Effect of oscillating interaction between magnetic impurities on the properties of dilute magnetic alloys

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We investigate the properties of dilute ferromagnetic alloys in which the exchange interaction of the impurity spins consist of a positive part that decreases exponentially with distance, and a part of the Kittel-Ruderman type, which oscillates with the distance. The oscillating interaction greatly decreases the molecular field at impurities that are located farther from the neighbors than the average distance between neighbors. Impurities located far away can be acted upon by a molecular field with direction opposite to that of the magnetization. This increases the contribution of the local spin flips to the thermodynamic and kinetic quantities, and their decrease with temperature becomes slower in comparison with the case of pure ferromagnetic interaction. In the limit of very low impurity concentrations, we obtain the number of impurities whose spins are directed opposite to the magnetization at T = 0, and establish a criterion for the existence of ferromagnetism.

1. INTRODUCTION

Paramagnetic metals with strong correlation of the electrons in a narrow d-band (pd, pt) exhibit ferromagnetism at low concentrations n of the magnetic impurities[1]. The ferromagnetism is due to indirect exchange interaction between the impurities, which is effected by the d-electrons in these metals and is described by the Heisenberg Hamiltonian

$$\mathcal{H} = -{}^1/_2 \sum_{ij} V(\mathbf{r}_i - \mathbf{r}_j)\mathbf{S}_i\mathbf{S}_j, \qquad (1)$$

where $\mathbf{S}_i$ is the spin localized at the point $\mathbf{r}_i$. The interaction energy V(r) in (1) at not very large distances is positive and decreases exponentially with r, and at large distances V(r) is determined by the Kittel-Ruderman formula[2,3]:

$$V(r) = V_0 e^{-r/R} + V_1 R^3 r^{-3} \cos 2k_F r, \quad r \gg R. \qquad (2)$$

In order of magnitude we have $V_1/V_0 = (k_F R)^{-3} \ll 1$. In expression (2) we have left out for simplicity the factor r^{-1} in the first term. At $r \gg R$ this simplification changes little the dependence of the ferromagnetic part of the interaction on r.

In an earlier study[4] of the properties of dilute ferromagnetic alloys, for which $\nu = (^4/_3)\pi R^3 n \ll 1$, we have neglected the oscillatory interaction. It was shown that the thermodynamic and kinetic properties of such alloys are strongly influenced by the spin flip of those impurities on which the weak molecular field acts. These impurities are separated from the neighbors by distances that are larger than the average distance between the impurities $r_{av} \approx n^{-1/3}$.

The oscillating potential at the remote impurities may become comparable with the positive potential, or may even exceed it. Consequently, at $T \ll T_C$ the molecular field at these impurities is weak or negative (directed opposite to the magnetization). It is precisely these impurities which play an important role at very low temperatures. Local spin flips of such impurities introduce contributions proportional to W(0)T into the magnetic specific heat and into the kinetic coefficients, and a contribution proportional to $(\partial W/\partial H)_{H=0}T^2$, into the temperature-dependent part of the magnetization, where W(H) is the distribution function of the molecular field H.

In the present paper we calculated W(0) in the limit of very low impurity concentrations and obtained the number of spins $n_\downarrow$ directed against the magnetization at T = 0, under the condition $n_\downarrow \ll n$. This inequality means that the alloy is ferromagnetic. The character of the magnetic ordering is determined in this case mainly by the positive part of the exchange interaction between the impurities. With decreasing impurity concentration, W(0) and $n_\downarrow$ increase, i.e., the role of the oscillating Kittel-Ruderman interaction increases. At sufficiently low impurity concentrations the oscillating interaction becomes predominant. As is well known, the character of the magnetic ordering in such a situation has not yet been determined[5]. It is most probable that the Kittel-Ruderman interaction leads to an ordering of the antiferromagnetic type. We shall therefore speak henceforth of vanishing of the ferromagnetism for those impurity concentrations, in which the principal role is assumed by the oscillating interaction. It turns out that the ferromagnetism can exist also at concentrations such that the amplitude of the oscillating potential exceeds the positive potential at distances on the order of r_{av}. At not too low temperatures, the most important are the impurities at which the molecular field is positive. The oscillating potential decreases the molecular field at such impurities. This leads to an increased contribution of the local flips to the thermodynamic and kinetic quantities in comparison with the pure ferromagnetic interaction.

The influence of the oscillating potential can be neglected at sufficiently high temperatures satisfying the inequality $T \gg {}^1/_2 S V_1 \nu^{1/2} \ln^{-3/2}(SV_0/T)$. In this case the results of[4] are valid.

2. STRONGLY DILUTE ALLOYS AT $T \to 0$

The average spin σ per impurity and the specific heat C_M per unit volume can be obtained by averaging the expressions for these quantities in the given field H over the distribution of the molecular field[4]:

$$\delta\sigma \equiv S-\sigma = S-\int_{-\infty}^{\infty} W(H)\left[\left(S+\frac{1}{2}\right)\operatorname{cth}\left(S+\frac{1}{2}\right)\beta H-\frac{1}{2}\operatorname{cth}\frac{\beta H}{2}\right]dH, \tag{3}$$

$$C_{\text{M}} = n\int_{-\infty}^{\infty} W(H)(\beta H)^2\left[\frac{1}{4}\operatorname{sh}^{-2}\frac{\beta H}{2}-\left(S+\frac{1}{2}\right)^2\operatorname{sh}^{-2}\left(S+\frac{1}{2}\right)\beta H\right]dH, \tag{4}$$

where $\beta = T^{-1}$, the temperature is measured in energy units; to simplify the notation, we shall henceforth write H in place of $\mu_e H$, where μ_e is the effective magneton.

Owing to the oscillating potential, the alloy contains impurities at which the molecular field is close to zero at low temperatures, i.e., $W(0) \neq 0$. Therefore as $T \to 0$ we obtain from (3) and (4)

$$\delta\sigma = 2S\frac{n_\downarrow}{n}+T^2\left(\frac{dW}{dH}\right)_{H=0}\int_0^\infty dx\,x^2\left[\frac{1}{4}\operatorname{sh}^{-2}\frac{x}{2}-\left(S+\frac{1}{2}\right)^2\operatorname{sh}^{-2}\left(S+\frac{1}{2}\right)x\right] = 2S\frac{n_\downarrow}{n}+\frac{2\pi^2 S}{3(2S+1)}\left(\frac{dW}{dH}\right)_{H=0}T^2, \tag{5}$$

$$C_{\text{M}} = n\frac{2\pi^2 S}{3(2S+1)}W(0)T. \tag{6}$$

Here

$$n_\downarrow = n\int_{-\infty}^{0} W(H)\,dH \tag{7}$$

is the number of spins directed opposite to the magnetization at T = 0.

To find the concentration dependence of C_M and $n_\downarrow$ it is necessary to know W(H) at $H \lesssim 0$. Small and negative fields, as already noted, are produced at remote impurities. If the distance is of the order of a certain r, then the probability of such fluctuations is obviously proportional to $\exp(-4\pi n r^3/3)$. It is therefore desirable to separate indirectly this factor in explicit form in the distribution function W(H). It is shown in the Appendix that the function W(H) can be represented in the form

$$W(H) = \int_0^\infty dt\,w(t)\,W(H,t). \tag{8}$$

Here t = r/R, w is the nearest-neighbor distribution function:

$$w(t) = 3\nu t^2 e^{-\nu t^3}; \tag{9}$$

$$W(H,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\rho\, e^{-i\rho H+i\rho V(t)S-\nu D(\rho,t)}, \tag{10}$$

where

$$D(\rho,t) = 3\int_t^\infty \left(1-e^{i\rho V(t_1)S}\right)t_1^2\,dt_1, \tag{11}$$

W(H, t) has the meaning of the distribution function of the molecular fields at impurities located at a distance t away from the nearest neighbor.

We shall henceforth consider in this section the case of extremely low concentrations, when $\nu t^2 \ll 1$ at the characteristic values of t. We can then expand the exponential in (11) in powers of ρ, up to terms proportional to ρ^2 (the validity of this expansion will be demonstrated below):

$$D(\rho,t) = 3\int_t^\infty \left\{-i\rho\left(SV_0e^{-t_1}+\frac{V_1S}{t_1^3}\cos 2k_FRt_1\right)+\frac{\rho^2}{2}\left(V_0Se^{-t_1}+\frac{V_1S}{t_1^3}\cos 2k_FRt_1\right)^2 t_1^2\right\}dt_1,$$

so that the function W(H, t) has a Gaussian form.

The $\nu(D(\rho, t)$ terms linear in ρ are smaller than the $\rho V(t)$ term in the argument of the exponential of the function W(H, t) by a factor νt^2. We shall show that near the maximum of the Gaussian function we have $\rho V(t) \ll 1$. The $D(\rho, t)$ terms linear in ρ can therefore be discarded. Owing to the fast oscillations of $\cos 2k_FRt$ at large t we can discard the member containing $\cos 2k_FRt$ in that term of the function $D(\rho, t)$ which is proportional to ρ^2, and replace $\cos^2 2k_FRt$ by ½.

Thus,

$$W(H,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\rho\,\exp\left\{-i\rho\left[H-SV_0e^{-t}-\frac{SV_1}{t^3}\cos 2k_FRt\right]-\frac{\nu\rho^2S^2}{4}\left(\frac{V_1^2}{t^3}+3V_0^2t^2e^{-2t}\right)\right\}. \tag{12}$$

Since we are interested in distances $t \gg (k_FR)^{-1}$ and $k_FR \gg 1$, we can average over the argument of the cosine in (12) at a given value of t[6]. The factor $[i\rho S(V_0e^{-t}+V_1t^{-3}\cos 2k_FRt)]$ under the integral sign in (12) is then replaced by $\exp(i\rho SV_0e^{-t})J_0(x)$, where $J_0(x)$ is a Bessel function and $x = \rho V_1 S/t^3$.

The characteristic values of ρ in the integral (12) are determined by the terms quadratic in ρ in the argument of the exponential. For these values of ρ, we have $x \simeq (\nu t^3)^{-1/2} \ll 1$, so that $J_0(x) = 1 + O(x^2)$. Integrating now with respect to ρ in (12) and using (8) and (9), we obtain ultimately for the function W(H):

$$W(H) = \frac{3}{S}\sqrt{\frac{\nu}{\pi}}\int_0^\infty dt\frac{t^{7/2}}{(V_1^2+3V_0^2t^5e^{-2t})^{1/2}}\times\exp\left\{-\frac{(H-V_0Se^{-t})^2}{\nu(V_1^2+3V_0^2t^5e^{-2t})S^2}t^3-\nu t^3\right\}. \tag{13}$$

We see that the alternating-sign potential does not change the average value of the molecular field at a given t, but only broadens the distribution.

We consider first W(0). The argument of the exponential of the integrand in (13) has a maximum when t_0 satisfies the relations

$$t_0e^{-2t_0} = {}^3/_2\nu^2V_1^2\,/\,V_0^2,\quad \nu t_0^2 \ll 1. \tag{14}$$

Near the maximum, the inequality $V_0^2t^5e^{-2t} \ll V_1^2$ is satisfied, and the Gaussian part of the argument of the exponential in (13) is of the order of $\nu t_0^2 \ll 1$, thus justifying the expansion of $D(\rho, t)$ in powers of ρ.

At $t < t_0$, the argument of the exponential in (13) decreases exponentially, and at $t > t_0$ it decreases in power-law fashion. Therefore the lower limit in (13) can be replaced by t_0, and we need retain only $-\nu t^3$ in the argument of the exponential. ,After integration we obtain

$$W(0) = \frac{t_0^{3/2}}{SV_1\sqrt{\pi\nu}}e^{-\nu t_0^3}. \tag{15}$$

Expression (15) together with (14) enables us to determine the concentration dependence of the thermodynamic and kinetic quantities at low temperatures, at concentrations satisfying the inequalities $\nu t_0^2 \ll 1$ and $\nu t_0^3 \gg 1$. It follows from (14) that t_0 decreases with increasing ν. Consequently, W(0) and the specific heat C_M decrease with increasing ν more slowly than exponentially.

We can analyze in similar fasion W(H) at small H. If $|H| < \nu V_1 S t_0^{-1/2}$, then the extremum point is determined by the same equation (14); the correction to t_0, due to H, is much smaller than unity. If $H > \nu V_1 S t_0^{-1/2}$, then $t_0 = \ln(SV_0/H)$ and W(H) does not depend on V_1.1)

Let us calculate the quantity η_1 defined in (7). As seen from (13), a contribution to the integral (7) is made by the values $|H| < SV_1\nu^{1/2}t^{-3/2}$. In the interval

$$\nu V_1 S t^{-1/2} < |H| < \nu^{1/2} V_1 S t^{-3/2}$$

the extremal point t(H) is determined from the equation

$$t(H)e^{-t(H)} = {}^3/_2\nu^2 V_1{}^2 S / V_0|H|, \tag{16}$$

and $\nu[t^3(H) - t_0^3] \ll 1$. We obtain for $n_\downarrow$ from (7), (13), and (16)

$$n_\downarrow = {}^1/_2 n e^{-\nu t_0{}^3}.$$

If $n_\downarrow \ll n$ at $T = 0$, then the alloy is ferromagnetic. Thus, a sufficient condition for the existence of ferromagnetism in an alloy with a positive and oscillating interaction is the inequality $\nu t_0^3 \ll 1$, where t_0 is determined by (14). At $\nu t_0^3 \approx 1$ the ferromagnetism vanishes. It follows from (14) that

$$\nu t_0{}^3 = \frac{V_0 e^{-t_0}}{V_1 t_0{}^{-3}} \sqrt{\frac{3}{2} t_0}. \tag{17}$$

Since $t_0^{1/2} \gg 1$, the inequality $\nu t_0^3 \gg 1$ can be satisfied even when $V_0 e^{-t_0} \lesssim V_1 t_0^{-3}$, i.e., ferromagnetism is possible in the alloys in question also when the amplitude of the oscillating potential at the average distance is larger than the positive potential. Putting $t_0^3 \approx \nu^{-1}$ in (17), we can estimate the critical concentration ν_0 for the vanishing of ferromagnetism. The quantity ν_0 depends little on V_1/V_0. For $V_1/V_0 \approx 0.2$—0.02 we have $\nu_0 \approx 10^{-3}$, which according to[4] corresponds to an impurity concentration $c \approx 10^{-2}\%$.

3. CONTRIBUTION OF IMPURITIES SITUATED IN A POSITIVE MOLECULAR FIELD

In this section we consider again relatively high temperatures (a criterion will be estimated below), when the main contribution to (3) and (4) is made by positive molecular fields H > T. Then

$$\delta\sigma = \int_{-\infty}^{\infty} e^{-\beta H} W(H)\,dH, \tag{18}$$

$$C_M = \int_{-\infty}^{\infty} (\beta H)^2 e^{-\beta H} W(H)\,dH. \tag{19}$$

It is now convenient for us to use for the function W(H) an expression analogous to[4]

$$W(H) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\rho\, e^{-i\rho H - D(\rho)},$$
$$D(\rho) = 3\nu \int_{d/R}^{\infty} dt\, t^2 \{1 - e^{i\rho V(t) S}\}, \tag{20}$$

where V(t) is determined by formula (2) and d is the lattice constant. We have introduced in the integral of (20) a cutoff radius, to avoid unphysical divergences connected with the oscillating potential. The cutoff radius does not enter in the final results. Just as in the preceding section, we can average in (20) over the argument of the cosine. We obtain

$$D(\rho) = 3\nu \int_{d/R}^{\infty} dt t^2 \left\{ 1 - J_0\left(\frac{\rho V_1 S}{t^3}\right) e^{i\rho S V_0 e^{-t}} \right\}. \tag{21}$$

Without loss of generality we can assume $W(H) = 0$ at $H \leq -H_0$, and H_0 is larger than all the characteristic energies in the problem. We can then transform (18) into

$$\delta\sigma = \int_0^{\infty} W_1(H) e^{-\beta H}\, dH,$$

where $W_1(H) = W(H + H_0)e^{-\beta H_0}$, and $W_1(H) = 0$ at $H \leq 0$. Using the connection between the Fourier transform of $W_1(H)$ and its Laplace transform, we obtain in analogy with[4]:

$$\delta\sigma = e^{-D(i\beta)}. \tag{22}$$

We integrate (21) by parts and introduce the variable $x = e^{-t}$. We obtain

$$D(i\beta) = \nu\beta V_0 S \int_0^{e^{-d/R}} dx\, \ln^3\frac{1}{x} e^{-\beta V_0 S x} \left[I_0\left(\frac{\beta V_1 S}{\ln^3 x^{-1}}\right) - \frac{3V_1}{V_0 x \ln^4 x} I_1\left(\frac{\beta V_1 S}{\ln^3 x^{-1}}\right) \right] \tag{23}$$

where $I_n(x)$ is a Bessel function of imaginary argument. At high temperatures we can expand the Bessel functions in (23) in powers of the argument and replace the upper limit in (23) by unity. Accurate to the exponentially small terms $\exp(-\beta V_0)$ we obtain:

$$D(i\beta) = \nu[f(\beta S V_0) - {}^1/_4 \beta^2 V_1{}^2 S^2 F(\beta S V_0)], \tag{24}$$

where

$$f(z) = \ln^3 z + 3C\ln^2 z + \left(3C^2 + \frac{\pi^2}{2}\right)\ln z + C\left(C^2 + \frac{\pi^2}{2}\right) - \psi''(1), \tag{25}$$

$$F(z) = \int_0^{\infty} \frac{dy e^{-y}}{\ln^3(z/y)}. \tag{26}$$

Here C is the Euler constant and ψ is the logarithmic derivative of the Γ function. The function F(z) at $\ln z \gg 1$ can be expanded in powers of $\ln^{-1} z$:

$$F(z) = \frac{1}{\ln^3 z}\left[1 - \frac{3C}{\ln z} + \frac{\pi^2 + 6C^2}{\ln^2 z} + O\left(\frac{1}{\ln^3 z}\right)\right]. \tag{27}$$

The first term in (24) is connected only with the ferromagnetic interaction. It was derived by us in[4]. The second term is due to the oscillating interaction.

The main contribution to (26) is made by $y \approx 1$. The argument of the Bessel functions in (23) is then of the order of $\beta V_1 S \ln^{-3}\beta V_0 S$. Consequently, expression (24) is valid for temperatures $T \gg T^*$ where T^* satisfies the equation

$$V_1 S / T^* = 2\ln^3 (V_0 S / T^*).$$

At such temperatures, the second term in (24) is smaller than the first and since $\nu \ln^3(SV_0/T) \gg 1$ at $T \ll T_C$, we get $D(i\beta) \gg 1$. It is seen from (3) that $\partial(\delta\sigma)/\partial\beta < 0$. Expression (22) for $\delta\sigma$ satisfies this inequality if $\partial D(i\beta)/\partial\beta > 0$, i.e., according to (24)—(27), if $T \gg T_1^*$, where T_1^* satisfies the equation

$$V_1 S / T_1{}^* = \sqrt{6} \ln^{5/2}(V_0 S / T_1{}^*).$$

Since $\ln(V_0 S/T) \gg 1$, it follows that $T_1^* > T^*$. At $T > T_1^*$, the main contribution to (18) is made by positive molecular fields.

The meaning of the inequality $T > T_1^*$ can be easily understood. If $V_1 = 0$, then the main contribution to $\delta\sigma$ at the temperature T is made by impurities that are located at a distance on the order of $R\mathcal{L} \gg r_{av}$ from the nearest neighbors, where $\mathcal{L} = \ln \beta S V_0$. The oscillating interaction, as already noted, does not change the average field acting on such an impurity, it only increases the scatter of the molecular fields from their mean value. Consequently, the oscillating interaction influences the remote impurity less than the ferromagnetic interaction, if the mean quadratic field produced by it

$$4\pi S^2 n \int_{R\mathcal{L}}^{\infty} V_1{}^2(r) r^2\, dr = \frac{\nu V_1{}^2 S^2}{\mathcal{L}^3}$$

is smaller than the corresponding field $\nu\beta^{-2}\mathscr{L}$ produced by the ferromagnetic interaction, which is equivalent to the inequality $T \gg T_1^*$.

Thus, the deviation of the magnetization from saturation at temperatures $T > T_1^*$ takes the form$^{2)}$

$$\delta\sigma = \exp \nu[-f(\beta SV_0) + {}^1/_4\beta^2V_1{}^2S^2F(\beta SV_0)]. \tag{28}$$

It is seen from (28) and (27) that the influence of the oscillating potential on the magnetization can be neglected if

$$T \gg \frac{SV_1}{2}\nu^{1/2}\ln^{-3/2}\frac{SV_0}{T}. \tag{29}$$

In this case the results obtained by us in[4] are valid. At lower temperatures, the second term in the exponent of (28) increases $\delta\sigma$ greatly.

In similar fashion we obtain for the specific heat at $T > T_1^*$

$$C_M = n\nu[f_1(1+\nu f_1) - 6(C + \ln(SV_0/T))]$$
$$\times \exp \nu[-f(\beta SV_0) + {}^1/_4\beta^2V_1{}^2S^2F(\beta SV_0)],$$

where

$$f_1 = 3\ln^2(SV_0/T) + 6C\ln(SV_0/T) + \pi^2/2 + 3C^2.$$

The pre-exponential factor does not depend on V_1. The exponential function, however, owing to the oscillating interaction, decreases with temperature more slowly than at $V_1 = 0$.

It is shown in[4] that even if we neglect the oscillating potential, $\delta\sigma$ and C_M at not too low temperature can decrease with temperature slowly, in a near-power-law fashion, with an exponent on the order of unity. The oscillating potential extends the region of slow decrease of $\delta\sigma$ and C_M towards lower temperatures, and increases the contribution made to these quantities by the local spin flips in comparison with the contribution of the spin waves. A quantitative comparison of the theoretical specific heat C_M with the experimental one was carried out in[4] at $T > 1.5$°K. At these temperatures, the inequality (2.9) is satisfied at $V_1/V_0 \ll 1$ for all the concentrations considered in[4].

If $T < T_1^*$, then $\partial D(i\beta)/\partial\beta < 0$, and the replacement of the hyperbolic functions in (3) and (4) in exponentials is not valid, i.e., the contribution of the positive fields $H > T$ to these integrals become insignificant. More negative fields, obviously, can likewise not contribute to (4) and to the temperature-dependent part of (3) since $W(H)$ decreases rapidly with decreasing H. Consequently, in the presently considered temperature region, the principal role is played by $|H| \lesssim T$, i.e., C_M and the temperature-dependent part of the $\delta\sigma$ are determined by formulas (6) and (5). Thus, at $T \ll T_1^*$ the specific heat due to local spin flips decreases with temperature more slowly than the specific heat connected with the spin waves. Therefore at not very large impurity concentrations the magnetic specific heat can be determined at all temperatures by the local spin flips, and not by the spin waves. The external magnetic field h can be easily taken into account by adding the term βh to $D(i\beta)$. Since the field h increases the number of spins oriented along the magnetization, it extends the temperature region in which the hyperbolic functions in (3) and (4) can be replaced by $e^{-\beta H}$. If $\mu_e h > T^*$; then such a replacement is possible also at $T < T^*$, since the new critical temperature $T^*(h)$ now satisfies the equation

$$\frac{\partial D(i\beta)}{\partial\beta} + \mu_e h = \frac{3}{\beta}\ln^2(SV_0\beta) - \frac{1}{2}\frac{\beta V_1{}^2S^2}{\ln^3 SV_0\beta} + \mu_e h = 0.$$

At temperatures $T \ll T^*$, the argument of the Bessel functions in (23) is large, and we can therefore use their asymptotic forms. We then have

$$D(i\beta) = \frac{\nu\beta SV_0}{\sqrt{2\pi\beta SV_1}}\int_0^{e-d/R} dx\ln^{9/2}\frac{1}{x}\exp\Big\{-\beta SV_0x$$
$$+\beta SV_1\ln^{-3}\frac{1}{x}\Big\}\left(1 - \frac{3V_1}{V_0x\ln^4 x}\right).$$

Since $\beta V_0 > 1$ and $\beta V_1 > 1$, the integral can be calculated by the saddle-point method. The saddle point x_0 satisfies the equation

$$x_0\ln^4 x_0 = 3V_1/V_0,$$

with $x_0 \ll 1$. Thus,

$$D(i\beta) = -\frac{3\nu}{2}\sqrt{3\beta SV_0}\frac{1}{x_0}\ln^{3/2}\frac{1}{x_0}\exp\left\{-\beta SV_0x_0\left(1 + \frac{1}{3}\ln x_0\right)\right\}. \tag{30}$$

i.e., $D(i\beta) < 0$. Consequently, at $T \gg T^* \gg T^*(h)$ we have

$$\delta\sigma = \exp\{-\nu D(i\beta) - \beta\mu_e h\}, \tag{31}$$

where $D(i\beta)$ is determined by formula (24) at $T > T^*$ and by formula (30) at $T^*(h) < T < T^*$.

If $T < T^*$ and $h = 0$, then $\delta\sigma \ll \exp(-\nu D(i\beta)) > 1$. Comparing this inequality with (31), we note that when a strong magnetic field is turned on in the temperature range $T^*(h) < T < T_1^*$, the quantity $\delta\sigma$ decrease much more slowly than by the factor $\exp(\beta\mu_e h)$. Since $-D(i\beta)$ increases with decreasing temperature when $T < T^*$, it follows that $\delta\sigma$ decreases with temperature at a fixed $h > T/\mu_e$ more slowly than $\exp(-\beta h\mu_e)$.

A formula similar to (31) can be written also for the specific heat. The qualitative conclusions from it are the same as for $\delta\sigma$.

The influence of the oscillating interaction on the kinetic phenomena in a ferromagnetic alloy can be analyzed in similar fashion. At $T \ll T_1^*$ and $h = 0$, the contributions made to the electric resistance, the thermal resistance, and the thermoelectric power by the local spin flips are proportional to $W(0)T$. At $T > T_1^*$ and $h = 0$, they are described by formulas (30)–(32) of[4], in which the argument of the exponential should be replaced by

$$\nu[f(\beta SV_0) - {}^1/_4\beta^2V_1{}^2S^2F(\beta SV_0)].$$

Finally, at $\mu_e h > T^*$ the kinetic coefficients, apart from the pre-exponential factors, vary with the field and with the temperature in analogy with $\delta\sigma$ (formula (31)).

APPENDIX

The distribution function $W(H)$ is given by[7]

$$W(H) = \frac{1}{2\pi v^N}\int_{-\infty}^{\infty} d\rho\int d\tau_1\,d\tau_2\ldots d\tau_N\exp\left\{-i\rho\left(H - S\sum_i V(r_i)\right)\right\}, \tag{A.1}$$

where v is the volume of the system and N is the number of particles in it. Using the identity

$$\sum_{i=1}^{N}\prod_{k=1}^{N}\Theta(r_i - r_k) = 1,$$

where

$$\Theta(x) = \begin{cases} 0 & \text{if } x \leqslant 0 \\ 1 & \text{if } x > 0' \end{cases}$$

and the symmetry of the ingegrand in (A.1) with respect to permutations of r_i, we obtain

$$W(H)=\frac{N}{2\pi v^N}\int\limits_{-\infty}^{\infty}d\rho\int\limits_{v}d\tau_1\int\limits_{v-\Omega_1}d\tau_2\dots d\tau_N\exp\left\{-i\rho\left(H-S\sum_i V(r_i)\right)\right\}. \tag{A.2}$$

Here Ω_1 is the space inside a sphere of radius r_1. We rewrite (A.2) in the following manner:

$$W(H)=\frac{1}{2\pi}\int\limits_{-\infty}^{\infty}d\rho\,\frac{N}{v}\int\limits_{0}^{\infty}d\tau_1 e^{-i\rho(H-SV(r_1))}\left(\frac{v-\Omega_1}{v}\right)^{N-1}$$
$$\times\left(\frac{1}{v-\Omega_1}\int\limits_{v-\Omega_1}d\tau e^{i\rho SV(r)}\right)^{N-1}.$$

Taking the limits as $N\to\infty$ and $v\to\infty$ with $N/v=n=\text{const}$, we obtain formulas (8)–(11) of the main text.

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[1] At $H>\nu V_1 S t_0^{-1/2}$ formula (13) yields the distribution of the molecular fields produced by one positive interaction, under the condition $\nu t^2\ll 1$, i.e., in the case when the field at the given impurity is produced by the nearest neighbor. It follows then from (13) that

$$W(H)=\frac{3\nu}{H}\ln^2\frac{SV_0}{H}\exp\left(-\nu\ln^3\frac{SV_0}{H}\right).$$

This expression can be obtained directly from (9) by putting $t=\ln(SV_0/H)$.

[2] In [4], the formulas (23)–(25) for $S-\sigma$ contain a superfluous factor 1/2.

[1] S. V. Vonsovskiĭ, Magnetizm (Magnetism), Nauka, 1971, Chap. 21.

[2] B. Giovannini, M. Peter, and J. R. Shrieffer, Phys. Rev. Lett., 12, 73 (1964).

[3] T. Moriya, Progr. Theor. Phys., 34, 329 (1965).

[4] I. Ya. Korenblit and E. F. Shender, Zh. Eksp. Teor. Fiz. 62, 1949 (1972) [Sov. Phys.-JETP 35, 1017 (1972)].

[5] A. A. Abrikosov, Usp. Fiz. Nauk 97, 403 (1969) [Sov. Phys.-Usp. 12, 168 (1969)].

[6] A. I. Larkin and D. E. Khmel'nitskiĭ, Zh. Eksp. Teor. Fiz. 58, 1789 (1970) [Sov. Phys.-JETP 31, 958 (1970)].

[7] S. Chandrasekhar, Revs. Mod. Phys. 15, 1 (1943).

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