THE PROBLEM OF LASER RADIATION SOURCES IN THE FAR ULTRAVIOLET AND X-RAY REGIONS

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As the active medium for a laser in the wavelength region $\lambda \sim 50$–150 Å it is proposed to use a plasma with multiply charged ions expanding into a neutral gas. Such a plasma can be produced by focusing of high-power laser pulses onto solid targets. To obtain gains $k \sim 1\ \text{cm}^{-1}$ in the region 50–150 Å in this case, it is necessary to use pumping pulses with an energy of several tens of joules. It is shown that the process of expansion of the hot plasma into a neutral gas can be used to create a powerful point source of nonlaser x rays in the region $\lambda \sim 1$–10 Å. The spectrum of this source should contain a small number of narrow lines. The conversion coefficient for pumping energy into a line spectrum of x rays may reach 1%.

1. Recently several articles have appeared which have been devoted to the problem of producing laser sources in the far ultraviolet and x-ray regions of the spectrum[1-4]. Attention is devoted to two main questions—the method of producing the active medium of the laser, and the resonators. In the present article we discuss in detail only the first of these questions, namely; the possibility of creating a population inversion with a gain $k[\text{cm}^{-1}] \sim 1$ in the wavelength region $\lambda \sim 200$ Å. At the same time we discuss several new possibilities of producing high-power nonlaser sources of x-ray with a line spectrum.

It is well known that the gain k at wavelength λ is related to the population inversion ΔN and the characteristics of the corresponding atomic transition $m \to n$ by the following relation:

$$k = \frac{\lambda^2 A_{mn}}{4\Delta\omega}\Delta N. \qquad (1)$$

Here A_{mn} and $\Delta\omega$ are the probability and width of the spontaneous radiation spectrum for the transition $m \to n$. Considering various media, we can assume $\Delta\omega \geq \Delta\omega_D = 2\pi\bar{v}/\lambda$, where $\Delta\omega_D$ is the Doppler width and $\bar{v}$ is the average velocity. Then, $\Delta N \lesssim N_m = q/A_m$, where q $(\text{cm}^{-3} \times \text{sec}^{-1})$ is the rate of excitation of the level m. Taking into account the obvious inequality $A_{mn}/A_m \leq 1$, we obtain

$$k \leq \frac{\lambda^3}{8\pi\bar{v}} q \approx \frac{\lambda^3}{8\pi\bar{v}}\frac{Q}{\delta E} \leq \frac{\lambda^4}{8\pi\bar{v}}\frac{Q}{2\pi\hbar c}, \qquad (2)$$

where $\delta E \gtrsim \hbar\omega = 2\pi\hbar c/\lambda$ is the average energy expended in producing one excited atom in the state m, and Q is the specific excitation power. According to Eq. (2), the quantities q and Q necessary for creation of an active medium with a given k rise rapidly with decreasing λ:

$$q \infty \lambda^{-3}, \quad Q \infty \lambda^{-4}. \qquad (3)$$

The difficulties of creating laser sources in the short-wavelength region of the spectrum are intrinsically related to this circumstance. Approximate values of q and Q for various wavelengths, estimated by means of Eq. (2) are

λ, Å:	1	10	100	1000
q, $\text{cm}^{-3}\cdot\text{sec}^{-1}$:	10^{32}	10^{29}	10^{26}	10^{23}
Q, W/cm^3:	10^{17}	10^{13}	10^{9}	10^{5}

At the present time it is difficult to find a method of pumping capable of providing specific powers $Q \gtrsim 10^{17}$ W/cm^3 ($\lambda \sim 1$ Å). Therefore it is more realistic to consider the spectral region $\lambda \gtrsim 10$ Å. Values of $Q \lesssim 10^{13}\ \text{W/cm}^3$ can in principle be provided under conditions of laser evaporation of solid targets and heating of the plasma produced by this[5]. For example, for a laser pulse energy of ~10 joules, a pulse duration of 10^{-9} sec, and a plasma volume of $10^{-3}\ \text{cm}^3$ we have $Q \sim 10^{13}\ \text{W/cm}^3$. Obviously values $Q \lesssim 10^{12}\ \text{W/cm}^3$ can be achieved not only in plasma obtained under conditions of laser heating but also by means of powerful pulsed discharges. However, for definiteness, we will discuss below for the most part the laser plasma.

2. Under the conditions of laser evaporation of solid targets containing atoms with a large number of electrons, a plasma is formed with a density close to the density of the solid material and a temperature of the order of 100 eV or several hundred eV. By choice of the target and the experimental conditions we can provide a multiplicity of ionization $z \sim 10$.

We will estimate the relative amount of thermal energy of such a plasma E_T and the energy expended in ionization E_i. Using the Thomas-Fermi model[6], we have $E_i \sim 16z^{7/3}N_i\ \text{eV/cm}^3$, where N_i is the concentration of ions. It is easy to see that the energy E_i is of the order of the plasma thermal energy $E_T = N_e kT + N_i kT = (z+1)N_i kT$, where $N_e = zN_i$ is the concentration of electrons. Therefore it is convenient to attempt to use the supply of energy E_i which can be liberated in electron-capture processes.

For the purpose of producing an active medium, we are most interested in the stage of expansion of the plasma, when the energy input is cut off and the plasma density rapidly drops. At the same time there is a decrease of the collision process probabilities $N_e\langle v\sigma_n^i\rangle$ and $N_e\langle v\sigma_{nn'}\rangle$, where σ_n^i and $\sigma_{nn'}$ are the ionization cross section of the level n and the cross section of the transition $n \to n'$. As is well known, the probability of radiative decay of a level with principal quantum number n falls rapidly with increasing n, roughly according to a law $A_n \propto n^{-4.5}$.[7] Therefore, beginning with some critical density N_e^{cr}, for all levels $n \leq n_c$ the lifetime will be determined only by radiative processes:

$$A_n > N_e^{cr} \langle v\sigma_n^i \rangle, \quad A_n > N_e^{cr} \langle v\sigma_{nn'} \rangle \tag{4}$$

and we can expect the production of a population inversion in the whole series of transitions $n \to n'$. To obtain large absolute concentrations of excited ions, it is necessary to provide sufficient efficiency of the electron-capture processes. In that case, if the plasma expands into a vacuum, electron capture can occur as the result of various recombination processes: collision, radiative, and two-electron processes. However, the probabilities for population of excited states as the result of all these processes are insufficiently large (we will say more about this below). Therefore the expansion of the plasma into a neutral gas is of greater interest. In this case population is possible of one (or several) excited states of ions as the result of a quasi-resonance charge-exchange process.

For definiteness let us consider charge exchange of an ion with charge z in helium atoms with an ionization potential I = 24.6 eV;

$$B_z + \mathrm{He}\,(1s^2) \to B_{z-1}(n_1) + \mathrm{He}^+(1s), \tag{5}$$

where B_z represents an ion with charge z. The cross section for this process depends strongly on n_i. Usually charge exchange occurs preferentially to one of the levels n_1 satisfying the condition

$$z^2\,\mathrm{Ry}/n_1^2 \geqslant I, \quad \mathrm{Ry} = 13.6\ \mathrm{eV}, \tag{6}$$

for which the probability of a Landau-Zener transition at the point of crossing of the terms is maximal. As an estimate of the cross section (see the Appendix) we can take

$$\sigma \approx \pi a_0^2 z^2 \approx 10^{-16} z^2\,\mathrm{cm}^2. \tag{7}$$

For $z \sim 10$ the cross sections (7) are very large. Therefore the population of the level n_1 as the result of process (5) is substantially more efficient than in the recombination processes listed above. We note that the possibility of using charge-exchange processes to produce an active medium in the optical region of the spectrum and in the ultraviolet region $\lambda \gtrsim 1000$ Å has been discussed in refs. 8–10.

The rate of excitation of the level n_1 as the result of process (5) is

$$q_1 = N_i N_0 \langle v\sigma \rangle \sim N_i N_0 v \cdot 10^{-16} z^2, \tag{8}$$

where N_0 is the density of the neutral gas and v is the ion velocity. If $n_1 \leq n_c$, levels $n < n_1$ are populated only as the result of radiative transitions and their population N_n can be expressed in terms of the Seaton cascade matrix[11]:

$$N_n = q_1 C_{n_1 n}/A_n. \tag{9}$$

Here and below for simplicity the evaluations are carried out for hydrogen-like ions. The quantities N_n/n^2 (the statistical weight of level n is $2n^2$) are given in Table I for $n_1 = 5$ and $n_1 = 7$. As can be seen, for a number of transitions $n \to n'$ the inversion condition $N_n/n^2 > N_{n'}/n'^2$ is satisfied. For what follows the

Table I

n	N_n/n^2				n	N_n/n^2			
	$n_1 = 5$		$n_1 = 7$			$n_1 = 5$		$n_1 = 7$	
2	0.021	—	0.021	—	5	0.35	0.54	0.056	0.133
3	0.03	0.125	0.027	0.12	6	—	—	0.095	0.195
4	0.059	0.133	0.037	0.112	7	—	—	0.78	1.12

Note. The populations of the levels n are given in units of $10^{-8} z^{-4} q_1$. The first column is given for an optically thin plasma, and the second for a plasma in which reabsorption takes place in the Lyman series transitions.

greatest interest is presented by a plasma in which reabsorption occurs in transitions of the Lyman series, and therefore in our evaluation below we will for the most part discuss just this case.

The gain k for these transitions, by means of Eqs. (8) and (9), can be represented in the form

$$k = \frac{\lambda^3}{8\pi} N_i N_0 \sigma \alpha_{nn'}, \tag{10}$$

$$\alpha_{nn'} = n^2 A_{nn'} [C_{n_1 n}/n^2 A_n - C_{n_1 n'}/n'^2 A_{n'}]. \tag{11}$$

Values of k for various transitions of hydrogen-like ions $n \to n'$ corresponding to $N_0 = 10^{18}\ \mathrm{cm}^{-3}$ and $N_i = 10^{17}\ \mathrm{cm}^{-3}$ are given in Table II. The concentrations N_0 and N_i are chosen with consideration of the following facts. The maximum permissible ion concentration $N_i = N_e/z$ is determined by the conditions (4). Estimates of the cross sections σ_n^i and $\sigma_{nn'}$ were made in accordance with Vainstein and Sobelman[12]. The concentration N_0 must satisfy the condition

$$\varkappa_\lambda = 10^{-17} (\lambda/\lambda_0)^3 N_0 \ll k_\lambda, \tag{12}$$

where κ_λ is the absorption coefficient due to the photoeffect in helium atoms, $\lambda_0 \approx 505$ Å is the photoabsorption edge, and k_λ is the gain for the transitions considered. The most severe limitation on the concentrations N_0 and N_i is due to the necessity of providing sufficiently large dimensions of the zone of mixing of ions and neutral atoms L, comparable with the size of the working volume. The most favorable conditions from this point of view are those of cylindrical expansion in which the working volume is $V = \pi r^2 l$. These conditions can be assured by appropriate choice of the system of focusing the laser beam at the target. The volume radius r can be taken as $r \lesssim 0.1$ cm. The volume length l can be chosen from the condition $kl \sim 1$.

It is easy to see that for transitions in the region

Table II. Wavelengths and gains in transitions corresponding to population in the charge-exchange process (5) of levels $n_1 = 5$ and $n_1 = 7$. The factor $\alpha_{nn'}$ is defined by Eq. (11)

	$n \to n'$					
	4→3	5→3	6→3	7→3	5→4	7→4
			$n_1 = 5,\ z = 10$			
λ, Å	186	130			420	
$\alpha_{nn'}$	0.011	0.228			0.272	
k, см$^{-1}$	2.8	20			800	
			$n_1 = 7,\ z = 13$			
λ, Å	111	76	65	59	245	128
$\alpha_{nn'}$	<0	0.072	0.021	0.163	0.014	0.15
k, см$^{-1}$	<0	0.22	0.39	2.2	14	22

$\lambda \approx 100$ Å ($5 \to 3$, $7 \to 4$) the situation is quite favorable. The value of the product $N_0 N_i$ in Eq. (10) can be reduced in Table II by an order of magnitude by providing the condition $kl \sim 1$ for $l \sim 1$ cm and simultaneously $L \sim r$. For the transition $7 \to 3$ ($\lambda = 59$ Å, gain $k \approx 2.2$) the situation is somewhat worse, but in this transition it is also possible to provide all of the necessary conditions.

The dimensions $l \sim 1$ cm and $r \sim 0.1$ cm correspond to a working volume $V \sim 3 \times 10^{-2}$ cm^3 and, consequently, a total number of ions $N_i^t \sim 3 \times 10^{15}$. The energy expended in ionization has an order of magnitude $E_i^t \sim 3 \times 10^{15} \times 16\, z^{7/3}$ Ev $\approx (1-2) \times 10^{19}$ eV $\approx 1-2$ joules ($z \sim 10-13$). Taking into account the thermal energy of the initial plasma $E_t = (z+1) N_i kT$ and the possible losses due to thermal conduction, reflection, and scattering of the laser beam and so forth, we can estimate the total laser pulse energy required as $E_L \approx (5-10) E_i^t \approx 10-20$ joules. The laser pulse duration Δt_L must be less than the time of expansion of the plasma to the necessary density N_i. For $l \sim 1$ cm and $r \sim 0.1$ cm we have $\Delta t_L \lesssim 10^{-9}$ sec.

The evaluations made of the parameters E_L and Δt_L naturally are very approximate, since the possibility of producing a plasma with a given total number of ions N_i^t and a specified z requires special choice of experimental conditions and, in particular, of the laser parameters. Above we have brought out in essence only the basic energetic possibilities. Just as in the general case, it is difficult to estimate the necessary values of specific power Q, since for this purpose, for chosen dimensions of the focusing spot l and r_0, it is necessary to know in addition the thickness of the evaporated layer δ. However, it is easy to see that for $l \sim 1$ cm and $r_0 \sim 10^{-2}$ cm the possible values of Q are consistent with the general estimates made above by means of Eq. (2). The maximum permissible value of the product $N_0 N_i$ in the charge-exchange region also depends substantially on the specific experimental conditions. In addition to the limitations on N_i and N_0 mentioned above, it is further necessary that in this region the shock wave in the initial neutral gas initiated by expansion of the plasma not produce ionization. Estimates show that this condition can be satisfied.

It is interesting to note that the charge-exchange probability $N_0\langle v\sigma\rangle$ exceeds the value τ_e^{-1}, where τ_e is the plasma expansion time. Therefore practically all ions are drawn into the charge-exchange process and the total number of photons which can be radiated in the working transition is $N_\lambda \sim N_i^t$, which corresponds to a rather high conversion of the primary laser radiation into short-wave radiation.

In view of the above, we can reach the conclusion that the charge-exchange process in the case of a plasma expanding into a neutral gas can provide gains $k \sim 1-10$ in the region $\lambda \approx 50-150$ Å with completely reasonable pump source parameters. In addition it is evident that on going to the region $\lambda \sim 5-10$ Å very serious difficulties are encountered. Since the product $N_i N_0$ in Eq. (10) cannot be increased because of difficulties with creation of the mixing zone, a decrease of the wavelength λ unavoidably leads to a sharp drop in the gain k. The fact that all of the estimates have been made for hydrogen-like ions has little effect on the results obtained. For example, in Eq. (10) in the case of nonhydrogen-like ions the only change is in the value of the coefficient $\alpha_{nn'}$, which cannot exceed unity.

It should be noted that the difficulties mentioned are not specific for the mechanism considered for creation of the population inversion. Other possible processes of electron capture into excited states of multiply-charged ions in hot plasma are less efficient than the charge-exchange process. Thus, for example, population of the level $n_1 = 7$ as the result of radiative recombination gives for the gain in the transition $7 \to 3$ the following result[13]:

$$k = 10^2 z^{-4}\ \text{cm}^{-1},\ \lambda = 10^4 z^{-2}\ \text{Å}. \tag{13}$$

For $z = 10$ we have $\lambda = 100$ Å, $k \approx 10^{-2}$ cm^{-1}. We note that other possibilities for producing an active medium (not involving use of a laser plasma) for the region $\lambda \sim 1-100$ Å also lead to very high levels of the necessary ideal pump power [1-3].

3. We will now discuss the possibility of using the charge-exchange mechanism in expansion of a plasma into a neutral gas to produce powerful nonlaser sources of x rays. As a consequence of the large value of the charge-exchange cross section $\sigma \sim \pi a_0^2 z^2$, the total probability of charge exchange during the time of the plasma expansion

$$W_{\text{ch.e.}}\tau_p = N_0\langle v\sigma\rangle r/v \approx N_0 r \cdot 10^{-16} z^2$$

reaches the order of magnitude of unity even for relatively low densities $N_0 \sim 10^{16}/rz^2$. Taking $z \sim 10$ and $r \sim 0.1-1$ cm, we obtain $N_0 \sim 10^{14}-10^{15}$. For such low densities N_0, naturally, no difficulties arise in creation of the mixing zone and we can assume that each of the ions undergoes charge exchange in the course of expansion of the plasma to a size r. This means that in the region of plasma densities satisfying Eq. (4) and also in the absence of reabsorption in the transitions being considered, the total number of photons radiated N_λ is roughly equal to the total number of ions N_i^t. Since we are now interested in the spontaneous radiation, we will consider transitions to the ground state, which corresponds to the shortest-wavelength radiation. The intensity distribution over the radiation lines can easily be found by means of the matrix $C_{n_1 n}$ (see Eq. (9)). As an example we have given in Table III some of the shortest-wavelength transitions to the ground state $n' = 1$ for $n_1 = 5$ and $n_1 = 7$, and also the number of photons

$$\eta_\lambda = C_{n_1 n} A_{n_1} / A_n, \tag{14}$$

radiated at wavelength λ, per charge-exchange event.

In accordance with the above, the radiation power I_λ can be evaluated as follows:

Table III. Wavelength λ and radiation quantum yield η_λ for charge exchange of an ion to the excited state n_1

	Transition $n \to n'$	λ, (Å)	η_λ
$z = 10,\ n_1 = 5$	$5 \to 1$	9.5	0.35
	$4 \to 1$	9.7	0.1
$z = 13,\ n_1 = 7$	$7 \to 1$	5.5	0.29
	$6 \to 1$	5.55	0.06

$$I_\lambda \sim N_i^t \eta_\lambda \hbar\omega/\tau_e. \qquad (15)$$

For the laser plasma parameters discussed in the preceding section, $N_i^t \sim 3\times 10^{15}$, $\tau_e \sim 10^{-7}$—10^{-6}, we have

$$I_\lambda \sim \eta_\lambda \cdot 5(10^5 \div 10^6)\ \mathrm{W}. \qquad (16)$$

Thus, the mechanism discussed permits obtaining a powerful pulsed source of x rays with a line spectrum containing a small number of lines. The widths of the spectral lines for the plasma densities considered in the charge-exchange region are determined by the Doppler effect and are of the order 10^{-4}—10^{-3} Å for the region $\lambda \sim 5$ Å.

4. Summing up, we can conclude that in the expansion of a hot plasma containing multiply charged ions into a neutral gas, formation is possible of an amplifying medium with gains $k \gtrsim 1\ \mathrm{cm}^{-1}$ in the far ultraviolet region of the spectrum down to wavelengths $\lambda \gtrsim 50$ Å. However, at the present time it is hard to find a method of producing an active medium in the wavelength region $\lambda \sim 1$—10 Å.

It should be noted that the problem of producing resonators for the region $\lambda \sim 50$—200 Å involves great difficulties. The various forms of resonators employing Bragg reflection discussed in the literature (see, for example, refs. 14—16) could in principle be used for the region $\lambda \sim 5$ Å, but are hardly suitable for $\lambda \sim 100$ Å. Reflection from metallic mirrors[17] can provide high reflection coefficients only for comparatively small deflection angles, which results in the necessity of using resonators with a large number of reflections. Here the requirements on the value of the gain are increased.

The discussion above also showed that the expansion of a hot plasma into a neutral gas can be used to produce a powerful point nonlaser source of x rays in the region $\lambda \sim 1$—10 Å. With appropriate choice of the experimental parameters, it is possible to obtain a line spectrum with a small number of narrow lines $\Delta\lambda/\lambda \sim 10^{-4}$. The spectral power of the radiation can reach values $P_\lambda \sim 10^8$ W/Å with a pulse duration $\sim 10^{-6}$—10^{-7} sec. The efficiency of conversion of the energy expended in production of the plasma into energy of the x-ray line spectrum can reach the order of 1%.

APPENDIX

A very simple estimate of the charge-exchange cross section can be obtained by means of the Brinkmann-Kramers[18] approximation. A general expression for the cross section for charge exchange of a proton in a hydrogen atom, $H^+ + H(n) \to H(n_1) + H^+$, has been obtained by May[19]. These results permit generalization to the case of collision of a completely ionized atom B with a neutral He atom:

$$B_z + \mathrm{He}(1s^2) \to B_{z-1}(n_1) + \mathrm{He}^+(1s), \qquad (A.1)$$

if hydrogen-like wave functions are used to describe the He atom. For quasiresonance charge exchange $I_1 = z^2/2n_1^2 \approx I$ (where $I \approx 0.9$ atomic units is the ionization potential of the He atom) we have

$$\sigma\left(n_1 = \frac{z}{\sqrt{2I}}\right) = z^2 \frac{2^7}{5} \frac{I^4}{v^2[v^2/8+I]^5}\pi a_0^2, \quad v > \sqrt{2I}, \qquad (A.2)$$

from which for $v \approx \sqrt{2I}$ we find

$$\sigma \approx 5.3 z^2 \pi a_0^2. \qquad (A.3)$$

As we go to velocities $v \lesssim \sqrt{2I}$ the Brinkmann-Kramers approximation becomes inapplicable. However in the case of quasiresonance collisions the cross section, as a rule, continues to rise down to extremely low velocities. We will estimate the cross section for the process (A.1) in the region $v < \sqrt{2I}$ by use of the Landau-Zener model (see, for example, ref. 20). According to this model the charge-exchange cross section in atomic units is determined by the formula

$$\sigma = 4R_x^2 G(\lambda)\pi a_0^2, \quad \lambda = \{2\pi V_{12}^2[v(V_{11}' - V_{22}')]^{-1}\}_{R=R_x}, \qquad (A.4)$$

where R_X is the point of crossing of the terms of the quasimolecule, $V_{12}(R)$ is the nondiagonal transition matrix element and $V_{11}(R)$ and $V_{22}(R)$ are the adiabatic values of the initial and final terms. The function $G(\lambda)$ gives the dependence of the cross section on the velocity v and has a rather sharp peak of 0.113 at $\lambda = 0.4$.

In the case of reaction (A.1), the initial term in the first approximation is horizontal, while the last is a hyperbola corresponding to the Coulomb repulsion of the reaction products, and therefore $R_X = (z-1)/(I_1 - I)$. For quasiresonance collisions $I_1 - I \ll I_1$ and, as can be seen from Eq. (A.4), the cross section at the maximum is of the order $z^2\pi a_0^2$. In order to find the location of the maximum, it is necessary to determine the value of the matrix element $V_{12}(R_X)$. Estimates show that a velocity $v \approx 10^7$ cm/sec for $z \approx 10$—13 corresponds to the maximum in the cross section for charge exchange to levels $n_1 \approx 4$—7. For lower levels $\lambda \gg 1$ and the charge-exchange cross section is exponentially small.

Thus, both the Brinkmann-Kramers approximation and the Landau-Zener model indicate preferential population of excited states with an effective cross section $\sigma \sim z^2\pi a_0^2$. Measurements of the total cross sections for charge exchange of doubly and triply charged ions of Ne, carried out by Flaks et al.[21,22] are consistent with this estimate. Finally, the possibility of preferential population of excited states in charge-exchange reactions has been confirmed experimentally[9,23].

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