyze the unsteady (transient) processes under conditions when the longitudinal-mode spectrum is nearly continuous (Sec. 3).

1. EQUATION OF MOTION OF A LASER WITH FREQUENCY DISPERSION

Without allowance for the frequency dispersion, the equation of motion of a laser in the semiclassical theory is

$$\frac{dE}{dt}+i\Omega E+\rho E=-\frac{1}{2\varepsilon_0}\frac{l}{L}\frac{dP}{dt}. \qquad (1.1)$$

Here Ω is the natural frequency of the resonator (the equation is written for one mode whose index is omitted), E is the component of the electric field intensity, P is the component of the polarization of the active medium, l is the length of the active medium, L is the path of the light in the resonator, and

$$\rho=\frac{1}{\tau}\ln\frac{1}{r}, \qquad (1.2)$$

where τ is the time of travel of the light in the resonator (in a ring resonator $\tau = L/c$, and in the linear one $\tau = 2L/c$), and r is the product of the amplitude reflection coefficients of the mirrors.

In the present section we generalize (1.1) to the case of frequency dispersion of the resonator mirrors, when r depends on the frequency, $r = r(\omega)$. In particular, the dispersion can be produced by additional mirrors. We point out immediately that the change of interest to us here occurs only in the last term of the left-hand side of (1.1), and consists in the substitution $\rho \to \hat{R}$, where $\hat{R}$ is a linear integral operator.

The form of the operator $\hat{R}$ is easiest to find in the approximation of plane waves and weak nonlinearity of the active medium[1)]. We consider, for example, a one-directional regime of ring-laser generation. We expand the electric field intensity E at each point z in a Fourier integral

$$E(z,t)=\int F(z,\omega)\exp(-i\omega t)\,d\omega. \qquad (1.3)$$

Writing down the cyclic condition for the individual frequencies with allowance for the dependence of ρ on ω and summing it over the frequencies, we get

$$\frac{dE}{dt}+i\Omega E+\hat{R}E=-\frac{1}{2\varepsilon_0}\frac{l}{L}\frac{dP}{dt}. \qquad (1.4)$$

Here

$$\hat{R}E=\int \rho(\omega)F(0,\omega)\exp(-i\omega t)\,d\omega \qquad (1.5)$$

(in the spectral representation) or else (in the time-domain representation)

$$\hat{R}E=\int_{-\infty}^{t}K(t-t')E(0,t')\,dt', \qquad (1.5a)$$

[1)]More accurately, we neglect the spatial modulation of the population inversion in the active medium, which is valid in any case near the generation threshold. It was found in [7] that allowance for this facotr has little effect on the characteristics of laser generation even when the pump exceeds the threshold appreciably. The same approximation is valid also within the framework of the "point model" [8].

where K(t) is the Fourier transform of $\rho(\omega)$. Equation (1.4) is derived for a field at a fixed point $z = 0$ corresponding to the coordinate of the boundary of the active layer. The method for recalculating the field for other points under the assumed approximations is obvious. We note that (1.4) retains its form for a standing-wave laser (linear resonator) and can be generalized for angular modes. In the case of generation at several modes of the principal resonator, it is necessary to write an expression of the type (1.4) for each of them.

Given the dispersion law $\rho(\omega)$ and Eq. (1.4) supplemented with (1.5) and with the equation connecting the polarization of the active medium with the field E, we can easily determine the characteristics of the single-particle regimes, by putting

$$F(0, \omega) = \mathscr{E}_0\delta(\omega - \omega_0), \quad E_0 = \mathscr{E}_0 \exp(-i\omega t). \tag{1.6}$$

Then (1.4) turns into a transcendental dispersion equation for the frequencies ω_0 and the amplitudes $\mathscr{E}_0$. The stability of the solutions can be investigated by the usual method of linearizing (1.4) with respect to small deviations from the stationary solution. We note that above the threshold it is necessary, in the general case, to solve the equations for the field deviations ΔE and ΔE^*. This circumstance is connected with the nonlinearity of the active medium (with the formation of combination tones). In the absence of resonator dispersion, it was investigated in detail by Kuznetsova and Rautian[10].

2. PULSATIONS OF SOLID-STATE LASER

As is well known[11], according to the rate equations for a single-mode solid-state laser, a single-frequency regime is established in the course of time (we denote this frequency by ω_0). This regime is reached in an oscillatory fashion in a ruby laser, with a pulsation frequency $\nu \approx 1$ MHz and a transient time $\sim 10^{-3}$ sec. At small deviations from the stationary regime, the field can be represented by a sum of three components with frequencies ω_0 and $\omega_0 \pm \nu$, and the damping of sideband components is small within one period of pulsations $(2\pi/\nu)$. This damping can be compensated for by frequency dispersion, if the loss in the sideband components is somewhat less than in the central component. We determine below the critical value of the dispersion needed to maintain undamped pulsations. For our purposes, it suffices to describe the connection between the polarization of the active medium and the field intensity, within the framework of the rate equation (the conditions for its applicability are indicated in[11]). Then

$$\left.\begin{aligned} &\frac{dx}{dt} + \hat{R}x = \frac{1}{2}\frac{\omega}{Q}nx \\ &\frac{dn}{dt} = \gamma_1(\eta - n) - \gamma_1|x|^2 \end{aligned}\right\}. \tag{2.1}$$

Here x is the dimensionless amplitude of the field, n is the ratio of the population difference to the threshold value, η is the ratio of the pump to its threshold value, and $\gamma_1 = T_1^{-1}$ is the relaxation rate.

In the absence of dispersion, $\hat{R} = \rho_0 = (1/2)\omega_0/Q$ determines the width of the resonator band for a mode of frequency ω_0, which we assume to coincide with the center of the contour, and with a figure of merit Q. In the stationary one-frequency regime we have here

$$n_0 = 1, \quad |x_0|^2 = \eta - 1 \tag{2.2}$$

Let us determine the influence of the dispersion on the stability of the stationary regime the parameters of the regime are obviously not altered at $\rho(\omega_0) = \rho_0$. To this end we put

$$x = x_0 + \Delta x, \quad n = 1 + \Delta n, \quad x_0 = \sqrt{\eta - 1} > 0 \tag{2.3}$$

and linearize (2.1) with respect to small deviations Δx and Δn in the form

$$\begin{aligned} &\Delta x = \Delta x_1 \exp(-i\nu t + \gamma t) + \Delta x_{-1} \exp(i\nu t + \gamma t), \\ &\Delta n = \Delta n_1 \exp(-i\nu t + \gamma t) + \Delta n_1^* \exp(i\nu t + \gamma t). \end{aligned} \tag{2.4}$$

The real quantities ν and γ are determined from the characteristic equation

$$\begin{vmatrix} p_1 & 0 & -x_0\rho_0 \\ 0 & p_{-1} & -x_0\rho_0 \\ \gamma_1 x_0 & \gamma_1 x_0 & p_0 \end{vmatrix} = p_0 p_1 p_{-1} + \gamma_1\rho_0 x_0^2 (p_1 + p_{-1}) = 0, \tag{2.5}$$

where

$$p = -i\nu + \gamma, \quad p_0 = p + \eta\gamma_1, \quad p_1 = p + \rho_1 - \rho_0, \quad p_{-1} = p + \rho_{-1}^* - \rho_0^*. \tag{2.6}$$

Here $\rho_{\pm 1}$ correspond to the values of $\rho(\omega)$ for the components of the field in (2.4) with respective amplitudes $\Delta x_{\pm 1}$. We shall assume henceforth that the dispersion is symmetrical, i.e.,

$$\rho_1 - \rho_0 = \rho_{-1}^* - \rho_0^*. \tag{2.7}$$

In particular, to satisfy (2.7) for a laser with an additional mirror (Sec. 4) it suffices to have $\omega_0\tau_1 = N_1$, where $\tau_1 = 2L_1/c$ and N_1 is an integer. Then $p_1 = p_{-1}$ and we obtain for the characteristic numbers p

$$p + \rho_1 - \rho_0 = 0, \tag{2.8}$$

$$p^2 + p(\eta\gamma_1 + \rho_1 - \rho_0) + 2\gamma_1\rho_0 x_0^2 + \eta\gamma_1(\rho_1 - \rho_0) = 0. \tag{2.9}$$

The roots of (2.8) correspond to the possible splitting of the frequencies of the linear resonator in the presence of noticeable dispersions. We are not interested in them now, assuming the dispersion to be sufficiently small. Equation (2.9) in the absence of dispersion $(\rho_1 = \rho_0)$ describes the usual damped pulsations. Neglecting the phase dispersion of $r(\omega)$ (Im $\rho(0) = 0$), which is immaterial at present, we find that the critical dispersion at which there is no damping is determined from the condition

$$\rho_1 - \rho_0 = -\eta\gamma_1. \tag{2.10}$$

We put

$$r(\omega_0 \pm \nu) = r(\omega_0)[1 + \varepsilon(1 - \cos(\nu\tau_1))] \approx r(\omega_0)(1 + {}^1/{}_2\varepsilon\nu^2\tau_1^2), \tag{2.11}$$

where $\epsilon \ll 1$ is the "depth of modulation" of the losses ($\epsilon > 0$, i.e., the losses are maximal at the frequency ω_0). We then get from (2.10)

$$\varepsilon_{cr} = 2\eta\gamma_1\tau(\nu\tau_1)^{-2}. \tag{2.12}$$

Putting $\tau \approx \tau_1$, which is typical of lasers with external mirrors, $\eta = 2$, $L = 1$ m, $\nu = 1$ MHz, and $\gamma_1 = 10^3$ sec^{-1}, we get $\epsilon_{cr} = 0.015$. This value can be exceeded in certain real laser schemes[5]. We note that undamped pulsations were obtained here with allowance for dispersion within the framework of one mode. Such a cause of pulsations was apparently not discussed earlier[9] 2).

3. CASE WHEN THE LONGITUDINAL-MODE SPECTRUM IS CLOSE TO CONTINUOUS

The frequency of the longitudinal mode with index N is determined from the condition

$$\omega_N\cdot\tau+\Phi=2\pi N, \tag{3.1}$$

where $\Phi = -\tau \operatorname{Im}\rho$ is the phase of the mirror reflection coefficient $r(\omega)$. A close frequency $\omega = \omega_N + \Delta\omega$ will also be a natural frequency if the additional phase shift $\Delta\omega\cdot\tau$ is offset by a change of the phase Φ as a result of frequency dispersion

$$\Delta\omega\cdot\tau+\Delta\Phi=0,\quad \Delta\Phi=\Phi(\omega_N+\Delta\omega)-\Phi(\omega_N). \tag{3.2}$$

From this we get the condition for the existence of a continuous spectrum

$$d\Phi/d\omega=-\tau. \tag{3.3}$$

If (3.3) is sufficiently accurately satisfied in the frequency interval in which the principal energy of the field is concentrated, something that can be attained at a fixed dispersion by choosing the proper resonator length, the spectrum should be close to continuous. We consider below the question of the influence of small deviations from (3.3).

For a gas laser, the relation between P and E can be regarded as algebraic. Near the generation threshold

$$-\frac{1}{2\varepsilon_0}\frac{l}{L}\frac{dP}{dt}\approx\frac{i\omega_0}{2\varepsilon_0}\frac{l}{L}P=\alpha E-\beta|E|^2E, \tag{3.4}$$

were ω_0 is a certain central frequency, which we assume to coincide with the center of the gain contour, α is the linear gain, and β is the saturation coefficient[12]. The slow radiation pulsations of interest to us here can be obtained in the following manner. We expand $\rho(\omega)$ in a Taylor series about ω_0 ($\omega = \omega_0 \neq \Delta\omega$):

$$\rho(\omega)=\frac{1}{\tau}\sum_{m=0}^{\infty}\frac{(-i)^m}{m!}a_m(\Delta\omega\cdot\tau_1)^m, \tag{3.5}$$

$$a_0=\tau\rho_0=\tau\rho(\omega_0),\quad a_m=i^m\frac{\tau}{\tau_1{}^m}\frac{d^m\rho}{d\omega^m}\bigg|_{\omega=\omega_0}. \tag{3.6}$$

Here τ_1 is the characteristic delay time in the dispersive system. Then

$$\hat{R}\mathcal{E}=\frac{1}{\tau}\sum_{m=0}^{\infty}\frac{1}{m!}\tau_1{}^m a_m\frac{d^m\mathcal{E}}{dt^m}, \tag{3.7}$$

where

$$E(t)=\mathcal{E}(t)\exp(-i\omega_0 t). \tag{3.8}$$

If ν is the characteristic frequency of the pulsations, then at $\nu\tau_1 \ll 1$ we can confine ourselves in (3.7) to a few first terms of the series. By the same token, we obtain a differential equation in place of an integro-differential equation. Such an approximation is valid only for sufficiently slow motions, i.e., in a limited region of initial conditions and generally speaking within a finite time interval.

Taking (3.4) and (3.7) into account, we represent the equation of motion in the form

$$M\frac{d^2\mathcal{E}}{dt^2}-(\alpha-\rho_0)\mathcal{E}+\beta\mathcal{E}|\mathcal{E}|^2=f\frac{d\mathcal{E}}{dt}-\frac{1}{\tau}\sum_{m=3}^{\infty}\frac{1}{m!}\tau_1{}^m a_m\frac{d^m\mathcal{E}}{dt^m}. \tag{3.9}$$

here

$$M=\frac{\tau_1{}^2}{2\tau}a_2,\quad f=\frac{\tau_1}{\tau}a_1-1. \tag{3.10}$$

We assume that the dispersion is also symmetrical (cf. (2.7))

$$\rho(-\Delta\omega^*)=\rho^*(\Delta\omega). \tag{3.11}$$

Then a_m, like α and β, are real. The condition (3.3) corresponds to $f = 0$. For "slow" pulsations, all the terms of the series in (3.9) will be small. Therefore under the indicated conditions the right-hand side of (3.9) can be regarded as a small perturbation.

The solution of the generating (unperturbed) equation

$$M\frac{d^2\mathcal{E}}{dt^2}-(\alpha-\rho_0)\mathcal{E}+\beta\mathcal{E}|\mathcal{E}|^2=0 \tag{3.12}$$

is obtained in the following manner. We introduce the real field amplitude and phase

$$\mathcal{E}(t)=A(t)\exp[i\varphi(t)]. \tag{3.13}$$

From (3.12) we get

$$A^2(t)\frac{d\varphi}{dt}=C=\text{const}, \tag{3.14}$$

$$\frac{d^2A}{dt^2}-\frac{C^2}{A^3}-\frac{\alpha-\rho_0}{M}A+\frac{\beta}{M}A^3=0. \tag{3.15}$$

Equation (3.15) describes oscillations of a nonlinear conservative pendulum in a potential field (Fig. 1)

$$V(A)=\frac{C^2}{2A^2}-\frac{\alpha-\rho_0}{2M}A^2+\frac{\beta}{4M}A^4. \tag{3.16}$$

It is easy to obtain a general solution of (3.15) in terms of elliptic functions, by determining the conserved "energy" integral W:

$$\frac{1}{2}\left(\frac{dA}{dt}\right)^2+V(A)=W=\text{const}. \tag{3.17}$$

Then

$$dA/\sqrt{W-V(A)}=\pm\sqrt{2}\,dt. \tag{3.18}$$

The motion is periodic with a period T that depends on the initial conditions (the constants C and W)

$$T=2^{3/2}\left(\frac{M}{\beta}\right)^{1/2}(I_2-I_0)^{-1/2}\times\mathrm{K}\left[\left(\frac{I_2-I_1}{I_2-I_0}\right)^{1/2}\right], \tag{3.19}$$

where K is a complete elliptic integral and $I_{0,1,2}$ are

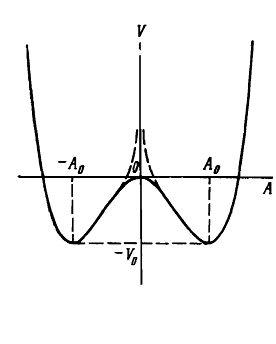

FIG. 1. The dashed line corresponds to $C \neq 0$ and the solid line to $C = 0$.

2)It was found in [5,6] that pulsations of solid-state lasers can occur when additional (parasitic) mirrors move relative to the principla mirrors of the resonator. The results presented above demonstrate the feasibility of pulsations in the presence of even immobile additional mirrors.

the roots of the cubic equation

$$-\frac{C^2}{2}+WI+\frac{\alpha-\rho_0}{2M}I^2-\frac{\beta}{4M}I^3=0. \quad (3.20)$$

We have assumed that

$$M>0,\quad \alpha-\rho_0>0,\quad I_0\leqslant 0<I_1<I_2,\quad W>W_{min}. \quad (3.21)$$

The intensity $I = A^2$ ranges from I_1 to I_2. In the important particular case $C = 0$, we obtain for motions near the bottom of the "potential well"

$$W=W_{min},\quad I_0=0,\quad I_1\approx I_2\approx I^{(0)}=(\alpha-\rho_0)/\beta \quad (3.22)$$

Small oscillations about the usual value of the intensity $I^{(0)}$ of a single-frequency regime with a period

$$T_0=2\pi\sqrt{M/(\alpha-\rho_0)},\quad \nu_0=2\pi/T_0=\sqrt{(\alpha-\rho_0)/M}. \quad (3.23)$$

When the "energy" W increases (larger distance from the bottom of the potential well), the period of motion increases and the spectrum becomes richer in harmonics having frequencies that are multiples of $\nu < \nu_0$.

The influence of the right-hand side of (3.9) can be estimated for motions near the bottom of the "potential well," by making the substitution $d^2/dt^2 \to -\nu^2$ in the sum (3.9). Allowance for terms with even indices m leads to a change in the "mass" M, and terms with odd m change the "friction coefficient" f. The presence of "friction" at $f < 0$ (dispersion less than critical) leads to "energy" dissipation and to a slow (near (3.3)) change of the amplitude and the period of the oscillations, so that the usual single-particle regime with frequency ω_0 and intensity $I^{(0)}$ is established in the course of time. At $f > 0$ (dispersion larger than critical), the "energy" increases, a fact that can be attributed to transition to generation at other frequencies that results from the frequency splitting in the presence of a definite dispersion. These premises are confirmed in the particular case of a three-mirror resonator, which is considered in the next section.

If (3.3) is sufficiently accurately satisfied, the spectrum of the laser is close to continuous and the transient times increase accordingly. These circumstances allow us to suggest that the cause of the "memory effect"[13] may be the presence of a definite frequency dispersion of the resonator. If we accept this point of view, we can present the following interpretation for the results of the experiments in[13]. During the first stage, the motion of the additional mirror "swings" the "pendulum" with a period corresponding to the Doppler frequency shift (proportional to the velocity of the mirror). As shown by the analysis of this section, when the spectrum is close to continuous the definite modulation can be appreciable even when the response signal from the moving mirror is of low amplitude, and this explains qualitatively the corresponding results[13]. When the modulation is turned off (the moving mirror is covered with an opaque screen), the "friction" can alter significantly the motion within times $T_{mem} \approx 2M/f$. If (3.3) is violated because L deviates from the required value by $\Delta L = 10^{-8}$ L, then $f = \Delta L/L = 10^{-8}$. At an upper limit of the "remembered" frequencies $\nu_0 = 10$ kHz and $\alpha - \rho_0 = 10$ kHz, using (3.23), we get $T_{mem} = 10^3$ sec.

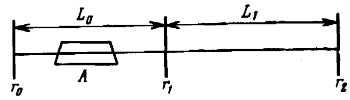

FIG. 2. A—active medium.

4. SPECTRUM OF LASER WITH ADDITIONAL MIRROR

The simplest and most widely used method of introducing frequency dispersion is to use an additional mirror. In the present section we investigate single-frequency regimes of a gas laser with a three-mirror resonator (Fig. 2).

We indicate first the features of the spectrum of a laser with a three-mirror resonator within the framework of the linear theory. We assume for simplicity that $L_1 = L_0$, $r_1 > 0$, and $r_2 < r_1$. The dispersion equation for the determination of the frequencies reduces in this case to a quadratic equation with respect to (see[14])

$$z=\exp(i\omega\tau_0)=\exp\left(i\frac{\omega}{c}L_0\right).$$

Analysis shows the existence of a critical value $r_2^{(cr)}$ (in[15] this value is called optimal), which depends on r_1

$$r_1=2r_2^{(cr)}/[1+(r_2^{(cr)})^2]. \quad (4.1)$$

At $r_2 < r_2^{(cr)}$, the resonator frequencies are the same as in the two-mirror resonator, $\omega = \omega_n$ (at $r_2 = 0$), i.e., there is no shift or splitting of the frequencies by the additional mirror. At $r_2 > r_2^{(cr)}$, the longitudinal modes of the two-mirror resonator are split into two components that are symmetrical with respect to ω_n [3)]. The sign of the "friction coefficient" f introduced in the last section coincides with the sign of the quantity $r_2 - r_2^{(cr)}$.

We now consider the question of the time dependence of small deviations of the field $\Delta\mathscr{E} = \mathscr{E} - \mathscr{E}_0$ from the stationary single-particle regime with unshifted frequency ω_0 (the amplitude $\mathscr{E}_0$ can be regarded as real without loss of generality, $\beta\mathscr{E}_0^2 = \alpha - \operatorname{Re}\rho_0$) at $r_2 < r_2^{(cr)}$ within the framework of (1.4) with a polarization (3.4) corresponding to the case of a gas laser. We have

$$\frac{d\Delta\mathscr{E}}{dt}+(\hat{R}-\rho_0)\Delta\mathscr{E}+\beta\mathscr{E}_0^2(\Delta\mathscr{E}+\Delta\mathscr{E}^*)=0. \quad (4.2)$$

We put

$$\Delta\mathscr{E}=\mathscr{E}_1e^{pt}+\mathscr{E}_{-1}e^{p^*t},\qquad p=-i\nu+\gamma, \quad (4.3)$$

$$\hat{R}e^{pt}=\rho_1e^{pt},\qquad \hat{R}e^{p^*t}=\rho_{-1}e^{p^*t}. \quad (4.4)$$

Then the characteristic number p should be determined from the equation

$$[p+\rho_1-\rho_0+\beta\mathscr{E}_0^2][p+\rho_{-1}^*-\rho_0^*+\beta\mathscr{E}_0^2]-[\beta\mathscr{E}_0^2]^2=0. \quad (4.5)$$

3)In contrast to our statement, Perel' and Rogova [15] admit of the possibility, in principle, of lasing at the unshifted frequencies ω_n at $r_2^{(cr)} < r_2 < r_1$; the apparent reason is that in [15] they took into account in this case only half of the solution of the dispersion equation. We note that if the length of one arm is an exact multiple of that of the other, $L_1 = mL_0$ with m an integer, there is also a splitting of the unshifted frequencies into two when r_2 increases to $r_2^{(cr)}$, where

$$r_2^{(cr)}=A-\sqrt{A^2-1},\quad A=\frac{1}{2}\left[\frac{m+1}{r_1}-(m-1)r_1\right].$$

With further increase of r_2, new components appear in the spectrum.

If the laser is exactly tuned to extremal losses we have

$$\rho_1 - \rho_0 = \rho_{-1}^* - \rho_0^* \equiv \rho_{1,0}. \tag{4.6}$$

We then get from (4.5)

$$p + \rho_{1,0} = 0, \tag{4.7}$$

$$p + \rho_{1,0} + 2\beta\mathscr{E}_0^2 = 0. \tag{4.8}$$

Equation (4.7) coincides with (2.8), and for the same reason we shall not be interested in the solutions of (4.7) (they can be obtained from the solutions of (4.8) as $\beta\mathscr{E}_0^2 \to 0$). Putting $\kappa = \exp(2\beta\mathscr{E}_0^2\tau_0)$, we represent (4.8) in the form

$$r^{(1)} / r^{(0)} = \varkappa \exp(p\tau_0), \tag{4.9}$$

where $r^{(0)}$ is the reflection coefficient of the mirror system 1–2 at the frequency ω_0 and $r^{(1)}$ is the reflection coefficient for the component of the deviation with an equal amplitude $\Delta\mathscr{E}_1$. For simplicity we put $r_0 = 1$ and neglect the absorption in the central mirror (with reflection coefficient r_1). Then $r(\omega)$ takes the form

$$r(\omega) = \frac{r_1 - r_2 \exp(i\omega\tau_1)}{1 - r_1 r_2 \exp(i\omega\tau_1)}. \tag{4.10}$$

Taking (4.10) into account, we transform (4.9) into

$$(1 - B_1\delta) / (1 - B_2\delta) = \varkappa \exp(p\tau_0), \tag{4.11}$$

where $\delta = \exp(-p\tau_1) - 1$

$$B_1 = \frac{r_2}{r_1 - r_2}, \quad B_2 = \frac{r_1 r_2}{1 - r_1 r_2}, \tag{4.12}$$

with $f = B_1 - B_2 - 1$. Here (4.11) reduces to a quadratic equation with respect to δ, with roots

$$\delta_{1,2} = [(1 + \varkappa B_2 - B_1) \pm \sqrt{D}] / 2B_1, \tag{4.13}$$

where

$$D = (1 + \varkappa B_2 - B_1)^2 - 4B_1(\varkappa - 1). \tag{4.14}$$

Accordingly

$$\exp(-p_{1,2}\tau) = (1 + \varkappa B_2 + B_1 \pm \sqrt{D}) / 2B_1. \tag{4.15}$$

From the condition $d = 0$ we obtain the pump interval within which $\operatorname{Im} p_{1,2} \neq 0$, i.e., there exist pulsations

$$\varkappa_{1,2} - 1 = \{[2B_1 - B_2(1 + B_2 - B_1)] \pm \sqrt{[2B_1 - B_2(1 + B_2 - B_1)]^2 - B_2^2(1 + B_2 - B_1)^2}\} / B_2^2. \tag{4.16}$$

At

$$|f| = |1 + B_2 - B_1| \ll 1, B_1, B_2 \tag{4.17}$$

we get

$$\varkappa_1 - 1 \approx \frac{(1 + B_2 - B_1)^2}{4B_1}, \quad \varkappa_2 - 1 \approx \frac{4B_1}{2B_2^2} \tag{4.18}$$

Thus, at pumps corresponding to the condition

$$\varkappa_1 < \varkappa < \varkappa_2 \tag{4.19}$$

there are emission pulsations that generally speaking attenuate. Let us calculate the rate of the damping of the pulsations under the condition (4.17)

$$-2\operatorname{Re} p \cdot \tau = \ln \frac{\varkappa(B_2 + 1)}{B_1} \approx \frac{\varkappa(B_2 + 1)}{B_1} - 1 > 0. \tag{4.20}$$

In the important particular case $f = 0$ near the generation threshold, when

$$\Delta\varkappa \equiv \varkappa - 1 \approx 2\beta\mathscr{E}_0^2\tau_0 \ll 1 \tag{4.21}$$

we get

$$\varkappa_1 = 1, \quad \operatorname{Im} p \cdot \tau = \sqrt{\Delta\varkappa / B_1}, \quad -2\operatorname{Re} p \cdot \tau = \Delta\varkappa. \tag{4.22}$$

It follows from (4.22) that the damping rate Re p is much less than the pulsation frequency Im p, which is proportional in accordance with (3.23) to the "strong-field" amplitude $\mathscr{E}_0$. Thus, in this case the situation is similar to that holding in solid-state lasers (Sec. 2). For a similar reason, the small perturbations of the problem (in our case, allowance for a small detuning and for deviations from the relation (4.6) can transform the damped pulsations into undamped ones.

We note in conclusion that the system of rate equations describing the solid-state laser is close to conservative, and therefore the role of the dispersion of the resonator in Sec. 2 reduces only to a "buildup" of the pulsations that are weakly damped in its absence. The necessary dispersion turns out to be relatively small (proportional to the damping rate). To obtain dispersion pulsations in a gas laser, on the other hand (Secs. 3 and 4), it is necessary first to make the system close to conservative, which calls already for a noticeable value of the dispersion, close to that necessary to start a splitting of the frequencies of the modes of the principal resonator.

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