# EIKONAL APPROXIMATION IN THE PROBLEM OF ELECTRON MOTION IN THE FIELD OF IMPURITY CENTERS

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Explicit expressions are obtained for the one-particle and two-particle electron Green's functions in the field of randomly distributed impurity centers. The case is considered when the electron wavelength is much smaller than the characteristic dimensions of the potential of a single impurity.

## 1. SINGLE-PARTICLE GREEN'S FUNCTION

THE present paper deals with the motion of an electron in a field of randomly-distributed impurities, in the case when the electron wavelength is much shorter than the characteristic dimensions of the potential of one impurity. It turns out that the problem can be solved in this case in general form, and closed expressions can be obtained for the single-particle and two-particle Green's functions of the electron. The obtained expressions are the analog of the eikonal approximation in scattering theory, so that the present method of calculating the Green's function will also be called the eikonal approximation.

We start with a calculation of the single-particle Green's function of one electron $G(\mathbf{p}, \omega)$ in the field of randomly distributed potentials $U(\mathbf{r})$. We assume that $U(\mathbf{r})$ decreases exponentially at infinity and, of course, at zero. For example,

$$U(\mathbf{r}) = V_0 e^{-\varkappa r}. \tag{1}$$

The question of the applicability of the formulas obtained below to a screened Coulomb potential will be discussed separately at the end of this section. The fundamental small parameter of the present theory is the quantity

$$\alpha = \varkappa / p \ll 1, \tag{2}$$

where p is the electron momentum. To calculate $G(\mathbf{p}, \omega)$ we employ the usual diagram technique for the electron Green's function in the field of random impurities (see, e.g., [1,2]). The characteristic diagram is shown in Fig. 1. The straight line corresponds to the zeroth electron Green's function

$$G_0(\mathrm{p}, \omega) = (\omega - p^2/2m + i\delta)^{-1}, \tag{3}$$

the wavy line corresponds to the Fourier transform of the potential $U(\mathbf{q})$, and the thick point, from which bundles of wavy lines emerge, corresponds to a factor equal to the impurity concentration n.

We note for future reference that the Fourier transform of the potential $U(\mathbf{q})$ differs noticeably from zero in the region of small $q < \kappa$. For example, the Fourier transform of the potential (1) is equal to

$$U(\mathbf{q}) = 8\pi V_0 \varkappa / (q^2 + \varkappa^2)^2. \tag{4}$$

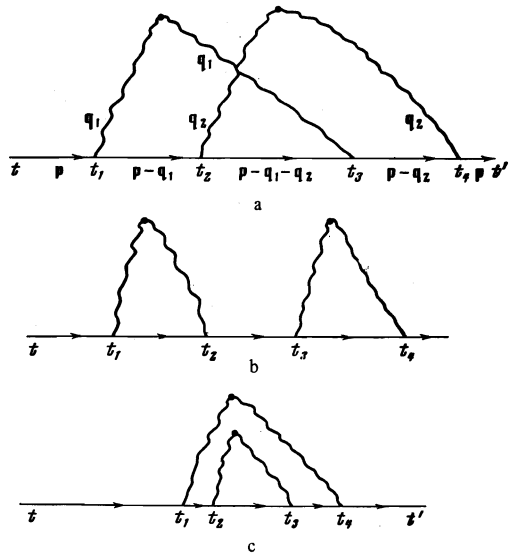


FIG. 1

It is therefore clear that when the condition (2) is satisfied we can neglect in expressions of the type

$$\int \frac{d\mathbf{q}}{(2\pi)^3} \frac{U^2(\mathbf{q})}{\omega - p^2/2m + \mathbf{pq}/m - q^2/2m + i\delta} \tag{5}$$

the term $q^2/2m$ in comparison with $p^2/2m$ and $\mathbf{p}\cdot\mathbf{q}/m$. It is quite obvious that a similar operation can be performed in all perturbation-theory diagrams, provided only that the condition (2) is satisfied. We assume this condition to be satisfied in the entire paper.

We now show how to sum the entire perturbation-theory series. We change over first to the time-dependent representation. In this representation, the diagram of Fig. 1a is equal to (with account of the fact that we have discarded all the terms of the type $\mathbf{q}_i \cdot \mathbf{q}_j$)

$$n^2 \int \frac{d\mathbf{q}_1\, d\mathbf{q}_2}{(2\pi)^6} U^2(\mathbf{q}_1) U^2(\mathbf{q}_2) (-i)^5 \int_0^t dt_1 \int_0^{t_1} dt_2 \int_0^{t_2} dt_3 \int_0^{t_3} dt_4 \exp\left\{ -i\frac{p^2}{2m} + i\frac{\mathbf{pq}_1}{m}(t_1 - t_2) + i\frac{\mathbf{p}(\mathbf{q}_1 + \mathbf{q}_2)}{m}(t_2 - t_3) + i\frac{\mathbf{pq}_2}{m}(t_3 - t_4) \right\}. \tag{6}$$

In the derivation of (6) we took into account the fact that

$$G_0(\mathbf{p}, t) = -i\vartheta(t) \exp(-ip^2 t / 2m) \tag{7}$$

and that the complete Green's function depends only on the time difference.

Let us examine in greater detail the expression in the exponential in (6). We see that most terms in the exponential were cancelled out; for example, from among the terms containing $\mathbf{p}\cdot\mathbf{q}_i$ we are left only with the terms

$$\mathbf{pq}_1(t_1-t_3)/m+\mathbf{pq}_2(t_2-t_4)/m.$$

We note now that $t_1$ and $t_3$ are the points of arrival and departure of the wavy line with momentum $\mathbf{q}_1$, while $t_2$ and $t_4$ are the corresponding points for $\mathbf{q}_2$. It is easy to prove that the same holds true also in more complicated diagrams. This makes it possible to integrate with respect to all $\mathbf{q}_i$. The diagram of Fig. 1a, e.g., is equal to

$$n(-i)^4 G_0(\mathbf{p},t)\int_0^t dt_1\int_0^{t_1} dt_2\int_0^{t_2} dt_3\int_0^{t_3} dt_4\,\rho_2\left(\frac{\mathbf{p}}{m}t_1,\frac{\mathbf{p}}{m}t_3\right)\rho_2\left(\frac{\mathbf{p}}{m}t_2,\frac{\mathbf{p}}{m}t_4\right), \quad (8)$$

where

$$\rho_k(\mathbf{r}_1\ldots\mathbf{r}_k)=\int d\mathbf{r}\,U(\mathbf{r}-\mathbf{r}_1)U(\mathbf{r}-\mathbf{r}_2)\ldots U(\mathbf{r}-\mathbf{r}_k). \quad (9)$$

We proceed now to summation of the diagrams. We sum first the diagrams corresponding to one bundle of wavy lines (see Fig. 2). Simple calculations yield the following contribution to the Green's function from one bundle:

$$\Delta G(\mathbf{p},t)=G_0(\mathbf{p},t)nL(\mathbf{p},t); \quad (10)$$

$$L(\mathbf{p},t)=\sum_{k=1}^{\infty}(-i)^k\int_0^t dt_1\ldots\int_0^{t_{k-1}} dt_k\,\rho_k\left(\frac{\mathbf{p}}{m}t_1\ldots\frac{\mathbf{p}}{m}t_k\right). \quad (11)$$

It is seen from (9) that $\rho_k$ is a symmetrical function of all its arguments. Taking this into account, expression (11) can be represented in the form

$$L(\mathbf{p},t)=\sum_{k=1}^{\infty}\frac{(-i)^k}{k!}\int_0^t dt_1\ldots\int_0^t dt_k\,\rho_k\left(\frac{\mathbf{p}}{m}t_1\ldots\frac{\mathbf{p}}{m}t_k\right)=\int d\mathbf{r}[e^{-i\xi(\mathbf{p},\mathbf{r},t)}- \quad (12)$$

$$\xi(\mathbf{p},\mathbf{r},t)=\int_0^t dt_1\,U\left(\mathbf{r}-\frac{\mathbf{p}}{m}t_1\right). \quad (13)$$

We now consider diagrams with two bundles, e.g., the diagrams shown in Fig. 1. The contribution of diagram 1a is written out in formula (8). The contribution of diagrams 1b and 1c differs from (8) only in the integration limits. Adding all three diagrams and taking into account the symmetry of $\rho_2$ with respect to its arguments, we obtain

$$\Delta G(\mathbf{p},t)=nG_0(\mathbf{p},t)\frac{(-i)^4}{8}\int d\mathbf{r}_1\,d\mathbf{r}_2\,\xi^2(\mathbf{p},\mathbf{r}_1,t)\xi^2(\mathbf{p},\mathbf{r}_2,t). \quad (14)$$

We have succeeded in summing the diagrams of Fig. 1 because, first, all the single-type diagrams (e.g., all the diagrams consisting of three bundles with definite numbers of lines in each bundle) differ only in the integration limits, and the integrands in them coincide; Second, all $\rho_k$ are symmetrical with respect to their arguments. It is easy to see that these properties are retained in all perturbation-theory diagrams. Using this circumstance, we obtain after straightforward calculations the following simple expression for the Green's function

$$G(\mathbf{p},t)=G_0(\mathbf{p},t)\exp\{nL(\mathbf{p},t)\},$$
$$G(\mathbf{p},\omega)=-i\int_0^{\infty}dt\exp\{i(\omega-p^2/2m)t+nL(\mathbf{p},t)\}. \quad (15)$$

If we introduce the function

$$\varphi(\mathbf{p},u)=\frac{1}{2\pi}\int_{-\infty}^{\infty}dt\exp\{iut+nL(\mathbf{p},t)\}, \quad (16)$$

then we obtain the following spectral representation for $G(\mathbf{p},\omega)$:

$$G(\mathbf{p},\omega)=\int_{-\infty}^{\infty}du\frac{\varphi(\mathbf{p},u)}{\omega-p^2/2m-u+i\delta}. \quad (17)$$

It is seen from (15), (12), and (13) that when $|\omega-p^2/2m|\gg p\kappa/m$ it is the small t that play an important role in (12), and then L(p, t) goes over into

$$L_0(t)=\int d\mathbf{r}[e^{-itu(r)}-1]. \quad (18)$$

On the other hand, it is well known that the function $\varphi_0(u)$ defined by formula (16) with L(p, t) replaced by $L_0(t)$ gives the distribution of the potentials from randomly distributed impurities with potential $U(\mathbf{r})$ and density n. The density of states

$$\rho(\omega)=-\frac{1}{\pi}\int\frac{d\mathbf{p}}{(2\pi)^3}\operatorname{Im}G(\mathbf{p},\omega), \quad (19)$$

determined from (17) is transformed into the well-known formula

$$\rho(\omega)=\int d\omega'\,\varphi_0(\omega-\omega')\rho_0(\omega'), \quad (20)$$

where $\rho_0(\omega)$ is the density of states in the absence of impurities, equal to

$$\rho_0(\omega)=2^{-1/2}\pi^{-2}m^{3/2}\omega^{1/2}\theta(\omega). \quad (21)$$

We note now that if we expand (12) in powers of $\xi$ up to $\xi^2$ inclusive and neglect the first term of the expansion, we obtain the formula

$$L(\mathbf{p},t)=-\frac{1}{2}\int_0^t dt_1\int_0^t dt_2\int d\mathbf{r}\,U\left(\mathbf{r}-\frac{\mathbf{p}}{m}t_1\right)U\left(\mathbf{r}-\frac{\mathbf{p}}{m}t_2\right), \quad (22)$$

which was investigated earlier (see[3,4]).

We consider now the Green's function at $|\omega-p^2/2m|\ll p\kappa/m$. This region corresponds to large times $pt\kappa/m\gg 1$ in formulas (12) and (13). We represent $L(\mathbf{pt})$ in the following form:

$$L(\mathbf{p},t)=\int d\mathbf{r}\int_0^t dt'\,U(\mathbf{r})\frac{\exp\{-i\xi(\mathbf{r}+\mathbf{p}t'/m,\mathbf{p},t)\}-1}{\xi(\mathbf{r}+\mathbf{p}t'/m,\mathbf{p},t)}; \quad (23)$$

inasmuch as the integration with respect to r extends over a region of order $\kappa^{-1}$, and $p\kappa t/m\gg 1$, we have approximately

$$\xi\left(\mathbf{r}+\frac{\mathbf{p}}{m}t',p,t\right)=\int_{-\infty}^{\infty}dt_1\,U\left(\mathbf{r}-\frac{\mathbf{p}}{m}t_1\right). \quad (24)$$

Introducing the notation

$$h(\rho)=\frac{m}{p}\int_{-\infty}^{\infty}dz\,U(\sqrt{\rho^2+z^2}), \quad (25)$$

we obtain

$$L(\mathbf{p},t)=\frac{pt}{m}\int d\rho[e^{-ih(\rho)}-1]. \quad (26)$$

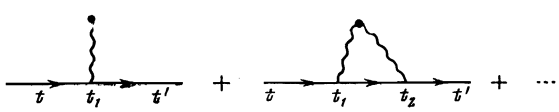


FIG. 2

The integral in (26) is the well-known eikonal expression for the forward scattering amplitude[5]. From (15) and (26) we get

$$G(\mathbf{p}, \omega) = [\omega - p^2/2m - \Sigma(\mathbf{p})]^{-1},$$
$$\Sigma(\mathbf{p}) = i\frac{np}{m}\int d\rho\,[e^{-ih(\rho)} - 1]. \quad (27)$$

Formula (27) is valid at $|\omega - p^2/2m| \ll p\kappa/m$. On the other hand, if $|\omega - p^2/2m| \sim p\kappa/m$, then $G(\mathbf{p}, \omega)$ must be calculated by numerical integration.

We note that cases are frequently encountered when there are two types of scatterers. If condition (2) is then satisfied for both types of scatterers, then we can readily see that

$$G(\mathbf{p}, t) = G_0(\mathbf{p}, t)\exp\{n_1 L_1(p, t) + n_2 L_2(p, t)\}, \quad (28)$$

where $L_i(\mathbf{p}, t)$ are determined by formulas (12) and (13) for different types of scatterers, and $n_i$ are their concentrations.

We now discuss the question of the applicability of the obtained formulas for a screened Coulomb potential. We note first that our method of diagram summation is not applicable directly in this case, since the diagrams containing bundles with many lines do not decrease rapidly enough at large q, this being a consequence of the singularity at $r \to 0$. However, as is easily seen from (12), (18) and (26), the integrals in the final expression for $L(\mathbf{p}, t)$ are determined by the regions of large $r \sim 1/\kappa$, since $L(\mathbf{p}, t) \to \infty$ as $\kappa \to 0$. For example, Eq. (26) takes the form

$$L(\mathbf{p}, t) = \frac{2\pi t p}{m}\int_0^\infty \rho\, d\rho\left\{\exp\left[-\frac{2ip_0}{p}K_0(\varkappa\rho)\right] - 1\right\}, \quad (29)$$

$$U(\mathbf{r}) = \frac{p_0}{mr}e^{-\varkappa r}$$

($p_0$ is the reciprocal Bohr radius, and we assume that $p_0 \gg \kappa$), while formula (18) becomes

$$L_0(t) = \int d\mathbf{r}\left\{\exp\left[-i\frac{p_0 t}{mr}e^{-\varkappa r}\right] - 1\right\}. \quad (29a)$$

It is seen from the formulas in (29) that the region of small r does not make an appreciable contribution to the integrals, but the exponential can still not be expanded in a series, since, e.g. in (29a), the third term of the expansion already becomes infinite. Thus, our difficulties are connected only with the divergence of the perturbation-theory series at short distances. Since we used in all our arguments only the fact that the integrals are in fact determined by the region of large $r \sim \kappa^{-1}$, it follows that our formulas can be applied also to a screened Coulomb potential, if, of course, the condition (2) is satisfied. We note that all these arguments are valid only for a continuous spectrum, while in the region of negative energies they are generally speaking incorrect.

We note in conclusion of this section, that, since the energy $\omega$ in our diagrams is a parameter, all the results are valid also for Matsubara Green's functions. To this end it is necessary only to replace $\omega$ by $i\omega_n$ in (17).

## 2. TWO-PARTICLE GREEN'S FUNCTION

We now consider the two-particle Green's function $K(\mathbf{p}, \mathbf{p}', \omega, \omega')$, which depends on two energies and two momenta. The determination of this function is clear

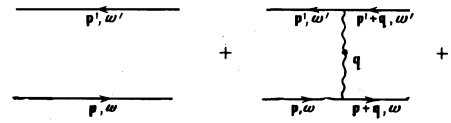


FIG. 3

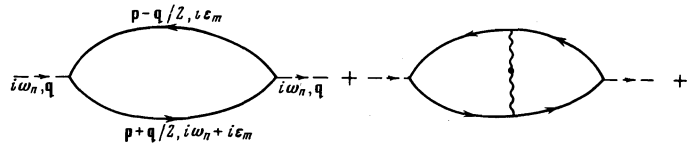


FIG. 4

from the simplest diagrams shown in Fig. 3. Integration is carried out here over all the $\mathbf{q}_i$. We assume that $\omega$ and $\omega'$ are arbitrary complex quantities. For concreteness, we put $\mathrm{Im}\,\omega > 0$ and $\mathrm{Im}\,\omega' < 0$. This means that in the t-representation the upper electron line corresponds to advanced Green's functions

$$G(\mathbf{p}', t') = i\vartheta(-t')\exp(-ip'^2t'/2m), \quad (30)$$

and the lower to retarded functions (see (7)).

Going through exactly the same steps as in the derivation of (17) for the single-particle Green's functions, we easily obtain the following expression for K

$$K(\mathbf{p}, \mathbf{p}', \omega, \omega') = \int_{-\infty}^{\infty} du \int_{-\infty}^{\infty} du' \frac{\varphi_1(u, u', \mathbf{p}, \mathbf{p}')}{(\omega - p^2/2m - u)(\omega' - p'^2/2m - u')}, \quad (31)$$

$$\varphi_1(u, u', \mathbf{p}, \mathbf{p}') = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \exp\{iut + iu't' + nL_1(t, t', \mathbf{p}, \mathbf{p}')\} \quad (32)$$

$$L_1(t, t', \mathbf{p}, \mathbf{p}') = \int d\mathbf{r}\,\{\exp[-i\xi(\mathbf{p}, \mathbf{r}, t) + i\xi(\mathbf{p}', \mathbf{r}, -t')] - 1\}. \quad (33)$$

It is easy to show that formulas (31)—(33) are valid for all complex $\omega$ and $\omega'$.

We now proceed to a concrete consideration of a retarded two-particle electron Green's function $K^R(\mathbf{q}, \omega)$, which is the Fourier transform of the two-particle electron Green's function of coinciding arguments. $K^R(\mathbf{q}, \omega)$ is an analytic continuation of the Matsubara function $K(\mathbf{q}, i\omega_n)$, the simplest diagrams for which are shown in Fig. 4. It is quite clear that it is precisely such a function which enters, e.g., in the expression for the conductivity or determines the polarization operator in the phonon Green's function (the dashed line in Fig. 4 represents the phonon Green's function). It is clear from the diagrams of Fig. 4 that

$$K(\mathbf{q}, i\omega_n) = -T\sum_{\varepsilon_m}\int \frac{d\mathbf{p}}{(2\pi)^3}K\left(\mathbf{p} + \frac{\mathbf{q}}{2}, \mathbf{p} - \frac{\mathbf{q}}{2}, i\omega_n + i\varepsilon_m, i\varepsilon_m\right). \quad (34)$$

Substituting (31) in (34) and changing the variables in $\varphi_1$, we obtain

$$K^R(\mathbf{q}, \omega) = \int \frac{d\mathbf{p}}{(2\pi)^3}\int_{-\infty}^{\infty} dv \int_{-\infty}^{\infty} dw\, \varphi_2(w, v, \mathbf{p}, \mathbf{q})$$
$$\times \frac{n(\xi_{\mathbf{p}+\mathbf{q}/2} + w + v/2) - n(\xi_{\mathbf{p}-\mathbf{q}/2} + w - v/2)}{\omega - \mathbf{pq}/m - v + i\delta}, \quad (35)$$

$$\varphi_2(w, v, \mathbf{p}, \mathbf{q}) = \varphi_1(w + v/2, w - v/2, \mathbf{p} + \mathbf{q}/2, \mathbf{p} - \mathbf{q}/2), \quad (36)$$

where $\xi_{\mathbf{p}} = p^2/2m - \mu$, $n(\omega)$ is the Fermi distribution function, and $\mu$ is the chemical potential. It is easily seen that

$$\varphi_2(w, v, \mathbf{p}, \mathbf{q}) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} dt \exp\{iw\tau + ivt + nL_2(\tau, t, \mathbf{p}, \mathbf{q})\}, \quad (37)$$

$$L_2(\tau, t, \mathbf{p}, \mathbf{q}) = L_1(\tau/2 + t, \tau/2 - t, \mathbf{p} + \mathbf{q}/2, \mathbf{p} - \mathbf{q}/2). \quad (38)$$

We note first that inasmuch as we have assumed throughout in the derivation of (33) that $p, p' \gg \kappa$ and, as will be seen subsequently, our answer will contain the momentum $\mathbf{q} = \mathbf{p} - \mathbf{p}'$, this means that (35) is valid only when

$$q \gg \varkappa. \quad (39)$$

We now consider the physical meaning of the result. We introduce to this end the function $T(\mathbf{r}, t)$, which is the Fourier transform of the function

$$T(\mathbf{q}, \omega) = \frac{K(\mathbf{q}, \omega) - K(\mathbf{q}, 0)}{i\omega}. \quad (40)$$

As we shall see below, the function $T(\mathbf{r}, t)$ determines the probability that an electron will move through a distance $\mathbf{r}$ after a time t. We consider $T(\mathbf{r}, t)$ in the classical limit, which is obtained in our formulas at $q \ll p$ and $\tau \ll t$, since $\tau$ is connected with the energy and t with the classical frequency, q is a quantity inverse to the distance, and $\mathbf{p}$ is the momentum. Expanding $L_2$ in terms of $\tau$ and $\mathbf{q}$, and $n(\xi_{\mathbf{p} \pm \mathbf{q}/2} + w \pm v/2)$ in $\mathbf{p} \cdot \mathbf{q}/2m + v/2$, we obtain at low temperatures, after simple transformations

$$T(\mathbf{r}, t) = \frac{\vartheta(t)}{2\pi} \int \frac{d\mathbf{p}\, d\mathbf{q}}{(2\pi)^6} \int_{-\infty}^{\infty} d\tau \times \exp\left\{ iq\left(\mathbf{r} - \frac{\mathbf{p}}{m} t\right) + i\left(\mu - \frac{p^2}{2m}\right)\tau + nL_{20}(\tau, t, \mathbf{p}, \mathbf{q}) \right\}, \quad (41)$$

$$L_{20}(\tau, t, \mathbf{p}, \mathbf{q}) = \int d\mathbf{r}\{\exp[-i\zeta(\tau, t, \mathbf{p}, \mathbf{q}, \mathbf{r})] - 1\}, \quad (42)$$

$$\zeta(\tau, t, \mathbf{p}, \mathbf{q}, \mathbf{r}) = \tau U(\mathbf{r}) + \frac{\mathbf{q}}{m} \int_0^t dt_1 (t - t_1) \mathbf{F}\left(\mathbf{r} + \frac{\mathbf{p}}{m} t_1\right),$$

where

$$\mathbf{F}(\mathbf{r}) = -\nabla U(\mathbf{r}). \quad (43)$$

Let now $pt\kappa/m \ll 1$. We can then neglect $\mathbf{p}t_1/m$ in comparison with $\mathbf{r}$ in the expression for $\zeta$, and we obtain

$$\zeta(\tau, t, \mathbf{p}, \mathbf{q}, \mathbf{r}) = \tau U(\mathbf{r}) + \mathbf{q}\mathbf{F}(\mathbf{r}) t^2/2m. \quad (44)$$

Then $T(\mathbf{r}, t)$ is given by

$$T(\mathbf{r}, t) = \vartheta(t) \int \frac{d\mathbf{p}}{(2\pi)^3} \int_{-\infty}^{\infty} du \int d\mathbf{F} \delta\left(\mu - \frac{p^2}{2m} - u\right) \times \delta\left(\mathbf{r} - \frac{\mathbf{p}}{m} t - \frac{\mathbf{F}t^2}{2m}\right) \Phi(u, \mathbf{F}), \quad (45)$$

where $\Phi(u, \mathbf{F})$ is the distribution function with respect to the potentials and forces in the field of randomly distributed impurities:

$$\Phi(u, \mathbf{F}) = \frac{1}{(2\pi)^4} \int_{-\infty}^{\infty} d\tau \int d\rho \exp\{iu\tau + i\rho\mathbf{F} + nL_3(\tau, \rho)\}, \quad (46)$$

$$L_3(\tau, \rho) = \int d\mathbf{r}\{\exp[-i\zeta_0(\tau, \rho, \mathbf{r})] - 1\},$$

$$\zeta_0(\tau, \rho, \mathbf{r}) = \tau U(\mathbf{r}) + \rho \mathbf{F}(\mathbf{r}).$$

The meaning of expression (45) is perfectly clear: It gives the probability that a particle situated at the initial instant of time at some point with potential u will be moved by a force $\mathbf{F}$, after a short time (during which the potential and force remain essentially unchanged), through a distance $\mathbf{r}$ averaged over the distribution of u and $\mathbf{F}$. The initial expression for $T(\mathbf{r}, t)$ also determines a probability of this kind, but under more general conditions, and in particular, the quantum-mechanical uncertainty is taken into account.

In conclusion we consider by way of an application the imaginary part of $K^R(\mathbf{q}, \omega)$, which determines, e.g., the phonon damping. We shall assume for simplicity a zero temperature and that the classical description is applicable, i.e., the phonon energy is much lower than the characteristic energies of the electron. In this case we easily obtain from (35)

$$\operatorname{Im} K^R(\mathbf{q}, \omega) = \frac{\omega}{\pi} \int \frac{d\mathbf{p}}{(2\pi)^3} \varphi_2\left(\mu - \frac{p^2}{2m}, \omega - \frac{\mathbf{pq}}{m}, \mathbf{p}, \mathbf{q}\right). \quad (47)$$

Since $\mathbf{q} \gg \kappa$, it follows that $p\kappa/m \ll |\omega - \mathbf{pq}/m|$. In this case we can use formula (44). We assume for simplicity that the number of impurities is large, meaning that $L_{20}$ in (42) must be expanded in powers of $\zeta$. Recognizing that the integrals of $\mathbf{F}(\mathbf{r})$ and $U(\mathbf{r})\mathbf{F}(\mathbf{r})$ with respect to $\mathbf{r}$ are equal to zero, we obtain, shifting the energy origin by a constant quantity

$$\varphi_2(w, v, \mathbf{p}, \mathbf{q}) = \varphi_0(w)\varphi_1(v, \mathbf{q}), \quad \varphi_0(w) = (2\pi\rho_0)^{-1/2} \exp\{-w^2/2\rho_0\}, \quad (48)$$

$$\varphi_1(v, \mathbf{q}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dt \exp\left\{ivt - \frac{\rho_1 q^2 t^4}{24m^2}\right\}; \quad (49)$$

$$\rho_0 = n \int d\mathbf{r}\, U^2(\mathbf{r}), \quad \rho_1 = n \int d\mathbf{r}\, F^2(\mathbf{r}).$$

Then

$$\operatorname{Im} K^R(\mathbf{q}, \omega) = \frac{\omega}{\pi} \int \frac{d\mathbf{p}}{(2\pi)^3} \varphi_0\left(\mu - \frac{p^2}{2m}\right) \varphi_1\left(\omega - \frac{\mathbf{pq}}{m}, \mathbf{q}\right). \quad (50)$$

If the momentum transfers are small, i.e. $|\omega| \gg |\mathbf{p} \cdot \mathbf{q}/m|$, then

$$\operatorname{Im} K^R(\mathbf{q}, \omega) = \omega\pi^{-1}\rho(\mu)\varphi_1(\omega, \mathbf{q}), \quad (51)$$

where $\rho(\mu)$ is the density of states on the Fermi surface. Formula (51) is simple enough for practical utilization. On the other hand, in more complicated cases a numerical calculation with the aid of the corresponding formulas is necessary, using (35) in the most general case and particular cases of this formula in simpler situations (e.g., formula (50) can be used at high impurity density and low phonon energy.

We note also the following. The condition (39) seems to impose significant limitations on the use of the present results to calculate the conductivity, since the wave vector is always very small. It can be shown, however, that in the case of high-frequency conductivity, when the electron can advance within the period $\omega^{-1}$ only a distance much shorter than the radius of the potential, i.e., when the condition $p_0\kappa/m\omega \ll 1$ is satisfied, where $p_0$ is the characteristic momentum of the electron, our formulas can also be used to calculate the conductivity, by setting, naturally, $\mathbf{q}$ equal to zero in these formulas.

We now consider the question of physical applications of the results of the article. Condition (2) is satisfied in many cases. Foremost among them are doped semiconductors in which, as is well known, the electrons behave like classical particles, since the Debye screening radius is many times larger than the wavelength. Another possibly more interesting application of the present theory concerns the motion of the electron in the field of critical fluctuations. We assume that a certain phase transition takes place in a substance in which there are free electrons; e.g., we have

a binary alloy of two metals, which can become ordered. Then large-scale critical fluctuations set in nea the ordering temperature. The parameter $\alpha$ in (2) can then be of the order of $10^{-3}$.

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