Spin waves in an Heisenberg ferromagnetic substance with single-ion anisotropy

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The Vaks-Larkin-Pikin (VLP) diagram technique is extended to the case of an Heisenberg ferrodielectric with single-ion crystallographic magnetic anisotropy. The temperature and field dependences of fluctuation decay of spin waves with small quasimomenta are investigated at temperatures close to the Curie temperature in a first approximation of the self-consistent field method.

1. INTRODUCTION

As is well known, the static and dynamic properties of magnetically-ordered crystals are determined substantially by the crystallographic magnetic anisotropy, one of the main sources of which is single-ion anisotropy. Assuming that the intracrystalline electric field has axial island symmetry, we can express the Hamiltonian of the generalized Heisenberg model of a ferrodielectric, following Tyablikov[1], in the form

$$\mathcal{H} = -\mu H \sum_{\mathbf{r}} S_{\mathbf{r}}^{z} - \frac{1}{2} \sum_{\mathbf{r}_1 \neq \mathbf{r}_2} V(\mathbf{r}_1 - \mathbf{r}_2) \mathbf{S}_{\mathbf{r}_1} \mathbf{S}_{\mathbf{r}_2} - \frac{D}{2} \sum_{\mathbf{r}} (S_{\mathbf{r}}^{z})^2 \qquad (1.1)$$

where $V(\mathbf{r}_1 - \mathbf{r}_2)$ is the potential of the exchange interaction between the spins S situated at sites $\mathbf{r}_1$ and $\mathbf{r}_2$ of the lattice; $D > 0$ is the anisotropy constant; **H** is the external magnetic field and is directed along the easiest-magnetization axis.

The magnetic properties of crystals having a Hamiltonian (1.1) were investigated at low temperatures by different workers, and the calculation results are summarized in[2]. The use of self-consistent-field methods and two-time temperature Green's functions[5,6] has made it possible to explain some properties of the systems (1.1) at arbitrary temperatures. Pikin[3] regarded single-ion anisotropy as a perturbation, while Ginzburg[4] took exact account of this perturbation in the zeroth approximation of the self-consistent-field method, and Potapkov[5] and Noskova[6] made it possible to take exact account of the single-ion anisotropy by using kinematic relations for the spin operators.

Our task is to generalize the Vaks, Larkin, and Pikin (VLP) diagram technique to permit exact allowance of the single-ion anisotropy in any approximation of the self-consistent-field method. It is then natural to represent the Hamiltonian (1.1) in the form

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\text{int}}, \qquad (1.2)$$

where the zeroth-approximation Hamiltonian

$$\mathcal{H}_0 = -\frac{y}{\beta} \sum_{\mathbf{r}} S_{\mathbf{r}}^{z} - \frac{D}{2} \sum_{\mathbf{r}} (S_{\mathbf{r}}^{z})^2 \qquad (1.3)$$

describes the behavior of the spin in a molecular field

$$y = \beta \mu H + \beta \sum_{\mathbf{r}_1} V(\mathbf{r}_1 - \mathbf{r}_2) \langle S_{\mathbf{r}_2}^{z} \rangle \qquad (1.4)$$

with allowance for the single-ion anisotropy, and

$$\mathcal{H}_{int} = -\frac{1}{2} \sum_{\mathbf{r}_1 \neq \mathbf{r}_2} V(\mathbf{r}_1 - \mathbf{r}_2) \{ (S_{\mathbf{r}_1}^{z} - \langle S^{z} \rangle)(S_{\mathbf{r}_2}^{z} - \langle S^{z} \rangle) + 2S_{\mathbf{r}_1}^{-} S_{\mathbf{r}_2}^{+} \} \qquad (1.5)$$

describes the pure spin interactions. Here $\beta = T^{-1}$ and the angle brackets denote averaging over the canonical ensemble with Hamiltonian $\mathcal{H}$.

Such a breakdown of the Hamiltonian is expedient because the multinode spin correlators that appear when the perturbation-theory series with the Hamiltonian $\mathcal{H}_{\text{int}}$ (1.5) are constructed are expressed in term of single-cell blocks, just as in the VLP diagram technique[7,8]. These blocks represent statistical mean values with Hamiltonian $\mathcal{H}_0$ of the T-ordered spin operators pertaining to one node.

2. GENERALIZATION OF THE VLP DIAGRAM TECHNIQUE

The connectivity theorem, according to which only connected diagrams need be taken into account in the calculation of the temperature correlation functions, remains valid in the developed diagram technique. Any connected diagram can be represented as an aggregate of connected single-cell blocks jointed by interaction lines.

We define, following[7], the Fourier component of an unconnected block with n emerging interaction lines, by the expression

$$\Gamma_n^{\alpha_1 \ldots \alpha_n}(\omega_1 \ldots \omega_n) = \frac{1}{\beta^n} \int_0^{\beta} \prod_{j=1}^{n} dt_j \exp(i\omega_j t_j) \left\langle \hat{T} \prod_{i=1}^{n} S^{\alpha_j}(t_j) \right\rangle_0, \qquad (2.1)$$

where

$$S^{\alpha}(t) = \exp(\mathcal{H}_0 t) S^{\alpha} \exp(-\mathcal{H}_0 t), \qquad (2.2)$$

T is the chronological-ordering operator, the symbol $\langle \ldots \rangle_0$ corresponds to averaging with the Hamiltonian $\mathcal{H}_0$, $i\omega_m = 2\pi i m T$ are the imaginary frequencies of the temperature diagram technique, t is the imaginary temperature time, and α takes on the values +, −, and z.

The expressions for the connected blocks are obtained by subtracting from (2.1) the sum of the products of all the possible blocks of lower rank.

To calculate the unconnected blocks (2.1) it is necessary to know the explicit form of the dependence of the spin operators on the "time." From (2.2) we obtain with the aid of the commutation relations $[S^Z, S^-] = -S^-$, $[S^Z, S^+] = S^+$, and $[S^+, S^-] = S^Z$

$$\begin{aligned} S^{+}(t) &= \exp[-(y - \gamma + 2\gamma S^{z})\beta^{-1}t] S^{+}, \\ S^{-}(t) &= \exp[(y + \gamma + 2\gamma S^{z})\beta^{-1}t] S^{-}, \\ S^{z}(t) &\equiv S^{z}, \quad \gamma = {}^{1}/_{2}\beta D. \end{aligned} \qquad (2.3)$$

Since the z-component of the spin is independent of the "time," the simplest blocks are of the type $\Gamma_n^{Z \ldots Z}(\omega_1 \ldots \omega_n)$. To calculate them, we obtain the partition function

$$Z = \sum_{m=-S}^{S} \exp(ym + \gamma m^2) = \sum_{n=0}^{\infty} \frac{\gamma^n}{n!} \frac{d^{2n}}{dy^{2n}} Z_0(y)$$

$$= \frac{1}{\sqrt{4\pi\gamma}} \int_{-\infty}^{\infty} \exp\left(-\frac{x^2}{4\gamma}\right) Z_0(y+x)\,dx = Z(y,\gamma), \qquad (2.4)$$

where

$$Z_0(y) = \sum_{m=-s}^{s} e^{ym} = \frac{\mathrm{sh}(S+1/2)y}{\mathrm{sh}\,1/2y}.$$

We regard the partition function Z as the generating function of the correlators

$$\langle (S^z)^n \rangle_0 = \frac{1}{Z}\frac{d^n}{dy^n} Z. \qquad (2.5)$$

We then obtain for the blocks $\Gamma_n^{z\ldots z}$ from (2.1), (2.4) and (2.5)

$$\Gamma_n^{z\ldots z}(\omega_1 \ldots \omega_n) = \frac{2}{Z}\delta_1 \ldots \delta_n \begin{cases} \sum_{m>0} m^n \exp(m^2\gamma)\,\mathrm{ch}\,my, & n = 2l, \\ \sum_{m>0} m^n \exp(m^2\gamma)\,\mathrm{sh}\,my, & n = 2l-1, \end{cases} \qquad (2.6)$$

where $\delta_i \equiv \delta_{\omega_i}$ is the Kronecker symbol and l can take on the values 1, 2, ...

Inclusion of the anisotropy in the zero-order Hamiltonian leads to the appearance in (2.3) of an operator "temporal" factor $\exp(2\gamma S^Z \beta^{-1})$. Therefore the analog of Wick's theorem for the spin operators[7] is missing from this model, and to calculate blocks containing transverse spin components it is necessary to use the definition of the $\hat{T}$-ordered product of operators:

$$\hat{T}(S^{\alpha_1}(t_1)\ldots S^{\alpha_n}(t_n)) = \sum_p \theta(t_1-t_2)\ldots\theta(t_{n-1}-t_n) S^{\alpha_1}(t_1)\ldots S^{\alpha_n}(t_n), \qquad (2.7)$$

where the summation is over all possible permutations of the "times" t_j; $\theta(t_i - t_j)$ are Heaviside unit step functions.

The Appendix gives the Fourier components of the simplest connected blocks only for the spin S = 1, since the corresponding formulas for arbitrary spin are too cumbersome.

Our renormalization of the blocks does not change the graphic structure of the diagrams[8] if the block Γ_n is represented as before by a point with n outgoing interaction lines. We shall henceforth not show the known diagrams, and present only the analytic expressions corresponding to them.

3. CURRIE TEMPERATURE OF ANISOTROPIC FERROMAGNET. MOLECULAR FIELD NEAR T_c

We shall show that single-ion magnetic anisotropy leads to a shift of the Curie temperature T_c and changes the magnitude of the molecular field y. We introduce the notation

$$\langle S^z \rangle_0 = b(y,\gamma) \equiv b. \qquad (3.1)$$

We then obtain from the general formula (2.5) at n = 1

$$b = \frac{2}{Z}\sum_{m>0} m \exp(m^2\gamma)\,\mathrm{sh}\,my. \qquad (3.2)$$

Assuming a field H = 0 and $\langle S^Z \rangle \approx b$, we obtain from the self-consistency condition

$$\langle S^z \rangle = (y - \beta\mu H)(\beta V_0)^{-1}, \qquad (3.3)$$

a transcendental equation for the phase-transition temperature:

$$\frac{T_c}{V_0} = \frac{2}{Z(0,\gamma_c)}\sum_{m>0} m^2 \exp(m^2\gamma_c). \qquad (3.4)$$

In the limiting case $\gamma_c = D/2T_c = 0$ we have

$$T_c^0 = 1/3 S(S+1)V_0, \qquad (3.5)$$

corresponding to the molecular-field approximation for an isotropic ferromagnet.

In the limiting case of a strongly anisotropic uniaxial crystal $\gamma_c \gg 1$ we obtain

$$T_c \approx S^2 V_0. \qquad (3.6)$$

In the opposite case $0 < \gamma_c < 1$, in the approximation linear in γ_c,

$$T_c \approx T_c^0 + 1/10 D(4T_c^0/V_0 - 1), \qquad (3.7)$$

i.e., the relative shift of the Curie temperature is

$$\Delta = \frac{T_c - T_c^0}{T_c} = \frac{\gamma_c}{15}(4S(S+1) - 3). \qquad (3.7')$$

At a spin S = 1/2, there is no contribution of the single-ion anisotropy. If S = 1, formulas (3.1)–(3.7) coincide with the corresponding results in[4,5].

It is easily shown that the quantities $\langle (S^Z)^n \rangle_0$ can be expressed in terms of the function b(y, γ) (3.1) and its derivatives with respect to y with the aid of relations (A.8) of[7]

$$\begin{aligned} \langle (S^z)^2 \rangle_0 &= b' + b^2, \\ \langle (S^z)^3 \rangle_0 &= b'' + 3bb' + b^3, \\ &\ldots\ldots\ldots\ldots \end{aligned} \qquad (3.8)$$

On the other hand, in the particular case S = 1 we obtain from (2.5)

$$\begin{aligned} \langle (S^z)^{2n} \rangle_0 &= \langle (S^z)^2 \rangle_0 = \frac{2e^\gamma \mathrm{ch}\,y}{1 + 2e^\gamma \mathrm{ch}\,y}, \\ \langle (S^z)^{2n+1} \rangle_0 &= \langle S^z \rangle_0 = \frac{2e^\gamma \mathrm{sh}\,y}{1 + 2e^\gamma \mathrm{ch}\,y}. \end{aligned} \qquad (3.9)$$

Substituting in (3.3) the expansion

$$\langle S^z \rangle \approx b \approx b'(0,\gamma)y + \frac{1}{3!} b'''(0,\gamma) y^3, \qquad (3.10)$$

where we have for S = 1, according to (3.4), (3.8), and (3.9),

$$\begin{aligned} b'(0,\gamma) &= \frac{T_c}{V_0} = \frac{2e^{\gamma_c}}{1 + 2e^{\gamma_c}}, \\ b'''(0,\gamma) &= b'(0,\gamma)[1 - 3b'(0,\gamma)], \end{aligned} \qquad (3.11)$$

we obtain an equation for the molecular field near T_c:

$$y^3 + 3a\tau y - 3ah = 0. \qquad (3.12)$$

Here

$$a = \frac{2T_c^0}{2T_c - T_c^0}, \quad \tau = \frac{T - T_c}{T_c}, \quad h = \frac{\mu H}{T_c}. \qquad (3.13)$$

The solutions of (3.12) take the following form in limiting cases:

a) case of weak fields $h^2 a^{-1} |\tau|^{-3} \ll 1$

$$y \approx \begin{cases} (3a|\tau|)^{1/2} + h/2|\tau|, & \tau < 0, \\ h/|\tau|, & \tau > 0; \end{cases} \qquad (3.14)$$

b) case of strong fields $h^4 a^{-1} |\tau|^{-3} \gg 1$

$$y \approx (3ah)^{1/4}[1 - (a/9h^2)^{1/4}\tau]. \qquad (3.15)$$

Since $h \ll 1$ in the usually realized experimental conditions, it follows from (3.14) and (3.15) that $y \gg h$ in the considered temperature region.

4. SPECTRUM OF COLLECTIVE EXCITATIONS

We determine the Fourier component of the temperature correlation function of the transverse spin components by the relation

$$K^{-+}(\mathbf{k}, i\omega_n)=\frac{1}{2\beta}\int_{-\beta}^{\beta}\exp(i\omega_n t)\,dt\cdot\sum_{r_2}\exp\{i\mathbf{k}(\mathbf{r}_1-\mathbf{r}_2)\}\langle\hat{T}S_{r_1}^{-}(t_1)S_{r_2}^{+}(t_2)\rangle. \tag{4.1}$$

Introducing the concept of the irreducible part $\Sigma^{-+}(\mathbf{k}, i\omega_n)$ as an assembly of diagrams that cannot be cut along one interaction line V_k, we obtain

$$K^{-+}(\mathbf{k}, i\omega_n)=\frac{\Sigma^{-+}(\mathbf{k}, i\omega_n)}{1-\beta V_k\Sigma^{-+}(\mathbf{k}, i\omega_n)}. \tag{4.2}$$

In the case of a large relative exchange-interaction radius r_0, the irreducible part is represented in the form of an expansion in its reciprocal powers:

$$\Sigma={}^{0}\Sigma+{}^{1}\Sigma+{}^{2}\Sigma+\ldots, \tag{4.3}$$

where the indices 0, 1, ... indicate the number of closed loops of the effective interaction lines

$$V^{\alpha\beta}(\mathbf{k}, i\omega_n)=V_k+\beta V_k^2K^{\alpha\beta}(\mathbf{k}, i\omega_n),\ (\alpha\beta)\to(+-),\ (zz). \tag{4.4}$$

The spin-wave spectrum is determined by the poles of the analytic continuation of the correlation function $K^{-+}(\mathbf{k}, i\omega_n)$. The question of the spectrum of the system (1.1) is discussed in a number of papers (see, e.g.,[2]). In the molecular-field approximation

$$\Sigma^{-+}={}^{0}\Sigma^{-+}=\Gamma_2^{-+} \tag{4.5}$$

the spectrum was investigated by Ginzburg[4], who was the first to show that in the general case the spectrum consists of 2S spin-wave branches. For S = 1, in particular,

$$\omega_{1,2}(\mathbf{q})=\mu H+bV_0-{}^1\!/_2bV_q\mp \mp[({}^1\!/_2bV_q)^2+(T\gamma)^2+T\gamma V_qP]^{1/2}, \tag{4.6}$$

where

$$P=\frac{2(1-e^{\gamma}\operatorname{ch}y)}{1+2e^{\gamma}\operatorname{ch}y}=\frac{2}{1+2e^{\gamma}\operatorname{ch}y}-\langle(S^z)^2\rangle_0. \tag{4.7}$$

A result similar to that of[4] was obtained independently in[5,6].

At a temperature T = 0 we obtain from (4.6)

$$\omega_1(\mathbf{k})=\mu H+V_0-V_k+{}^1\!/_2D,\qquad \omega_2=\mu H+V_0-{}^1\!/_2D. \tag{4.8}$$

It is obvious from (4.8) that the optical branch of the spectrum ω_2 does not depend on the quasimomentum and corresponds to local excitations of the spin at the node[9,6]. An analysis of the expressions (4.6) shows that the mode ω_2 remains much less collectivized than the low-frequency mode ω_1 to temperatures close to T_c^0.

To determine the low-temperature corrections $\Delta\omega_{1,2}$ to the spectrum $\omega_{1,2}$ we take into account the contribution made to the irreducible part by diagrams with one loop of the effective interaction lines[8]; we then have for the first branch

$$\Delta\omega_1(\mathbf{k})=-\sum_q n_{1q}\left[\frac{D(V_q+V_k)}{V_q+V_k-D}+V_0+V_{k-q}-V_q-V_k\right], \tag{4.9}$$

where

$$n_{1q}=(\exp[\beta\omega_1(\mathbf{q})]-1)^{-1},$$

and for the optical branch the correction $\Delta\omega_2$ turns out to be exponentially small.

In the particular case of weak anisotropy, $D\ll V_q$, $V_k\sim V_0$, we have

$$\Delta\omega_1(\mathbf{k})\approx\sum_q n_{1q}[V_0+V_{k-q}-V_q-V_k+D]. \tag{4.10}$$

Therefore in the approximation of quadratic isotropic dispersion we obtain for the acoustic branch

$$\omega_1(\mathbf{k})=\omega_s(\mathbf{k})+\frac{D}{2}\left[1-2\left(\frac{3T}{2\pi V_0}\right)^{3/2}\frac{1}{r_0^3}Z_{3/2}(\beta\mu H+\gamma)\right], \tag{4.11}$$

where $\omega_S(\mathbf{k})$ is the dispersion law for the spin waves of an isotropic ferromagnet at low temperatures[8], and

$$Z_\alpha(x)=\sum_{n=1}^{\infty}n^{-\alpha}e^{-nx}. \tag{4.12}$$

The second term in (4.11), which is due to single-ion anisotropy, can be represented as a linearized expansion of $\langle S^Z\rangle^2$ in terms of the quantity $\bar{n}=\sum_q n_{1q}$; then

$$\omega_1(\mathbf{k})=\omega_s(\mathbf{k})+{}^1\!/_2D\langle S^z\rangle^2. \tag{4.13}$$

An analysis of the expressions (4.6) for the collective-excitation frequencies at arbitrary temperatures was carried out earlier[5] for the particular case H = 0 and q = 0. It is curious to note that if the molecular field is $y=\gamma$ then, according to (4.6), the frequency of the acoustic mode vanishes, and

$$\omega_2(\mathbf{k})={}^1\!/_2D(2-V_k/V_0). \tag{4.14}$$

If H = 0 and $D\ll V_0$, then the condition $y=\gamma$ is satisfied at $T\approx T_c^0$. For fields $\mu HT^{-1}=h>\gamma$ there exists no temperature at which one of the frequencies vanishes.

We shall assume henceforth that in the high-temperature region of interest to us the magnetizing field is strong enough ($h>\gamma$) to be able to assume the sample to be single-domain, and at the same time weak enough for the molecular-field equation (3.12) to have the solutions (3.14) and (3.15). We assume thus that

$$\gamma<h\ll y\ll 1. \tag{4.15}$$

In this case we get from (4.6)

$$\omega_{1k}\approx\mu H+T_cV_0^{-1}y(V_0-V_k),\qquad \omega_{2k}\approx\mu H+T_cy(1+\beta\gamma(V_0-V_k)). \tag{4.16}$$

We see from this, in particular, that in a temperature region not too close to the Curie temperature T_c, the dependence of the optical branch of the spectrum on the quasimomentum is weak.

5. FLUCTUATION DAMPING OF SPIN WAVES

The damping $\Gamma(\omega, \mathbf{k})$ of the collective excitations in a system with Hamiltonian (1.1) is governed by two mechanisms, scattering of the spin waves by the $\langle S^Z\rangle$ fluctuations, and spin-wave processes with participation of four and more magnons. In the first-order approximation of the self-consistent-field method, there is only the fluctuation damping $\Gamma_f(\omega, \mathbf{k})$.

Using for the irreducible part Σ^{-+} the single-loop diagrams shown in Fig. 1 of[8] and the analytic expressions for connected blocks at S = 1 (see the Appendix), we write out the terms of ${}^1\Sigma$ with nonzero imaginary parts:

$$\Sigma'(\mathbf{k}, i\Omega)=\sum_q(1-\beta b'V_q)^{-1}(1-\beta V_{q-k}\Gamma_1^{+-}(\Omega,\Omega))^{-1}$$
$$\times\{b'V_q^2[\varphi_1G_{\Omega_1}^3+\varphi_2G_{\Omega_2}^3-\beta V_{k-q}\varphi_1\varphi_2G_{\Omega_1}G_{\Omega_2}(G_{\Omega_1}-G_{\Omega_2})^2]+\Gamma_2^{+-}(\Omega,\Omega)V_{k-q}$$
$$\times[\varphi_3G_{\Omega_1}^2+\varphi_4G_{\Omega_2}^2-2\varphi_5G_{\Omega_1}G_{\Omega_2}-\beta V_q(\varphi_3b'-\varphi_6^2)(G_{\Omega_1}-G_{\Omega_2})^2]$$
$$-2V_qV_{k-q}(\varphi_1G_{\Omega_1}^2+\varphi_2G_{\Omega_2}^2)(\varphi_6G_{\Omega_1}+\varphi_7G_{\Omega_2})\}. \tag{5.1}$$

Following the analytic continuation $i\Omega\to\omega$, we obtain for the damping of the first mode

$$\Gamma_f(\omega_{1k})=\Gamma_{f1}=\frac{V_kG_{11}^{-1}G_{12}^{-1}\operatorname{Im}\Sigma'(\mathbf{k}, \omega_{1k})}{\omega_{1k}-\omega_{2k}}, \tag{5.2}$$

$$G_{11}^{-1} = (y + \gamma - \omega_{1k}\beta), \quad G_{12}^{-1} = (y - \gamma - \omega_{1k}\beta).$$

A general expression for the fluctuation damping of the second mode is obtained by making the substitutions $\omega_{1k} \rightleftarrows \omega_{2k}$ in (5.2).

In the absence of single-ion anisotropy we have $G_{11} = G_{12} = (y - i\beta\omega)^{-1}$, and then, using formulas (A.1) of the Appendix, we obtain from (5.2) the known result[8] for the fluctuation damping of the spin waves of an isotropic ferromagnet.

To calculate the main contribution made to the damping by magnons with small quasimomenta, we can confine ourselves to a replacement of the Fourier components of the exchange interaction in (5.2) by V_0. In this approximation,

$$\Gamma_{f1} \approx \frac{\pi\gamma^2 V_0^3 (A - \beta V_0 B)}{(\omega_{1k} - \omega_{2k})^2 (1 - \beta b' V_0)} \sum_q \delta(\omega_{1k} - \omega_{1q}), \tag{5.3}$$

where

$$\begin{aligned} A &= \varphi_1\varphi_4 G_{11} + \varphi_2\varphi_3 G_{12}, \\ B &= \varphi_1\varphi_2 G_{11} G_{12} + (\varphi_1 G_{11} + \varphi_2 G_{12})(\varphi_3 b_1' - \varphi_6^2). \end{aligned} \tag{5.4}$$

Allowance for the fluctuation damping is of greatest interest at temperatures close to the Curie point. Noting that when condition (4.15) is satisfied we have

$$G_{11} \approx G_{12} \approx y^{-1}, \; G_{21} \approx -G_{22} \approx \gamma^{-1}, \; 1 - \beta b' V_0 = \tau, \tag{5.5}$$

and changing over to integration over the quasimomenta, we obtain from (4.16), (5.3), and (5.5)

$$\Gamma_{f1} \approx \frac{3\sqrt{3}}{2\pi r_0^3} \left(\frac{T}{T_c}\right)^2 \left(\frac{\Delta}{y}\right)^2 |\tau|^{-1} V_0 k. \tag{5.6}$$

Here ks is the dimensionless quasimomentum (see[8]), and Δ is the relative shift of the Curie temperature (3.7′) for S = 1. Similarly,

$$\Gamma_{f2} \approx \frac{\sqrt{3}}{2\pi r_0^3} \left(\frac{T}{T_c}\right)^2 \frac{|\tau|^{-1} V_0}{\Delta y} k. \tag{5.7}$$

Substituting in (5.6) and (5.7) the molecular field y defined by expressions (3.14) and (3.15), we obtain for the fluctuation damping of the acoustic and optical branches of the spectrum:

a) weak fields $h^2 a^{-1} |\tau|^{-3} \ll 1$

$$\left.\begin{aligned} \Gamma_{f1} &\sim \Delta^2 |\tau|^{-2} k \\ \Gamma_{f2} &\sim \Delta^{-1} |\tau|^{-3/2} k \end{aligned}\right\}, \; \tau < 0, \qquad \left.\begin{aligned} \Gamma_{f1} &\sim \Delta^2 h^{-2} \tau k \\ \Gamma_{f2} &\sim \Delta^{-1} h^{-1} k \end{aligned}\right\}, \; \tau > 0; \tag{5.8}$$

b) strong fields $h^2 a^{-1} |\tau|^{-3} \gg 1$

$$\Gamma_{f1} \sim \Delta^2 h^{-2/3} |\tau|^{-1} k, \; \Gamma_{f2} \sim \Delta^{-1} h^{-1/3} |\tau|^{-1} k. \tag{5.9}$$

Comparison of the results (5.8) and (5.9) for Γ_{f1} with the corresponding formulas for an isotropic ferromagnet[8] shows that the fluctuation damping in an anisotropic crystal is essentially determined by the anisotropy constant and depends linearly on the quasimomentum. Since the spin-wave spectrum has a gap, the relative damping $\Gamma'_{f1} = \Gamma_{f1}\omega_{1k}^{-1}$ of the long-wave magnons is weak. For example, in the region of weak fields and $\tau < 0$ they exist for $k \ll h r_0^3 \Delta^{-2} \tau^2$. The fluctuation damping Γ_{f2} of the optical magnons is also linear in k, but near T_c the region of quasimomenta in which they exist is narrower. At $\tau < 0$ in the region of weak fields Γ'_{f2} is small at $k \ll r_0^3 \Delta \tau^2$. A comparison of the relative damping of the two modes at a fixed value of k shows Γ'_{f1} is weaker than Γ'_{f2} that near T_c by a factor $\Delta^3 h^{-1}$ both in weak and in strong fields.

Since the scattering by magnetic inhomogeneities with transformation of acoustic magnons into optical ones are forbidden by the conservation laws in the absence of a dipole-dipole interaction, one should expect the fluctuation damping of all spin-wave branches to be linear in k near T_c for arbitrary values of the spin S.

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APPENDIX

The Fourier components of the simplest connected blocks are given in the case S = 1 by the analytic expressions

$$\begin{aligned}
&\Gamma_2^{zz}(12) = b'\delta_1\delta_2, \; \Gamma_2^{+-}(12) = \{\varphi_1 G_{11} + \varphi_2 G_{12}\}\delta_{1-2}, \\
&\Gamma_3^{+-z}(123) = -\{\varphi_1 G_{11} G_{21} + \varphi_2 G_{12} G_{22}\}\delta_{1-2+3} \\
&\quad - \{\varphi_6 G_{11} + \varphi_7 G_{12}\}\delta_{1-2}\delta_3, \\
&\Gamma_4^{+-zz}(1234) = \{\varphi_1 G_{11} G_{21}(G_{1+3,1} + G_{1+4,1}) \\
&\quad + \varphi_2 G_{12} G_{22}(G_{1+3,2} + G_{1+4,2})\}\delta_{1-2+3+4} \\
&\quad + \{\varphi_6 G_{11} G_{21} + \varphi_7 G_{12} G_{22}\}(\delta_{1-2+3}\delta_4 + \delta_{1-2+4}\delta_3) \\
&\quad + \{\varphi_8 G_{11} + \varphi_9 G_{12}\}\delta_{1-2}\delta_3\delta_4, \\
&\Gamma_4^{++--}(1234) = \{\varphi_1 (G_{1+2,\,1+2} - 2G_{1+2,\,1+1})(G_{11} + G_{21})(G_{31} + G_{41}) \\
&\quad + \varphi_2 (G_{1+2,\,1+2} - 2G_{1+2,\,2+2})(G_{12} + G_{22})(G_{32} + G_{42})\}\delta_{1+2-3-4} \\
&\quad + \{\varphi_3 G_{11} G_{21} + \varphi_4 G_{12} G_{22} - \varphi_5 (G_{11} G_{22} + G_{12} G_{21})\} \cdot \\
&\quad \cdot (\delta_{1-3}\delta_{2-4} + \delta_{1-4}\delta_{2-3}); \\
&G_{ij} = (a_j - i\omega_i\beta)^{-1}, \quad a_1 = y + \gamma, \quad a_2 = y - \gamma; \\
&\varphi_1 = Z^{-1}(e^{y+\gamma} - 1), \quad \varphi_2 = Z^{-1}(1 - e^{\gamma - y}), \\
&Z = 1 + 2e^{\gamma} \operatorname{ch} y, \\
&\varphi_3 = 2Z^{-1} + \varphi_1(1 - \varphi_1), \quad \varphi_4 = 2Z^{-1} - \varphi_2(1 + \varphi_2), \\
&\varphi_5 = \varphi_1\varphi_2 + Z^{-1}, \quad \varphi_6 = \varphi_5 - \varphi_3, \quad \varphi_7 = \varphi_5 - \varphi_4, \\
&\varphi_8 = \varphi_1(1 - 2b + b^2 - b') + (1 - 2b)Z^{-1}, \\
&\varphi_9 = \varphi_2(1 + 2b + b^2 - b') - (1 + 2b)Z^{-1}.
\end{aligned}$$

From these formulas we get the relations

$$\begin{aligned}
&\varphi_1 + \varphi_2 = \langle S^z \rangle_0 = b = 2Z^{-1} e^{\gamma} \operatorname{sh} y, \\
&\varphi_6 + \varphi_7 = -(\varphi_3 + \varphi_4 - 2\varphi_5) = -b', \quad \varphi_8 + \varphi_9 = b''.
\end{aligned} \tag{A.1}$$

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