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RELAXATION IN A TWO-TEMPERATURE PLASMA WITH DIRECTED ELECTRON MOTION

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Temperature and energy relaxation of the directed motion of particles in a two-temperature plasma with directed electron motion is investigated in the presence and absence of an external magnetic field. It is shown that the rate of variation of the particle energy in such a plasma may strongly differ from that for the particle energy in a plasma without directed electron motion.

1. Temperature relaxation in a plasma consisting of hot electrons and cold ions has been studied in sufficient detail both in the absence and in the presence of an external magnetic field (Spitzer[1], Ramazashvili, Rukhadze and Silin[2], Ichimaru and Rosenbluth[3]).

The literature that we are acquainted with fails, however, to throw any light on the question of temperature relaxation in such a plasma in the presence of directed electron motion. Meanwhile, the plasmas that we often have to deal with in the laboratory are precisely those whose electrons are in motion relative to the ions. The study of the effect of directed electron motion on temperature relaxation in a two-temperature plasma is the object of this paper.

We determine the variation (per unit time) of the total energy of the electron and ion components of the plasma $\dot{\mathscr{E}}_\alpha$ (α = e, i) and of the energy of the directed electron motion $\dot{\mathscr{E}}_D$ both in the absence and in the presence of an external magnetic field. It is shown that even when the velocity of the electron current $\mathbf{u}$ is small the quantities $\dot{\mathscr{E}}_\alpha$ can differ greatly from the corresponding quantities calculated for $u = 0$. The parameter that characterizes the effect of directed electron motion on the rate of energy transfer is $w/\Delta T$, where w is the energy of directed electron motion and ΔT is the difference between the electron and ion temperatures.

If we know the quantities $\dot{\mathscr{E}}_\alpha$ and $\dot{\mathscr{E}}_D$, we can estimate the temperature relaxation time and the slowing-down time of the directed particle motion in the case of a closed system (an example of such a system is a torus in which directed electron motion is produced along the axis at some instant of time). Analogously, in the case of a system in which the energy and directed velocity of the particles are maintained constant at one boundary, knowledge of these quantities makes it possible to estimate the "relaxation length," which is the distance from the boundary at which the relaxation occurs (an example of such a system is a long cylinder with a current of hot electrons injected through one of the bases).

2. We consider a plasma in which the electrons move relative to the ions with a mean velocity $\mathbf{u}$ directed along the constant homogeneous magnetic field $\mathbf{B}_0$. We start with the equations of motion

$$\frac{d\mathbf{v}}{dt}=\frac{e_\alpha}{m_\alpha}\mathbf{E}(\mathbf{r},t)+\frac{e_\alpha}{m_\alpha c}[\mathbf{v}\mathbf{B}_0],\quad \frac{d\mathbf{r}}{dt}=\mathbf{v},\tag{1}$$

where $\mathbf{E}$ is the fluctuating electric field, and e_α and m_α are the charge and mass of the particle of type α (α = e or i for electrons or ions, respectively). By solving these equations and averaging over the fluctuations we can obtain the Fokker-Planck coefficients that enter in the kinetic equations for the particles, namely the diffusion coefficient D_{ij} and the friction coefficient $\mathbf{F}$. Omitting the cumbersome general expressions for the coefficients D_{ij} and $\mathbf{F}$, we confine ourselves to the most interesting cases, those of weak ($\omega_\alpha \ll \Omega_\alpha$) and strong ($\omega_\alpha \gg \Omega_\alpha$) magnetic fields ($\omega_\alpha = |e_\alpha|B_0/m_\alpha c$ is the gyrofrequency and $\Omega_\alpha = (4\pi e_\alpha^2 nm_\alpha^{-1})^{1/2}$ is the plasma frequency of the particles type α; n is the particle density). For $\omega_\alpha \ll \Omega_\alpha$ we have

$$D_{ij}=\frac{e^2}{(2\pi)^2 m_e^2}\int d^3k\,d\omega\,\frac{k_ik_j}{k^2}\langle E^2\rangle_{\mathbf{k}\omega}\delta(\omega-\mathbf{kv}),\tag{2}$$

$$\mathbf{F}=\frac{e^2}{8\pi^2 m_e^2}\int d^3k\,d\omega\,\mathbf{k}\left[\frac{1}{2\pi}\frac{\partial}{\partial\omega}\langle E^2\rangle_{\mathbf{k}\omega}+\frac{4m}{k^2}\operatorname{Im}\frac{1}{\varepsilon}\right]\delta(\omega-\mathbf{kv}),\tag{3}$$

where $\langle E^2\rangle_{\mathbf{k}\omega}$ is the Fourier component of the electric-field correlator, $\epsilon = 1 + 4\pi(\kappa_e = \kappa_i)$ and κ_α is the plasma longitudinal dielectric constant and the longitudinal electric susceptibility of the α-th plasma component (e is the electron charge; $e_i = -e$). For $\omega_\alpha \gg \Omega_\alpha$ the Fokker-Planck coefficients are defined by formulas (2) and (3), in which one should make the substitutions $\delta(\omega - \mathbf{k}\cdot\mathbf{v}) \to \delta(\omega - k_\parallel v_\parallel)$ and regard κ and ϵ as the longitudinal components of the corresponding tensors in the magnetic field, for example $\kappa = k^{-2}k_ik_j\kappa_{ij}$ ($k_\parallel$ and $v_\parallel$ are the components of the vectors $\mathbf{k}$ and $\mathbf{v}$ along $\mathbf{B}_0$).

Since we are interested in the case of a plasma with a Maxwellian particle velocity distribution (characterized by the temperatures T_α and the directed electron velocity $\mathbf{u}$), we can use the following well-known expression for the electric field correlator (see, for example[4]):

$$\langle E^2\rangle_{\mathbf{k}\omega}=\frac{16\pi}{|\varepsilon|^2}\left[\frac{T_e}{\omega-\mathbf{ku}}\operatorname{Im}\varkappa_e+\frac{T_i}{\omega}\operatorname{Im}\varkappa_i\right].\tag{4}$$

If we know the Fokker-Planck coefficients $\hat{D}$ and $\mathbf{F}$, it is easy to determine the variation (per unit time) of the energy of the type-α particles:

$$\dot{\mathscr{E}}_e=-\dot{\mathscr{E}}_i=-\frac{2}{\pi^2 n}\int d^3k\,d\omega\,\frac{\operatorname{Im}\varkappa_i\operatorname{Im}\varkappa_e}{|\varepsilon|^2(\omega-\mathbf{ku})}[\omega(T_e-T_i)+\mathbf{ku}T_i].\tag{5}$$

In the case of directed electron motion it is natural to represent the quantity $\dot{\mathscr{E}}_e$ in the form

$$\dot{\mathscr{E}}_e=\dot{\mathscr{E}}_T+\dot{\mathscr{E}}_D,$$

where $\dot{\mathscr{E}}_T$ and $\dot{\mathscr{E}}_D$ are the energies of the random and directed electron motions. We then obtain for the quantity $\dot{\mathscr{E}}_D$

$$\dot{\mathscr{E}}_D = -\frac{2}{\pi^2 n}\int d^3k\, d\omega \frac{\operatorname{Im}\varkappa_i \operatorname{Im}\varkappa_e}{|\varepsilon|^2(\omega - \mathbf{ku})}\frac{\mathbf{ku}}{\omega}[\omega(T_e - T_i) + \mathbf{ku}T_i]. \quad (6)$$

3. When expressions (5) and (6) are integrated with respect to ω and k the main contribution comes from the region $k > a^{-1}$ ($a_\alpha = T_\alpha^{1/2}(4\pi e_\alpha n)^{-1/2}$ is the Debye radius for type-α particles). The upper limit of integration with respect to k must be chosen equal to r_{min}^{-1}, where r_{min}^{-1} is the minimal impact parameter or the de Broglie wavelength (see, for example,[5]).

In the case of a weak magnetic field ($\omega_e \ll \Omega_e$) we get for $\dot{\mathscr{E}}_\alpha$ and $\dot{\mathscr{E}}_D$

$$\dot{\mathscr{E}}_e = -\dot{\mathscr{E}}_i = -\frac{4\pi n e^4\Lambda}{m_i u}\left[\Phi\left(\sqrt{\frac{w}{T_e}}\right) - 2\sqrt{\frac{w}{\pi T_e}}\frac{T_i}{T_e}e^{-w/T_e}\right], \quad (7)$$

$$\dot{\mathscr{E}}_D = -\frac{4\pi n e^4\Lambda}{m_e u}\left[\Phi\left(\sqrt{\frac{w}{T_e}}\right) - 2\sqrt{\frac{w}{\pi T_e}}e^{-w/T_e}\right], \quad (8)$$

where $w = (1/2)m_e u^2$ is the energy of the directed electron motion and Λ is the Coulomb logarithm ($\Phi(x)$ is the error integral). These formulas are valid all the way to $W \sim T_e$. At large values of w the plasma ceases to be stable as a result of the buildup of electron Langmuir oscillations; in this case expression (4) in formulas (2) and (3) should be replaced by expressions for the correlator $\langle E^2\rangle_{\mathbf{k}\omega}$ in which allowance is made for turbulent fluctuations. In this paper we shall not deal with the variation of particle energy in a turbulent plasma.

In the case of low directed electron velocities ($w \ll T_e$) expressions (7) and (8) take the form

$$\dot{\mathscr{E}}_e = -\dot{\mathscr{E}}_i = -4\sqrt{2\pi}\frac{ne^4}{m_i}\sqrt{\frac{m_e}{T_e}}\Lambda\left[\frac{\Delta T}{T_e} - \frac{w}{3T}\left(1 - \frac{3T_i}{T_e}\right)\right], \quad (9)$$

$$\dot{\mathscr{E}}_D = -\frac{8}{3}\sqrt{2\pi}\frac{ne^4}{m_e}\sqrt{\frac{m_e}{T_e}}\Lambda\frac{w}{T_e}, \quad (10)$$

where $\Delta T = T_e - T_i$.

Of course, if u = 0 formulas (9) and (10) reduce to the well-known expressions for the rate of change of particle energy in the absence of directed electron motion (see[1,5]). It is, however, important that the relative contribution of the directed electron motion to the quantities $\dot{\mathscr{E}}_\alpha$ is proportional to $w/\Delta T$ (and not to w/T_α). Therefore, even when the directed electron velocities are low, as in the case of an almost isothermal plasma, this term can be significant. In particular, if $0 < \Delta T < \Delta T_0$, where $\Delta T_0 = (1/3)m_i u^2$, the energy of the random electron motion does not decrease (as in the case of u = 0), but increases. The energy of directed electron motion always decreases, and, in order of magnitude

$$\dot{\mathscr{E}}_D \sim -\frac{m_i}{m_e}\dot{\mathscr{E}}_i \min\left\{1; \frac{w}{\Delta T}\right\}.$$

In the case $w \sim T_e$ the contribution of the directed electron motion to the quantities $\dot{\mathscr{E}}_\alpha$ is significant for any ratio of electron and ion temperatures. Values of the ratio $R = \dot{\mathscr{E}}_\alpha/\dot{\mathscr{E}}_{\alpha 0}$ ($\dot{\mathscr{E}}_{\alpha 0} \equiv \dot{\mathscr{E}}_\alpha$ (w = 0)) at $w = T_e$ are shown below:

T_e/T_i:	1	1.1	2	4	10	∞
R:	∞	4.1	1.14	0.89	0.79	0.75

Note that when $w \sim T_e$ the energy of the random electron motion always increases.

4. In the case of a strong magnetic field ($\omega_i \gg \Omega_i$) the quantities $\dot{\mathscr{E}}_\alpha$ and $\dot{\mathscr{E}}_D$ are of the form

$$\dot{\mathscr{E}}_e = -\dot{\mathscr{E}}_i = -2\sqrt{2\pi}\frac{ne^4}{m_i}\sqrt{\frac{m_e}{T_e}}\Lambda e^{-w/T_e}\left[1 - \frac{T_i}{T_e}\left(1 - \frac{w}{T_e}\right)\right], \quad (11)$$

$$\dot{\mathscr{E}}_D = -4\sqrt{2\pi}\frac{ne^4}{m_e}\sqrt{\frac{m_e}{T_e}}\Lambda\frac{w}{T_e}e^{-w/T_e}. \quad (12)$$

When $w \ll T_e$ we obtain from (11) and (12)

$$\dot{\mathscr{E}}_e = -\dot{\mathscr{E}}_i = -2\sqrt{2\pi}\frac{ne^4}{m_i}\sqrt{\frac{m_e}{T_e}}\Lambda\left[\frac{\Delta T}{T_e} - \frac{w}{T_e}\left(1 - \frac{3T_i}{T_e}\right)\right], \quad (13)$$

$$\dot{\mathscr{E}}_D = -4\sqrt{2\pi}\frac{ne^4}{m_e}\sqrt{\frac{m_e}{T_e}}\Lambda\frac{w}{T_e}. \quad (14)$$

Expressions (11) to (14) have the same structure and the same order of magnitude as expressions (7) to (10) which describe the rate of change of particle energy in the absence of a magnetic field. The same conclusions about the relative contribution of the directed electron motion and about the direction of energy transfer are therefore valid in the case of a strong magnetic field as in the absence of a field. The energy of random electron motion increases (rather than decreases as in the case of u = 0) when $0 < \Delta T < \Delta T_0$, where $\Delta T_0 = m_i u^2$.

The values $R = \dot{\mathscr{E}}_\alpha/\dot{\mathscr{E}}_{\alpha 0}$ at $w = T_e$ in the case of the strong magnetic field, for different values of T_e/T_i, are:

T_e/T_i:	1	1.1	2	4	10	∞
R:	∞	4.1	0.74	0.49	0.41	0.37

We consider, finally, the case of intermediate magnetic fields when the electrons are strongly magnetized and the ions are weakly magnetized ($\omega_e \gg \Omega_e$, $\omega_i \ll \Omega_i$). From (5) and (6) we get

$$\dot{\mathscr{E}}_e = -\dot{\mathscr{E}}_i = -2\sqrt{2\pi}\frac{ne^4}{m_i}\sqrt{\frac{m_e}{T_e}}\Lambda e^{-w/T_e}\left[1 - \frac{T_i}{T_e}\left(1 - \frac{2w}{T_e}\right)\right]L, \quad L = \ln\frac{m_i T_e}{m_e T_i}, \quad (15)$$

$$\dot{\mathscr{E}}_D = -4\sqrt{2\pi}\frac{ne^4}{m_e}\sqrt{\frac{m_e}{T_e}}\Lambda\frac{w}{T_e}e^{-w/T_e}. \quad (16)$$

We call attention to the fact that in this case (as also in the absence of directed electron motion[5]) another large parameter L enters the expressions for $\dot{\mathscr{E}}_\alpha$ along with the Coulomb logarithm. Thus the losses of the energy of directed electron motion prove to be relatively less than in the cases of a weak or strong magnetic field.

When $w \ll T_e$ the quantity $\dot{\mathscr{E}}_D$ is determined by the same formula (14) as in the strong-field case. As to the quantities $\dot{\mathscr{E}}_\alpha$, the corresponding expressions are obtained from (13) with the substitution $\Lambda \to \Lambda L$. The temperature difference ΔT at which the energy of random electron motion begins to increase is in this case less than in the cases studied above, $T_0 = m_i u^2 L^{-1}$.

5. As already pointed out, when the quantities $\dot{\mathscr{E}}_\alpha$ and $\dot{\mathscr{E}}_D$ are calculated from the general expressions (5) and (6), the main contribution comes from the region $k > a_i^{-1}$. Physically, the contribution of this region corresponds to energy transfer by close (pair) collisions of particles. When calculating the rate of energy transfer through distant collisions one must take into account the contribution of the poles of the integrands in formulas (5) and (6) (corresponding to the energy transfer when collective plasma oscillations are excited and absorbed). The largest term turns out here to be the one that describes the interaction of particles with the ion (or magnetic) sound.

As shown in[2], in a plasma with electrons at rest the contribution of the ion sound to $\mathscr{E}_\alpha$ is, in general, insignificant. This contribution increases with increasing T_e/T_i, but even when $T_e/T_i \sim 100$ it amounts to only a few per cent. It is easily shown that an analogous situation holds in the case of a plasma with directed electron motion.

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